# INSIGHT

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## *A Study of Human Understanding*

*by*

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τὰ μὲν οὖν εἴδη τὸ νοητικὸν ἐν τοῖς
φαντάσμασι νοεῖ.
ARISTOTLE, *De Anima*, III, 7

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# PREFACE

In the ideal detective story the reader is given all the clues yet fails to spot the criminal. He may advert to each clue as it arises. He needs no further clues to solve the mystery. Yet he can remain in the dark for the simple reason that reaching the solution is not the mere apprehension of any clue, not the mere memory of all, but a quite distinct activity of organizing intelligence that places the full set of clues in a unique explanatory perspective.

By insight, then, is meant not any act of attention or advertence or memory but the supervening act of understanding. It is not any recondite intuition but the familiar event that occurs easily and frequently in the moderately intelligent, rarely and with difficulty only in the very stupid. In itself it is so simple and obvious that it seems to merit the little attention that commonly it receives. At the same time, its function in cognitional activity is so central that to grasp it in its conditions, its working, and its results, is to confer a basic yet startling unity on the whole field of human inquiry and human opinion. Indeed, this very wealth of implications is disconcerting, and I find it difficult to state in any brief and easy manner what the present book is about, how a single author can expect to treat the variety of topics listed in the table of contents, why he should attempt to do so in a single work, and what good he could hope to accomplish even were he to succeed in his odd undertaking.

Still, a preface should provide at least a jejune and simplified answer to such questions and, perhaps, I can make a beginning by saying that the aim of the work is to convey an insight into insight. Mathematicians seek insight into sets of elements. Scientists seek insight into ranges of phenomena. Men of common sense seek insight into concrete situations and practical affairs. But our concern is to reach the act of organizing intelligence that brings within a single perspective the insights of mathematicians, scientists, and men of common sense.

It follows at once that the topics listed in the table of contents are not so disparate as they appear on a superficial reading. If anyone wishes to become a mathematician or a scientist or a man of common sense, he will derive no direct help from the present work. As physicists study

the shape of waves and leave to chemists the analysis of air and water, so we are concerned not with the objects understood in mathematics but with mathematicians' acts of understanding, not with objects understood in the various sciences but with scientists' acts of understanding, not with the concrete situations mastered by common sense but with the acts of understanding of men of common sense.

Further, while all acts of understanding have a certain family likeness, a full and balanced view is to be reached only by combining in a single account the evidence obtained from different fields of intelligent activity. Thus, the precise nature of the act of understanding is to be seen most clearly in mathematical examples. The dynamic context in which understanding occurs can be studied to best advantage in an investigation of scientific methods. The disturbance of that dynamic context by alien concerns is thrust upon one's attention by the manner in which various measures of common nonsense blend with common sense.

However, insight is not only a mental activity but also a constituent factor in human knowledge. It follows that insight into insight is in some sense a knowledge of knowledge. Indeed, it is a knowledge of knowledge that seems extremely relevant to a whole series of basic problems in philosophy. This I must now endeavour to indicate even though I can do so only in the abrupt and summary fashion that leaves terms undefined and offers arguments that fall far short of proof.

First, then, it is insight that makes the difference between the tantalizing problem and the evident solution. Accordingly, insights seem to be the source of what Descartes named clear and distinct ideas and, on that showing, insight into insight would be the source of the clear and distinct idea of clear and distinct ideas.

Secondly, inasmuch as it is the act of organizing intelligence, insight is an apprehension of relations. But among relations are meanings, for meaning seems to be a relation between sign and signified. Insight, then, includes the apprehension of meaning, and insight into insight includes the apprehension of the meaning of meaning.

Thirdly, in a sense somewhat different from Kant's, every insight is both *a priori* and synthetic. It is *a priori*, for it goes beyond what is merely given to sense or to empirical consciousness. It is synthetic, for it adds to the merely given an explanatory unification or organization. It seems to follow that insight into insight will yield a synthetic and *a priori* account of the full range of synthetic, *a priori* components in our cognitional activity.



Fourthly, a unification and organization of other departments of knowledge is a philosophy. But every insight unifies and organizes. Insight into insight, then, will unify and organize the insights of mathematicians, scientists, and men of common sense. It seems to follow that insight into insight will yield a philosophy.

Fifthly, one cannot unify and organize knowing without concluding to a unification and organization of the known. But a unification and organization of what is known in mathematics, in the sciences, and by common sense is a metaphysics. Hence, in the measure that insight into insight unifies and organizes all our knowing, it will imply a metaphysics.

Sixthly, the philosophy and metaphysics that result from insight into insight will be verifiable. For just as scientific insights both emerge and are verified in the colours and sounds, tastes and odours, of ordinary experience, so insight into insight both emerges and is verified in the insights of mathematicians, scientists, and men of common sense. But if insight into insight is verifiable, then the consequent philosophy and metaphysics will be verifiable. In other words, just as every statement in theoretical science can be shown to imply statements regarding sensible fact, so every statement in philosophy and metaphysics can be shown to imply statements regarding cognitional fact.

Seventhly, besides insights there are oversights. Besides the dynamic context of detached and disinterested inquiry in which insights emerge with a notable frequency, there are the contrary dynamic contexts of the flight from understanding in which oversights occur regularly and one might almost say systematically. Hence, if insight into insight is not to be an oversight of oversights, it must include an insight into the principal devices of the flight from understanding.

Eighthly, the flight from understanding will be seen to be anything but a peculiar aberration that afflicts only the unfortunate or the perverse. In its philosophic form (which is not to be confused with its psychiatric, moral, social, and cultural manifestations) it appears to result simply from an incomplete development in the intelligent and reasonable use of one's own intelligence and reasonableness. But though its origin is a mere absence of full development, its consequences are positive enough. For the flight from understanding blocks the occurrence of the insights that would upset its comfortable equilibrium. Nor is it content with a merely passive resistance. Though covert and devious, it is resourceful and inventive, effective and extraordinarily

plausible. It admits a vast variety of forms and, when it finds some untenable, it can resort to others. If it never refuses to supply superficial minds with superficial positions, it is quite competent to work out a philosophy so acute and profound that the elect strive in vain and for centuries to lay bare its real inadequacies.

Ninthly, just as insight into insight yields a clear and distinct idea of clear and distinct ideas, just as it includes an apprehension of the meaning of meaning, just as it exhibits the range of the *a priori*, synthetic components in our knowledge, just as it involves a philosophic unification of mathematics, the sciences, and common sense, just as it implies a metaphysical account of what is to be known through the various departments of human inquiry, so also insight into the various modes of the flight from understanding will explain:

(1) the range of really confused yet apparently clear and distinct ideas,
(2) aberrant views on the meaning of meaning,
(3) distortions in the *a priori*, synthetic components in our knowledge,
(4) the existence of a multiplicity of philosophies, and
(5) the series of mistaken metaphysical and anti-metaphysical positions.

Tenthly, there seems to follow the possibility of a philosophy that is at once methodical, critical, and comprehensive. It will be comprehensive because it embraces in a single view every statement in every philosophy. It will be critical because it discriminates between the products of the detached and disinterested desire to understand and, on the other hand, the products of the flight from understanding. It will be methodical because it transposes the statements of philosophers and metaphysicians to their origins in cognitional activity and it settles whether that activity is or is not aberrant by appealing, not to philosophers, not to metaphysicians, but to the insights, methods, and procedures of mathematicians, scientists, and men of common sense.

The present work, then, may be said to operate on three levels. It is a study of human understanding. It unfolds the philosophic implications of understanding. It is a campaign against the flight from understanding. These three levels are solidary. Without the first there would be no base for the second and no precise meaning for the third. Without the second the first could not get beyond elementary statements and



there could be no punch to the third. Without the third the second would be regarded as incredible and the first would be neglected.

Probably I shall be told that I have tried to operate on too broad a front. But I was led to do so for two reasons. In constructing a ship or a philosophy one has to go the whole way; an effort that is in principle incomplete is equivalent to a failure. Moreover, against the flight from understanding half measures are of no avail. Only a comprehensive strategy can be successful. To disregard any stronghold of the flight from understanding is to leave intact a base from which a counter-offensive promptly will be launched.

If, however, these considerations are granted, it still will be urged that what I have attempted could be executed properly only by the organized research of specialists in many different fields. This, of course, I cannot but admit. I am far from competent in most of the many fields in which insights occur, and I could not fail to welcome the impressive assembly of talent and the comforting allocation of funds associated with a research project. But I was not engaged in what commonly is meant by research. My aim was neither to advance mathematics nor to contribute to any of the specialized branches of science but to seek a common ground on which men of intelligence might meet. It seemed necessary to acknowledge that the common ground I envisaged was rather impalpable at a time when neither mathematicians nor scientists nor men of common sense were notably articulate on the subject of insight. What had to be undertaken was a preliminary, exploratory journey into an unfortunately neglected region. Only after specialists in different fields had been given the opportunity to discover the existence and significance of their insights, could there arise the hope that some would be found to discern my intention where my expression was at fault, to correct my errors where ignorance led me astray, and with the wealth of their knowledge to fill the dynamic but formal structures I tried to erect. Only in the measure that this hope is realized, will there be initiated the spontaneous collaboration that commonly must precede the detailed plans of an organized investigation.

There remains the question, What practical good can come of this book? The answer is more forthright than might be expected. For insight is the source not only of theoretical knowledge but also of all its practical applications and, indeed, of all intelligent activity. Insight into insight, then, will reveal what activity is intelligent, and insight into oversights will reveal what activity is unintelligent. But to be practical

is to do the intelligent thing and to be unpractical is to keep blundering about. It follows that insight into both insight and oversight is the very key to practicality.

Thus, insight into insight brings to light the cumulative process of progress. For concrete situations give rise to insights which issue into policies and courses of action. Action transforms the existing situation to give rise to further insights, better policies, more effective courses of action. It follows that if insight occurs, it keeps recurring; and at each recurrence knowledge develops, action increases its scope, and situations improve.

Similarly, insight into oversight reveals the cumulative process of decline. For the flight from understanding blocks the insights that concrete situations demand. There follow unintelligent policies and inept courses of action. The situation deteriorates to demand still further insights and, as they are blocked, policies become more unintelligent and action more inept. What is worse, the deteriorating situation seems to provide the uncritical, biased mind with factual evidence in which the bias is claimed to be verified. So in ever increasing measure intelligence comes to be regarded as irrelevant to practical living. Human activity settles down to a decadent routine, and initiative becomes the privilege of violence.

Unfortunately, as insight and oversight commonly are mated, so also are progress and decline. We reinforce our love of truth with a practicality that is equivalent to an obscurantism. We correct old evils with a passion that mars the new good. We are not pure. We compromise. We hope to muddle through. But the very advance of knowledge brings a power over nature and over men too vast and terrifying to be entrusted to the good intentions of unconsciously biased minds. We have to learn to distinguish sharply between progress and decline, learn to encourage progress without putting a premium upon decline, learn to remove the tumour of the flight from understanding without destroying the organs of intelligence.

No problem is at once more delicate and more profound, more practical and perhaps more pressing. How, indeed, is a mind to become conscious of its own bias when that bias springs from a communal flight from understanding and is supported by the whole texture of a civilization? How can new strength and vigour be imparted to the detached and disinterested desire to understand without the reinforcement acting as an added bias? How can human intelligence hope to deal with the



unintelligible yet objective situations which the flight from understanding creates and expands and sustains? At least, we can make a beginning by asking what precisely it is to understand, what are the dynamics of the flow of consciousness that favours insight, what are the interferences that favour oversight, what, finally, do the answers to such questions imply for the guidance of human thought and action.

I must conclude. There will be offered in the *Introduction* a more exact account of the aim and structure of this book. Now I have to make a brief acknowledgement of my manifold indebtedness, and naturally I am led to think in the first place of the teachers and writers who have left their mark upon me in the course of the twenty-eight years that have elapsed since I was introduced to philosophy. But so prolonged has been my search, so much of it has been a dark struggle with my own flight from understanding, so many have been the half-lights and detours in my slow development, that my sincere gratitude can find no brief and exact yet intelligible expression. I turn, accordingly, to list more palpable benefactors: the staff of L'Immaculée Conception in Montreal where the parallel historical investigation* was undertaken; the staff of the Jesuit Seminary in Toronto where this book was written; the Rev. Eric O'Connor of Loyola College, Montreal, who was ever ready to allow me to draw upon his knowledge of mathematics and of science; the Rev. Joseph Wulftange, the Rev. Joseph Clark, the Rev. Norris Clarke, the Rev. Frederick Crowe, the Rev. Frederick Copleston, and the Rev. André Godin who kindly read the typescript and by their diversified knowledge, encouraging remarks and limited criticisms permitted me to feel that I was not entirely wrong; the Rev. Frederick Crowe who has undertaken the tedious task of compiling an index.

## NOTE TO SECOND EDITION

I have recast the last fifteen lines on page 66, the first two on page 67, lines 15 to 31 on page 98, the first twelve lines on page 111, and the passage running from line 24 page 340 to line 12 page 341.

* 'The Concept of *Verbum* in the Writings of St. Thomas Aquinas.' *Theological Studies* (Woodstock, Md.), VII (1946), 349–92; VIII (1947), 35–79, 404–44; X (1949), 3–40, 359–93.

# INTRODUCTION

The aim of the present work may be bracketed by a series of disjunctions. In the first place, the question is not whether knowledge exists but what precisely is its nature. Secondly, while the content of the known cannot be disregarded, still it is to be treated only in the schematic and incomplete fashion needed to provide a discriminant or determinant of cognitive acts. Thirdly, the aim is not to set forth a list of the abstract properties of human knowledge but to assist the reader in effecting a personal appropriation of the concrete, dynamic structure immanent and recurrently operative in his own cognitional activities. Fourthly, such an appropriation can occur only gradually, and so there will be offered, not a sudden account of the whole of the structure, but a slow assembly of its elements, relations, alternatives, and implications. Fifthly, the order of the assembly is governed, not by abstract considerations of logical or metaphysical priority, but by concrete motives of pedagogical efficacy.

The programme, then, is both concrete and practical, and the motives for undertaking its execution reside, not in the realm of easy generalities, but in the difficult domain of matters of fact. If, at the end of the course, the reader will be convinced of those facts, much will be achieved; but at the present moment all I can do is to clarify my intentions by stating my beliefs. I ask, accordingly, about the nature rather than about the existence of knowledge because in each of us there exist two different kinds of knowledge. They are juxtaposed in Cartesian dualism with its rational '*Cogito, ergo sum*' and with its unquestioning extroversion to substantial extension. They are separated and alienated in the subsequent rationalist and empiricist philosophies. They are brought together again to cancel each other in Kantian criticism. If these statements approximate the facts, then the question of human knowledge is not whether it exists but what precisely are its two diverse forms and what are the relations between them. If that is the relevant question, then any departure from it is, in the same measure, the misfortune of missing the point. But whether or not that is the relevant question, can be settled only by undertaking an arduous exploratory journey through the many fields in which men succeed in knowing or attempt the task but fail.

Secondly, an account of knowing cannot disregard its content, and its content is so extensive that it mocks encyclopaedias and overflows libraries; its content is so difficult that a man does well devoting his life to mastering some part of it; yet even so, its content is incomplete and subject to further additions, inadequate and subject to repeated, future revisions. Does it not follow that the proposed exploratory journey is, not merely arduous, but impossible? Certainly it would be impossible, at least for the writer, if an acquaintance with the whole range of knowledge were a requisite in the present inquiry. But, in fact, our primary concern is not the known but the knowing. The known is extensive, but the knowing is a recurrent structure that can be investigated sufficiently in a series of strategically chosen instances. The known is difficult to master, but in our day competent specialists have laboured to select for serious readers and to present to them in an adequate fashion the basic components of the various departments of knowledge. Finally, the known is incomplete and subject to revision, but our concern is the knower that will be the source of the future additions and revisions.

It will not be amiss to add a few corollaries, for nothing disorientates a reader more than a failure to state clearly what a book is not about. Basically, then, this is not a book on mathematics, nor a book on science, nor a book on common sense, nor a book on metaphysics, indeed, in a sense, it is not even a book about knowledge. On a first level, the book contains sentences on mathematics, on science, on common sense, on metaphysics. On a second level, the meaning of all these sentences, their intention and significance, are to be grasped only by going beyond the scraps of mathematics or science or common sense or metaphysics to the dynamic, cognitional structure that is exemplified in knowing them. On a third level, the dynamic, cognitional structure to be reached is not the transcendental *ego* of Fichtean speculation, nor the abstract pattern of relations verifiable in Tom and Dick and Harry, but the personally appropriated structure of one's own experiencing, one's own intelligent inquiry and insights, one's own critical reflection and judging and deciding. The crucial issue is an experimental issue, and the experiment will be performed not publicly but privately. It will consist in one's own rational self-consciousness clearly and distinctly taking possession of itself as rational self-consciousness. Up to that decisive achievement, all leads. From it, all follows. No one else, no matter what his knowledge or his eloquence, no matter what his logical rigour or his persuasiveness, can do it for you. But



though the act is private, both its antecedents and its consequents have their public manifestation. There can be long series of marks on paper that communicate an invitation to know oneself in the tension of the duality of one's own knowing; and among such series of marks with an invitatory meaning the present book would wish to be numbered. Nor need it remain a secret whether such invitations are helpful or, when helpful, accepted. Winter twilight cannot be mistaken for the summer noonday sun.

In the third place, then, more than all else, the aim of the book is to issue an invitation to a personal, decisive act. But the very nature of the act demands that it be understood in itself and in its implications. What on earth is meant by rational self-consciousness? What is meant by inviting it to take possession of itself? Why is such self-possession said to be so decisive and momentous? The questions are perfectly legitimate, but the answer cannot be brief.

However, it is not the answer itself that counts so much as the manner in which it is read. For the answer cannot but be written in words; the words cannot but proceed from definitions and correlations, analyses and inferences; yet the whole point of the present answer would be missed if a reader insisted on concluding that I must be engaged in setting forth lists of abstract properties of human knowing. The present work is not to be read as though it described some distant region of the globe, which the reader never visited, or some strange and mystical experience, which the reader never shared. It is an account of knowledge. Though I cannot recall to each reader his personal experiences, he can do so for himself and thereby pluck my general phrases from the dim world of thought to set them in the pulsing flow of life. Again, in such fields as mathematics and natural science, it is possible to delineate with some accuracy the precise content of a precise insight; but the point of the delineation is not to provide the reader with a stream of words that he can repeat to others or with a set of terms and relations from which he can proceed to draw inferences and prove conclusions. On the contrary, the point here, as elsewhere, is appropriation; the point is to discover, to identify, to become familiar with the activities of one's own intelligence; the point is to become able to discriminate with ease and from personal conviction between one's purely intellectual activities and the manifold of other, 'existential' concerns that invade and mix and blend with the operations of intellect to render it ambivalent and its pronouncements ambiguous.

At this juncture, however, many a potential reader will expostulate. The illustrations offered in the first five chapters do not lie within the orbit of his interests. Intelligence and reasonableness are marks common to all instances of *homo sapiens*. But my initial concentration on mathematics and natural science seems unduly to narrow the effective range of the invitation that I issue to an appropriation of one's own rational self-consciousness.

Perhaps an explanation of the motives that guided my decision in this matter will serve, not only to explain my procedure, but also to enable each reader to estimate for himself the measure in which the earlier chapters have to be understood if he is to be in a position to profit from the book as a whole. In the first place, it is essential that the notion of insight, of the accumulation of insights, of higher viewpoints, and of their heuristic significance and implications, not only should be grasped clearly and distinctly but also, in so far as possible, should be identified in one's own personal intellectual experience. The precise nature of such an identification will be clarified in the chapter on Self-affirmation for, as seems clear, it is both easy and common to conceive introspection and intellectual experience in a fashion that, when submitted to scrutiny, proves to be meaningless. Still, if that account of our awareness of the levels of consciousness is to be intelligible, it has to be preceded by a grasp, both precise and firm, of the successive types of activity that serve to mark and to define the successive levels of consciousness. In turn, if one's apprehension of those activities is to be clear and distinct, then one must prefer the fields of intellectual endeavour in which the greatest care is devoted to exactitude and, in fact, the greatest exactitude is attained. For this reason, then, I have felt obliged to begin my account of insight and its expansion with mathematical and scientific illustrations and, while I would grant that essentially the same activities can be illustrated from the ordinary use of intelligence that is named common sense, I also must submit that it would be impossible for common sense to grasp and say what precisely common sense happens to illustrate.

But further considerations are no less operative. For the present enterprise is concerned to unravel an ambiguity and to eliminate an ambivalence. St. Augustine of Hippo narrates that it took him years to make the discovery that the name, real, might have a different connotation from the name, body. Or, to bring the point nearer home, one might say that it has taken modern science four centuries to make the discovery that the objects of its inquiry need not be imaginable entities



moving through imaginable processes in an imaginable space-time. The fact that a Plato attempted to communicate through his dialogues, the fact that an Augustine eventually learnt from the writers whom, rather generally, he refers to as Platonists, has lost its antique flavour and its apparent irrelevance to the modern mind. Even before Einstein and Heisenberg it was clear enough that the world described by scientists was strangely different from the world depicted by artists and inhabited by men of common sense. But it was left to twentieth-century physicists to envisage the possibility that the objects of their science were to be reached only by severing the umbilical cord that tied them to the maternal imagination of man.

As the reader will have divined, the relevance of mathematics and mathematical physics to the present investigation is not only the transference of their clarity and precision to the account of insight but also the significance of the transition from the old mechanism to relativity and from the old determinism to statistical laws. In earlier periods the thinker that would come to grips with his thinking could be aided by the dialogues of Plato and, on a more recondite level, he could appeal to what M. Gilson would call the experiment of history in ancient, medieval, and modern philosophy. But today there are at his disposal both the exactitude and the impressive scale of a complementary historical experiment that began with the blending of scientific principles and philosophic assumptions in Galileo and has ended with their sharp segregation in our own day. What a Plato laboured to communicate through the effort in appropriation of his artistic dialogues, what the intelligence of an Augustine only slowly mastered in the throes of a religious conversion, what led a Descartes to a method of universal doubt and prompted a Kant to undertake a Critique of Pure Reason, has cast a shadow, no less momentous but far more sharply defined, in the realm of exact science. Clearly in a contemporary effort to resolve the duality in man's knowledge it would be foolhardy to ignore, if not the most striking, at least the most precise element in the evidence available on the issue.

But there is also a third purpose that I hope to achieve through an appropriation of the modes of scientific thought. For such thought is methodical and the scientist pins his faith, not on this or that scientific system or conclusion, but on the validity of scientific method itself. But what ultimately is the nature and ground of method but a reflective grasp and specialized application of the object of our inquiry,

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But there is also a third purpose that I hope to achieve through an appropriation of the modes of scientific thought. For such thought is methodical and the scientist pins his faith, not on this or that scientific system or conclusion, but on the validity of scientific method itself. But what ultimately is the nature and ground of method but a reflective grasp and specialized application of the object of our inquiry,

namely, of the dynamic structure immanent and recurrently operative in human cognitional activity? It follows that empirical science as methodical not merely offers a clue for the discovery but also exhibits concrete instances for the examination of the larger, multiform dynamism that we are seeking to explore. Accordingly, it will be from the structural and dynamic features of scientific method that we shall approach and attempt to cast into the unity of a single perspective such apparently diverse elements as:

(1) Plato's point in asking how the inquirer recognizes truth when he reaches what, as an inquirer, he did not know,

(2) the intellectualist (though not the conceptualist) meaning of the abstraction of form from material conditions,

(3) the psychological manifestation of Aquinas' natural desire to know God by his essence,

(4) what Descartes was struggling to convey in his incomplete *Regulae ad directionem ingenii*,

(5) what Kant conceived as *a priori* synthesis, and

(6) what is named the finality of intellect in J. Maréchal's vast labour on *Le Point de départ de la métaphysique*.

I have been insisting on the gravity of the motives that led me to begin this essay in aid of self-appropriation with a scrutiny of mathematical physics. But if I am to avoid overstatement, I must hasten to add that the significance of the scrutiny is, so to speak, psychological rather than logical. For the present work falls into two parts. In the first part, insight is studied as an activity, as an event that occurs within various patterns of other related events. In the second part, insight is studied as knowledge, as an event that, under determinate conditions, reveals a universe of being. The first part deals with the question, What is happening when we are knowing? The second part moves to the question, What is known when that is happening? Were there no psychological problem, the first part could be reduced to sets of definitions and clarifications for, from a logical viewpoint, the first judgment that occurs in the whole work is the judgment of self-affirmation in the eleventh chapter. But the hard fact is that the psychological problem exists, that there exist in man two diverse kinds of knowing, that they exist without differentiation and in an ambivalent confusion until they are distinguished explicitly and the implications of the distinction are drawn explicitly. The hard fact is that the personal psychological problem



cannot be solved by the ordinary procedure of affirming the propositions that are true and denying the propositions that are false, for the true meaning of the true propositions always tends to be misapprehended by a consciousness that has not yet discovered its need of discovering what an Augustine took years and modern science centuries to discover.

It remains that something be said on the last two of the five disjunctions by which we proposed to bracket the aim of this book. As has been noted, we are concerned not with the existence of knowledge but with its nature, not with what is known but with the structure of the knowing, not with the abstract properties of cognitional process but with a personal appropriation of one's own dynamic and recurrently operative structure of cognitional activity. There is now to be explained the fourth disjunction, for the labour of self-appropriation cannot occur at a single leap. Essentially, it is a development of the subject and in the subject and, like all development, it can be solid and fruitful only by being painstaking and slow.

Now it would be absurd to offer to aid a process of development and yet write as though the whole development were already an accomplished fact. A teacher of geometry may be convinced that the whole of Euclid is contained in the theory of the *n*-dimensional manifold of any curvature. But he does not conclude that Euclid is to be omitted from the elementary programme and that his pupils should begin from the tensor calculus. For even though Euclid is a particular case, still it is the particular case that alone gives access to the general case. And even though Euclidean propositions call for qualification when the more general context is reached, still an effective teacher does not distract his pupils with qualifications they will understand only vaguely, when it is his business to herd them, as best he can, across the *pons asinorum*.

In similar fashion this book is written, not from above downwards, but from below upwards. Any coherent set of statements can be divided into definitions, postulates, and conclusions. But it does not follow that between the covers of a single book there must be a single coherent set of statements. For the single book may be written from a moving viewpoint, and then it will contain, not a single set of coherent statements, but a sequence of related sets of coherent statements. Moreover, as is clear, a book designed to aid a development must be written from a moving viewpoint. It cannot begin by presupposing that a

reader can assimilate at a stroke what can be attained only at the term of a prolonged and arduous effort. On the contrary, it must begin from a minimal viewpoint and a minimal context; it will exploit that minimum to raise a further question that enlarges the viewpoint and the context; it will proceed with the enlarged viewpoint and context only as long as is necessary to raise still deeper issues that again transform the basis and the terms of reference of the inquiry; and clearly, this device can be repeated not merely once or twice but as often as may be required to reach the universal viewpoint and the completely concrete context that embraces every aspect of reality.

However, if this procedure alone is adapted to the aim of the present work, I must beg to stress, once and for all, that its implications are not to be overlooked. If Spinoza wrote his *Ethics* in what, in his day, was thought to be the geometric style, it is not to be inferred that I am endeavouring to walk in his footsteps, that I never heard of Gödel's theorem, that I am not operating from a moving viewpoint that successively sets up contexts only to go beyond them. If the inference is not to be made, the further implications of such an inference are not to be assumed. The premises from which my own position can be deduced are not complete in the first section of the first chapter when a brief description endeavours to fix the meaning of the name, insight. The context is enlarged but not completed when a study of mathematical development makes the notion of insight more precise. There is the broader context of a mathematicized world of events that has appeared by the end of the fifth chapter, but it has to be included within the still fuller context of the world of common sense to be depicted in chapters six and seven. The eighth chapter adds things, which, though previously disregarded, never were denied. The ninth and tenth chapters add reflection and judgment, which neither were excluded from earlier considerations nor, on the other hand, were they capable of making a systematic entry. In the eleventh chapter there occurs the first judgment of self-affirmation but only in the twelfth chapter is it advanced that that judgment is knowledge and only in the thirteenth is it explained in what sense such knowledge is to be named objective. The four chapters on metaphysics follow to sweep all that has been seen into the unity of a larger perspective, only to undergo a similar fate, first, in the account of general transcendent knowledge and, again, in the approach to special transcendent knowledge.

Clearly, then, if anyone were to offer to express my meaning within a briefer compass than I have been able to attain, he must bear in mind that earlier statements are to be qualified and interpreted in the light of later statements.



Nor is this all. For already it has been pointed out that the present work is concerned with the known only in the schematic and incomplete fashion that is needed to clarify the nature and affirm the existence of different departments of knowing. This extremely general qualification has to be combined with the qualification of earlier statements by later and, I suggest, the combination can be effected systematically in the following manner.

Gödel's theorem is to the effect that any set of mathematical definitions and postulates gives rise to further questions that cannot be answered on the basis of the definitions and postulates. Consider, then, a series of sets of definitions and postulates, say $P, Q, R, \ldots$ such that, if $P$ is assumed, there arise questions that can be answered only by assuming $Q$, if $Q$ is assumed, there arise questions that can be answered only by assuming $R$, and so forth. Then besides the successive lower contexts, $P, Q, R, \ldots$ there also is the upper context in which Gödel's theorem is expressed. Moreover, inasmuch as the theorem is quite general, the upper context is independent of the content of any particular contexts such as, $P, Q, R, \ldots$. Finally, since there is no last, lower context that is definitive, since $R$ will demand a context $S$, and $S$ a context $T$, and $T$ a context $U$, and so on indefinitely, the really significant context is the upper context; all lower contexts, $P, Q, R, S, T, U, \ldots$ are provisional; and they attain a definite significance only in the measure that they give access to the upper context.

Now let us go beyond Gödel's theorem, not in the direction of greater abstractness, but in the direction of greater concreteness, and not to greater concreteness on the side of the object (which is vast and difficult and open to further additions and revisions) but to greater concreteness on the side of the subject. Besides the *noêma* or *intentio intenta* or *pensée pensée*, illustrated by the lower contexts, $P, Q, R, \ldots$ and by the upper context that is Gödel's theorem, there also is the *noêsis* or *intentio intendens* or *pensée pensante* that is constituted by the very activity of inquiring and reflecting, understanding and affirming, asking further questions and reaching further answers. Let us say that this noetic activity is engaged in a lower context when it is doing mathematics or following scientific method or exercising common sense. Then it will be moving towards an upper context when it scrutinizes mathematics

or science or common sense in order to grasp the nature of noetic activity. And if it comes to understand and affirm what understanding is and what affirming is, then it has reached an upper context that logically is independent of the scaffolding of mathematics, science, and common sense. Moreover, if it can be shown that the upper context is invariant, that any attempt to revise it can be legitimate only if the hypothetical reviser refutes his own attempt by invoking experience, understanding, and reflection in an already prescribed manner, then it will appear that, while the *noëma* or *intentio intenta* or *pensée pensée* may always be expressed with greater accuracy and completeness, still the immanent and recurrently operative structure of the *noësis* or *intentio intendens* or *pensée pensante* must always be one and the same.

In other words, not only are we writing from a moving viewpoint but also we are writing about a moving viewpoint. Not only are earlier statements to be qualified by later statements, but also the later qualification is to the effect that earlier statements tend to be mere scaffolding that can be subjected to endless revision without implying the necessity of any revision of one's appropriation of one's own intellectual and rational self-consciousness.

In the fifth place, to turn to the final disjunction, the order in which the moving viewpoint assembles the elements for an appropriation of one's own intellectual and rational self-consciousness is governed, not by considerations of logical or metaphysical priority, but by considerations of pedagogical efficacy.

Now this fifth disjunction would be superfluous if I could not anticipate that among potential readers there might be men already in possession of a logical or a metaphysical scheme of things. Accordingly, though it will be the constant rule of the present work to deal with issues in their proper generality and at their proper place and time, it seems necessary to depart for a moment from that rule to envisage some of the points on which logicians or metaphysicians are going to find it obvious that, on their already established criteria, I must be utterly on the wrong track.

From a logical viewpoint it might seem that enough has been said, but two points merit special attention. In the course of Chapter XIV or, at least, by Chapter XVII the reader will be able to hold in a single coherent view the totality of contradictory positions on knowledge, objectivity, and reality. But such a perspective is dialectical or metalogical. It cannot be produced by the logical arts of definition, postulation, and inference. It can be mediated by a book only in so far as there is a communication of insights that in some remote fashion is analogous to the evocation of images or to the suggestion of feelings. Hence, particularly in our first ten chapters which deal with the genesis of concepts and judgments, of terms and propositions, the only possible vehicle for the essential content of our analysis is a prelogical and even preconceptual mode of communication.



Secondly, our goal is insight into insight and that goal is reached inasmuch as the insight that is sought rises upon a differentiated series of illustrative insights. But the illustrative insights have to be elementary. We cannot reproduce whole treatises and, if we could and did, we should defeat our purpose. Hence, our illustrations have to be simple insights stripped from their context of further complementary insights that correct, qualify, adjust, and refine. Now such stripping will pain specialist readers. If they miss our point entirely, it may even convince them that insight itself is as superficial as our illustrations. However, specialists have in their own understanding the remedy for their pain, for they always can bring to light the complementary insights by asking themselves *why* our illustrations are unsatisfactory. Moreover, if they do so, they can advance rapidly towards an insight into insight while, if they merely grumble that this set of words is wrong and that set misleading, they risk encouraging an oversight of insight and even a flight from understanding.

To turn from logical to metaphysical considerations is to envisage a quite different circle of possible readers. Among contemporary Scholastics there is a broad agreement on metaphysical issues and, at the same time, a strongly contrasting divergence on epistemological questions. This disparity may lend my work an appearance of wrong-headedness for instead of approaching what is doubtful from what is assured I begin from knowledge and reach metaphysics only as a conclusion.

I am far from certain, however, that this is a correct perspective. The broad agreement of Scholastics in metaphysics is matched by an equally broad agreement in epistemology, and the divergence of Scholastic views in epistemology is matched by the no less impressive array of disputed questions in metaphysics. It follows that the real problem is to advance from a mere broad agreement in metaphysics and epistemology to a precise and detailed agreement in both, and to that end an obvious means is our attempt to reach a fresh and fuller view of the relevant facts.

To conclude, our aim regards:

(1) not the fact of knowledge but a discrimination between two facts of knowledge,

(2) not the details of the known but the structure of the knowing,

(3) not the knowing as an object characterized by catalogues of abstract properties but the appropriation of one's own intellectual and rational self-consciousness,

(4) not a sudden leap to appropriation but a slow and painstaking development, and

(5) not a development indicated by appealing either to the logic of the as yet unknown goal or to a presupposed and as yet unexplained ontologically structured metaphysics, but a development that can begin in any sufficiently cultured consciousness, that expands in virtue of the dynamic tendencies of that consciousness itself, and that heads through an understanding of all understanding to a basic understanding of all that can be understood.

The last phrase has the ring of a slogan and, happily enough, it sums up the positive content of this work. *Thoroughly understand what it is to understand, and not only will you understand the broad lines of all there is to be understood but also you will possess a fixed base, an invariant pattern, opening upon all further developments of understanding.*

For the appropriation of one's own rational self-consciousness, which has been so stressed in this Introduction, is not an end in itself but rather a beginning. It is a necessary beginning, for unless one breaks the duality in one's knowing, one doubts that understanding correctly is knowing. Under the pressure of that doubt, either one will sink into the bog of a knowing that is without understanding, or else one will cling to understanding but sacrifice knowing on the altar of an immanentism, an idealism, a relativism. From the horns of that dilemma one escapes only through the discovery (and one has not made it yet if one has no clear memory of its startling strangeness) that there are two quite different realisms, that there is an incoherent realism, half animal and half human, that poses as a half-way house between materialism and idealism and, on the other hand, that there is an intelligent and reasonable realism between which and materialism the half-way house is idealism.

The beginning, then, not only is self-knowledge and self-appropriation but also a criterion of the real. If to convince oneself that knowing is understanding, one ascertains that knowing mathematics is understanding and knowing science is understanding and the knowledge of common sense is understanding, one ends up not only with a detailed account of understanding but also with a plan of what there is to be known. The many sciences lose their isolation from one another; the chasm between science and common sense is bridged; the structure of the universe proportionate to man's intellect is revealed; and as that revealed structure provides an object for a metaphysics, so the initial self-criticism provides a method for explaining how metaphysical and anti-metaphysical affirmations arise, for selecting those that are correct, and for eliminating those that patently spring from a lack of accurate self-knowledge. Further, as a metaphysics is derived from the known structure of one's knowing, so an ethics results from knowledge of the compound structure of one's knowing and doing; and as the metaphysics, so too the ethics prolongs the initial self-criticism into an explanation of the origin of all ethical positions and into a criterion for passing judgment on each of them. Nor is this all. Still further questions press upon one. They might be ignored if knowing were not understanding or if understanding were compatible with the obscurantism that arbitrarily brushes questions aside. But knowing is understanding, and understanding is incompatible with the obscurantism that arbitrarily brushes questions aside. The issue of transcendent knowledge has to be faced. Can man know more than the intelligibility immanent in the world of possible experience? If he can, how can he conceive it? If he can conceive it, how can he affirm it? If he can affirm it, how can he reconcile that affirmation with the evil that tortures too many human bodies, darkens too many human minds, hardens too many human hearts? Such are the questions of the last two chapters, but further comment on the answers offered there will be more intelligible in an Epilogue than in an Introduction.



As the reader shortly will discover, this is not an erudite work. Prior to all writing of history, prior to all interpretation of other minds, there is the self-scrutiny of the historian, the self-knowledge of the interpreter. That prior task is my concern. It is a concern that has its origins and background, its dependences and affiliations; they might be worth recounting; but they would be worth recounting only because of the worth of the prior concern; and they would be interpreted correctly only if the prior concern were successful in accomplishing the prior task.

So it is that my references are few and unessential. In the analysis of empirical science I thought that it would be helpful to select a single book in which a reader could find an account of topics that arose; for this reason, then, and without any intention of suggesting some unique authoritativeness I regularly refer to Lindsay and Margenau's frequently reprinted *Foundations of Physics*. Again, scattered throughout the work, there occur bold statements on the views of various thinkers. May I express the hope that they will not cause too much annoyance? As the lengthy discussion of the truth of interpretation in Chapter XVII will reveal, they can hardly pretend to be verdicts issued by the court of history, whose processes labour under much longer delays than the worst of the courts of law. Their primary significance is simply that of an abbreviated mode of speech that has a fair chance of communicating rapidly what otherwise could hardly be said at all. And, perhaps, to that primary meaning there could be added a suggestion that, in the measure that the principles of this work are accepted, the significance that we happen to have underlined may provide a starting-point for further inquiry.

In the Introduction to his *Treatise of Human Nature*, David Hume wrote that one does not conquer a territory by taking here an outpost and there a town or village but by marching directly upon the capital and assaulting its citadel. Still, correct strategy is one thing; successful execution is another; and even after the most successful campaign there remains a prolonged task of mopping up, of organization, and of consolidation. If I may be sanguine enough to believe that I have hit upon a set of ideas of fundamental importance, I cannot but acknowledge that I do not possess the resources to give a faultless display of their implications in the wide variety of fields in which they are relevant. I can but make the contribution of a single man and then hope that others, sensitive to the same problems, will find that my efforts shorten their own labour and that my conclusions provide a base for further developments.

# *PART ONE*

# INSIGHT AS ACTIVITY

CHAPTER 1

# ELEMENTS

In the midst of that vast and profound stirring of human minds, which we name the Renaissance, Descartes was convinced that too many people felt it beneath them to direct their efforts to apparently trifling problems. Again and again, in his *Regulae ad directionem ingenii*, he reverts to this theme. Intellectual mastery of mathematics, of the departments of science, of philosophy, is the fruit of a slow and steady accumulation of little insights. Great problems are solved by being broken down into little problems. The strokes of genius are but the outcome of a continuous habit of inquiry that grasps clearly and distinctly all that is involved in the simple things that anyone can understand.

I thought it well to begin by recalling this conviction of a famous mathematician and philosopher, for our first task will be to attain familiarity with what is meant by insight, and the only way to achieve this end is, it seems, to attend very closely to a series of instances all of which are rather remarkable for their banality.

## 1. A DRAMATIC INSTANCE

Our first illustrative instance of insight will be the story of Archimedes rushing naked from the baths of Syracuse with the cryptic cry, 'Eureka!' King Hiero, it seems, had had a votive crown fashioned by a smith of rare skill and doubtful honesty. He wished to know whether or not baser metals had been added to the gold. Archimedes was set the problem and in the bath had hit upon the solution. Weigh the crown in water! Implicit in this directive were the principles of displacement and of specific gravity.

With those principles of hydrostatics we are not directly concerned. For our objective is an insight into insight. Archimedes had his insight by thinking about the crown; we shall have ours by thinking about Archimedes. What we have to grasp is that insight

(1) comes as a release to the tension of inquiry,
(2) comes suddenly and unexpectedly,

(3) is a function not of outer circumstances but inner conditions,
(4) pivots between the concrete and the abstract, and
(5) passes into the habitual texture of one's mind.*

First, then, insight comes as a release to the tension of inquiry. This feature is dramatized in the story by Archimedes' peculiarly uninhibited exultation. But the point I would make does not lie in this outburst of delight but in the antecedent desire and effort that it betrays. For if the typical scientist's satisfaction in success is more sedate, his earnestness in inquiry can still exceed that of Archimedes. Deep within us all, emergent when the noise of other appetites is stilled, there is a drive to know, to understand, to see why, to discover the reason, to find the cause, to explain. Just what is wanted, has many names. In what precisely it consists, is a matter of dispute. But the fact of inquiry is beyond all doubt. It can absorb a man. It can keep him for hours, day after day, year after year, in the narrow prison of his study or his laboratory. It can send him on dangerous voyages of exploration. It can withdraw him from other interests, other pursuits, other pleasures, other achievements. It can fill his waking thoughts, hide from him the world of ordinary affairs, invade the very fabric of his dreams. It can demand endless sacrifices that are made without regret though there is only the hope, never a certain promise, of success. What better symbol could one find for this obscure, exigent, imperious drive, than a man, naked, running, excitedly crying, 'I've got it'?

Secondly, insight comes suddenly and unexpectedly. It did not occur when Archimedes was in the mood and posture that a sculptor would select to portray 'The Thinker'. It came in a flash, on a trivial occasion, in a moment of relaxation. Once more there is dramatized a universal aspect of insight. For it is reached, in the last analysis, not by learning rules, not by following precepts, not by studying any methodology. Discovery is a new beginning. It is the origin of new rules that supplement or even supplant the old. Genius is creative. It is genius precisely because it disregards established routines, because it originates the novelties that will be the routines of the future. Were there rules for discovery, then discoveries would be mere conclusions. Were there precepts for genius, then men of genius would be hacks. Indeed, what is true of discovery, also holds for the transmission of discoveries by

* A profusion of instances of insight is offered by E. D. Hutchinson in three articles originally published in *Psychiatry* and reprinted in *A Study of Interpersonal Relations* [edited by P. Mullahy, New York 1949].



teaching. For a teacher cannot undertake to make a pupil understand. All he can do is present the sensible elements in the issue in a suggestive order and with a proper distribution of emphasis. It is up to the pupils themselves to reach understanding, and they do so in varying measures of ease and rapidity. Some get the point before the teacher can finish his exposition. Others just manage to keep pace with him. Others see the light only when they go over the matter by themselves. Some finally never catch on at all; for a while they follow the classes but, sooner or later, they drop by the way.

Thirdly, insight is a function, not of outer circumstances, but of inner conditions. Many frequented the baths of Syracuse without coming to grasp the principles of hydrostatics. But who bathed there without feeling the water, or without finding it hot or cold or tepid? There is, then, a strange difference between insight and sensation. Unless one is deaf, one cannot avoid hearing. Unless one is blind, one has only to open one's eyes to see. The occurrence and the content of sensation stand in some immediate correlation with outer circumstance. But with insight, internal conditions are paramount. Thus, insight depends upon native endowment and so, with fair accuracy, one can say that insight is the act that occurs frequently in the intelligent and rarely in the stupid. Again, insight depends upon a habitual orientation, upon a perpetual alertness ever asking the little question, 'Why?' Finally, insight depends on the accurate presentation of definite problems. Had Hiero not put his problem to Archimedes, had Archimedes not thought earnestly, perhaps desperately, upon it, the baths of Syracuse would have been no more famous than any others.

Fourthly, insight pivots between the concrete and the abstract. Archimedes' problem was concrete. He had to settle whether a particular crown was made of pure gold. Archimedes' solution was concrete. It was to weigh the crown in water. Yet if we ask what was the point to that procedure, we have to have recourse to the abstract formulations of the principles of displacement and of specific gravity. Without that point, weighing the crown in water would be mere eccentricity. Once the point is grasped, King Hiero and his golden crown become minor historical details of no scientific importance. Again the story dramatizes a universal aspect of insight. For if insights arise from concrete problems, if they reveal their value in concrete applications, none the less they possess a significance greater than their origins and a relevance wider than their original applications.

Because insights arise with reference to the concrete, mathematicians need pen and paper, teachers need black-boards, pupils have to perform experiments for themselves, doctors have to see their patients, trouble-shooters have to travel to the spot, people with a mechanical bent take things apart to see how they work. But because the significance and relevance of insight goes beyond any concrete problem or application, men formulate abstract sciences with their numbers and symbols, their technical terms and formulae, their definitions, postulates, and deductions. Thus, by its very nature, insight is the mediator, the hinge, the pivot. It is insight *into* the concrete world of sense and imagination. Yet what is known by insight, what insight adds to sensible and imagined presentations, finds its adequate expression only in the abstract and recondite formulations of the sciences.

Fifthly, insight passes into the habitual texture of one's mind. Before Archimedes could solve his problem, he needed an instant of inspiration. But he needed no further inspiration when he went to offer the king his solution. Once one has understood, one has crossed a divide. What a moment ago was an insoluble problem, now becomes incredibly simple and obvious. Moreover, it tends to remain simple and obvious. However laborious the first occurrence of an insight may be, subsequent repetitions occur almost at will. This, too, is a universal characteristic of insight and, indeed, it constitutes the possibility of learning. For we can learn inasmuch as we can add insight to insight, inasmuch as the new does not extrude the old but complements and combines with it. Inversely, inasmuch as the subject to be learnt involves the acquisition of a whole series of insights, the process of learning is marked by an initial period of darkness in which one gropes about insecurely, in which one cannot see where one is going, in which one cannot grasp what all the fuss is about; and only gradually, as one begins to catch on, does the initial darkness yield to a subsequent period of increasing light, confidence, interest, absorption. Then, the infinitesimal calculus or theoretical physics or the issues of philosophy cease to be the mysterious and foggy realms they had seemed. Imperceptibly we shift from the helpless infancy of the beginner to the modest self-confidence of the advanced student. Eventually we become capable of taking over the teacher's role and complaining of the remarkable obtuseness of pupils that fail to see what, of course, is perfectly simple and obvious to those that understand.



## 2. DEFINITION

As every schoolboy knows, a circle is a locus of co-planar points equidistant from a centre. What every schoolboy does not know is the difference between repeating that definition, as a parrot might, and uttering it intelligently. So, with a sidelong bow to Descartes' insistence on the importance of understanding very simple things, let us inquire into the genesis of the definition of the circle.

### 2.1 *The Clue*

Imagine a cart-wheel with its bulky hub, its stout spokes, its solid rim.

Ask a question. Why is it round?

Limit the question. What is wanted is the immanent reason or ground of the roundness of the wheel. Hence a correct answer will not introduce new data such as carts, carting, transportation, wheelwrights, or their tools. It will refer simply to the wheel.

Consider a suggestion. The wheel is round because its spokes are equal. Clearly, that will not do. The spokes could be equal yet sunk unequally into the hub and rim. Again, the rim could be flat between successive spokes.

Still, we have a clue. Let the hub decrease to a point; let the rim and spokes thin out into lines; then, if there were an infinity of spokes and all were exactly equal, the rim would have to be perfectly round; inversely, were any of the spokes unequal, the rim could not avoid bumps or dents. Hence, we can say that the wheel necessarily is round, inasmuch as the distance from the centre of the hub to the outside of the rim is always the same.

A number of observations are now in order. The foregoing brings us close enough to the definition of the circle. But our purpose is to attain insight, not into the circle, but into the act illustrated by insight into the circle.

The first observation, then, is that points and lines cannot be imagined. One can imagine an extremely small dot. But no matter how small a dot may be, still it has magnitude. To reach a point, all magnitude must vanish, and with all magnitude there vanishes the dot as well. One can imagine an extremely fine thread. But no matter how fine a thread may be, still it has breadth and depth as well as length. Remove from the image all breadth and depth, and there vanishes all length as well.

### 2.2 *Concepts*

The second observation is that points and lines are concepts.

Just as imagination is the playground of our desires and our fears, so conception is the playground of our intelligence. Just as imagination can create objects never seen or heard or felt, so too conception can create objects that cannot even be imagined. How? By supposing. The imagined dot has magnitude as well as position, but the geometer says, Let us suppose it has only position. The imagined line has breadth as well as length, but the geometer says, Let us suppose it has only length.

Still, there is method in this madness. Our images and especially our dreams seem very random affairs, yet psychologists offer to explain them. Similarly, the suppositions underlying concepts may appear very fanciful, yet they too can be explained. Why did we require the hub to decrease to a point and the spokes and rim to mere lines? Because we had a clue—the equality of the spokes—and we were pushing it for all it was worth. As long as the hub had any magnitude, the spokes could be sunk into it unequally. As long as the spokes had any thickness, the wheel could be flat at their ends. So we supposed a point without magnitude, and lines without thickness to obtain a curve that would be perfectly, necessarily round.

Note, then, two properties of concepts. In the first place, they are constituted by the mere activity of supposing, thinking, considering, formulating, defining. They may or may not be more than that. But if they are more, then they are not merely concepts. And if they are no more than supposed or considered or thought about, still that is enough to constitute them as concepts. In the second place, concepts do not occur at random; they emerge in thinking, supposing, considering, defining, formulating; and that many-named activity occurs, not at random, but in conjunction with an act of insight.

### 2.3 *The Image*

The third observation is that the image is necessary for the insight.

Points and lines cannot be imagined. But neither can necessity or impossibility be imagined. Yet in approaching the definition of the circle, there occurred some apprehension of necessity and of impossibility. As we remarked, if all the radii are equal, the curve must be perfectly round; and if any radii are unequal, the curve cannot avoid bumps or dents.

Further, the necessity in question was not necessity in general but a necessity of roundness resulting from these equal radii. Similarly, the impossibility in question was not impossibility in the abstract but an impossibility of roundness resulting from these unequal radii. Eliminate the image of the centre, the radii, the curve, and by the same stroke there vanishes all grasp of necessary or of impossible roundness.



But it is that grasp that constitutes the insight. It is the occurrence of that grasp that makes the difference between repeating the definition of a circle, as a parrot might, and uttering it intelligently, uttering it with the ability to make up a new definition for oneself.

It follows that the image is necessary for the insight. Inversely, it follows that the insight is the act of catching on to a connection between imagined equal radii and, on the other hand, a curve that is bound to look perfectly round.

### 2.4 *The Question*

The fourth observation adverts to the question.

There is the question as expressed in words. Why is the wheel round?

Behind the words there may be conceptual acts of meaning, such as 'wheel', 'round', etc.

Behind these concepts there may be insights in which one grasps how to use such words as 'wheel', 'round', etc.

But what we are trying to get at is something different. Where does the 'Why?' come from? What does it reveal or represent? Already we had occasion to speak of the psychological tension that had its release in the joy of discovery. It is that tension, that drive, that desire to understand, that constitutes the primordial 'Why?' Name it what you please, alertness of mind, intellectual curiosity, the spirit of inquiry, active intelligence, the drive to know. Under any name, it remains the same and is, I trust, very familiar to you.

This primordial drive, then, is the pure question. It is prior to any insights, any concepts, any words, for insights, concepts, words, have to do with answers; and before we look for answers, we want them; such wanting is the pure question.

On the other hand, though the pure question is prior to insights, concepts, and words, it presupposes experiences and images. Just as insight is into the concretely given or imagined, so the pure question is about the concretely given or imagined. It is the wonder which Aristotle claimed to be the beginning of all science and philosophy. But no one just wonders. We wonder about something.

### 2.5 *Genesis*

A fifth observation distinguishes moments in the genesis of a definition.

When an animal has nothing to do, it goes to sleep. When a man has nothing to do, he may ask questions. The first moment is an awakening to one's intelligence. It is release from the dominance of biological drive and from the routines of everyday living. It is the effective emergence of wonder, of the desire to understand.

The second moment is the hint, the suggestion, the clue. Insight has begun. We have got hold of something. There is a chance that we are on the right track. Let's see.

The third moment is the process. Imagination has been released from other cares. It is free to co-operate with intellectual effort, and its co-operation consists in endeavouring to run parallel to intelligent suppositions while, at the same time, restraining supposition within some limits of approximation to the imaginable field.

The fourth moment is achievement. By their co-operation, by successive adjustments, question and insight, image and concept, present a solid front. The answer is a patterned set of concepts. The image strains to approximate to the concepts. The concepts, by added conceptual determinations, can express their differences from the merely approximate image. The pivot between images and concepts is the insight. And setting the standard which insight, images, and concepts must meet is the question, the desire to know, that could have kept the process in motion by further queries, had its requirements not been satisfied.

### 2.6 *Nominal and Explanatory Definition*

A sixth observation distinguishes different kinds of definition. As Euclid defined a straight line as a line lying evenly between its extremes, so he might have defined a circle as a perfectly round plane curve. As the former definition, so also the latter would serve to determine unequivocally the proper use of the names, straight line, circle. But, in fact, Euclid's definition of the circle does more than reveal the proper use of the name, circle. It includes the affirmation that in any circle all radii are exactly equal; and were that affirmation not included in the definition, then it would have had to be added as a postulate.

To view the same matter from another angle, Euclid did postulate that all right angles are equal. Let us name the sum of two adjacent right angles a straight angle. Then, if all right angles are equal, necessarily all straight angles will be equal. Inversely, if all straight angles are equal, all right angles must be equal. Now if straight lines are really straight, if they never bend in any direction, must not all straight angles be equal? Could not the postulate of the equality of straight angles be included in the definition of the straight line, as the postulate of the equality of radii is included in the definition of the circle?



At any rate, there is a difference between nominal and explanatory definitions. Nominal definitions merely tell us about the correct usage of names. Explanatory definitions also include something further that, were it not included in the definition, would have to be added as a postulate.

What constitutes the difference? It is not that explanatory definitions suppose an insight while nominal definitions do not. For a language is an enormously complicated tool with an almost endless variety of parts that admit a far greater number of significant combinations. If insight is needed to see how other tools are to be used properly and effectively, insight is similarly needed to use a language properly and effectively.

Still, this yields, I think, the answer to our question. Both nominal and explanatory definitions suppose insights. But a nominal definition supposes no more than an insight into the proper use of language. An explanatory definition, on the other hand, supposes a further insight into the objects to which language refers. The name, circle, is defined as a perfectly round plane curve, as the name, straight line, is defined as a line lying evenly between its extremes. But when one goes on to affirm that all radii in a circle are equal or that all right angles are equal, one no longer is talking merely of names. One is making assertions about the objects which names denote.

### 2.7 *Primitive Terms*

A seventh observation adds a note on the old puzzle of primitive terms.

Every definition presupposes other terms. If these can be defined, their definitions will presuppose still other terms. But one cannot regress to infinity. Hence, either definition is based on undefined terms or else terms are defined in a circle so that each virtually defines itself.

Fortunately, we are under no necessity of accepting the argument's supposition. Definitions do not occur in a private vacuum of their own. They emerge in solidarity with experiences, images, questions, and insights. It is true enough that every definition involves several terms, but

it is also true that no insight can be expressed by a single term, and it is not true that every insight presupposes previous insights.

Let us say, then, that for every basic insight there is a circle of terms and relations, such that the terms fix the relations, the relations fix the terms, and the insight fixes both. If one grasps the necessary and sufficient conditions for the perfect roundness of this imagined plane curve, then one grasps not only the circle but also the point, the line, the circumference, the radii, the plane, and equality. All the concepts tumble out together, because all are needed to express adequately a single insight. All are coherent, for coherence basically means that all hang together from a single insight.

Again, there can be a set of basic insights. Such is the set underlying Euclidean geometry. Because the set of insights is coherent, they generate a set of coherent definitions. Because different objects of definition are composed of similar elements, such terms as point, line, surface, angle, keep recurring in distinct definitions. Thus, Euclid begins his exposition from a set of images, a set of insights, and a set of definitions; some of his definitions are merely nominal; some are explanatory; some are derived partly from nominally and partly from explanatorily defined terms.

### 2.8 *Implicit Definition*

A final observation introduces the notion of implicit definition.

D. Hilbert has worked out *Foundations of Geometry* that satisfy contemporary logicians. One of his important devices is known as implicit definition. Thus, the meaning of both point and straight line is fixed by the relation that two and only two points determine a straight line.

In terms of the foregoing analysis, one may say that implicit definition consists in explanatory definition without nominal definition. It consists in explanatory definition, for the relation that two points determine a straight line is a postulational element such as the equality of all radii in a circle. It omits nominal definition, for one cannot restrict Hilbert's point to the Euclidean meaning of position without magnitude. An ordered pair of numbers satisfies Hilbert's implicit definition of a point, for two such pairs determine a straight line. Similarly, a first degree equation satisfies Hilbert's implicit definition of a straight line, for such an equation is determined by two ordered pairs of numbers.

The significance of implicit definition is its complete generality. The omission of nominal definitions is the omission of a restriction to the objects which, in the first instance, one happens to be thinking about. The exclusive use of explanatory or postulational elements concentrates attention upon the set of relationships in which the whole scientific significance is contained.



## 3. HIGHER VIEWPOINTS

The next significant step to be taken in working out the nature of insight is to analyse development. Single insights occur either in isolation or in related fields. In the latter case, they combine, cluster, coalesce, into the mastery of a subject; they ground sets of definitions, postulates, deductions; they admit applications to enormous ranges of instances. But the matter does not end there. Still further insights arise. The shortcomings of the previous position become recognized. New definitions and postulates are devised. A new and larger field of deductions is set up. Broader and more accurate applications become possible. Such a complex shift in the whole structure of insights, definitions, postulates, deductions, and applications, may be referred to very briefly as the emergence of a higher viewpoint. Our question is, Just what happens?

Taking our clue from Descartes' insistence on understanding simple things, we select as our pilot instance the transition from arithmetic to elementary algebra. Moreover, to guard against possible misinterpretations, let us say that by arithmetic is meant a subject studied in elementary school.

### 3.1 *Positive Integers*

A first step is to offer some definition of the positive integers, 1, 2, 3, 4. . . .

Let us suppose an indefinite multitude of instances of 'one'. They may be anything anyone pleases, from sheep to instances of the act of counting or ordering.

Further, let us suppose as too familiar to be defined, the notions of 'one', 'plus', and 'equals'.

Then, there is an infinite series of definitions for the infinite series of positive integers, and it may be indicated symbolically by the following:

| | | | | |
|---|---|---|---|---|
| 1 | + | 1 | = | 2 |
| 2 | + | 1 | = | 3 |
| 3 | + | 1 | = | 4 |
| etc., | | etc., | | etc. . . . |

This symbolic indication may be interpreted in any of a variety of manners. It means one plus one equals two, or two is one more than one, or the second is the next after the first, or even the relations between classes of groups each with one, or two, or three, etc., members. As the acute reader will see, the one important element in the above series of definitions, is the etc., etc., etc. . . . Without it, the positive integers cannot be defined; for they are an indefinitely great multitude; and it is only in so far as some such gesture as etc., etc., etc., is really significant, that an infinite series of definitions can occur. What, then, does the etc., etc., mean? It means that an insight should have occurred. If one has had the relevant insight, if one has caught on, if one sees how the defining can go on indefinitely, no more need be said. If one has not caught on, then the poor teacher has to labour in his apostolate of the obvious. For in defining the positive integers there is no alternative to insight.

Incidentally, it may not be amiss to recall what already has been remarked, namely, that a single insight is expressed in many concepts. In the present instance, a single insight grounds an infinity of concepts.

### 3.2 *Addition Tables*

A second step will consist in making somewhat more precise the familiar notion of equality. Let us say that when equals are added to equals, the results are equal; that one is equal to one; and that, therefore, an infinite series of addition tables can be constructed.

The table for adding 2 is constructed by adding one to each side of the equations that define the positive integers. Thus,

| | |
|---|---|
| From the table | $2+1=3$ |
| Adding 1 | $2+1+1=3+1$ |
| Hence, from the table | $2+2=4.$ |

In like manner the whole table for adding 2 can be constructed. From this table, once it is constructed, there can be constructed a table for adding 3. From that table it will be possible to construct a table for adding 4. Etc., etc., etc., which again means that an insight should have occurred.

Thus, from the definitions of the positive integers and the postulate about adding equals to equals, there follows an indefinitely great deductive expansion.



### 3.3 *The Homogeneous Expansion*

A third step will be to venture into a homogeneous expansion. The familiar notion of addition is to be complemented by such further notions as multiplication, powers, subtraction, division and roots. This development, however, is to be homogeneous and by that is meant that no change is to be involved in the notions already employed.

Thus, multiplication is to mean adding a number to itself so many times, so that five by three will mean the addition of three fives. Similarly, powers are to mean that a number is multiplied by itself so many times, so that five to the third will mean five multiplied by five with the result multiplied again by five. On the other hand, subtraction, division, and roots will mean the inverse operations that bring one back to the starting-point.

By a few insights, that need not be indicated, it will be seen that tables for multiplication and for powers can be constructed from the addition tables. Similarly, tables for subtraction, division, and roots can be constructed from the tables for addition, multiplication and powers.

The homogeneous expansion constitutes a vast extension of the initial deductive expansion. It consists in introducing new operations. Its characteristic is that the new operations involve no modification of the old.

### 3.4 *The Need of a Higher Viewpoint*

A fourth step will be the discovery of the need of a higher viewpoint. This arises when the inverse operations are allowed full generality, when they are not restricted to bringing one back to one's starting-point. Then, subtraction reveals the possibility of negative numbers, division reveals the possibility of fractions, roots reveal the possibility of surds. Further, there arise questions about the meaning of operations. What is multiplication when one multiplies negative numbers or fractions or surds? What is subtraction when one subtracts a negative number? etc., etc., etc. Indeed, even the meaning of 'one' and of 'equals' becomes confused, for there are recurring decimals and it can be shown that point nine recurring is equal to one.*

| | |
|---|---|
| * Let | $x = 0{\cdot}\dot{9}$ |
| then | $10x = 9{\cdot}\dot{9}$ |
| hence | $9x = 9$ |
| and so | $x = 1.$ |

3.5 *Formulation of the Higher Viewpoint*

A fifth step will be to formulate a higher viewpoint. Distinguish

(1) rules,
(2) operations, and
(3) numbers.

Let numbers be defined implicitly by operations, so that the result of any operation will be a number and any number can be the result of an operation.

Let operations be defined implicitly by rules, so that what is done in accord with rules is an operation.

The trick will be to obtain the rules that fix the operations which fix the numbers.

The emergence of the higher viewpoint is the performance of this trick. It consists in an insight that

(1) arises upon the operations performed according to the old rules, and

(2) is expressed in the formulation of the new rules.

Let me explain. From the image of a cart-wheel we proceeded by insight to the definition of the circle. But, while the cart-wheel was imagined, the circle consists of a point and a line, neither of which can be imagined. Between the cart-wheel and the circle there is an approximation but only an approximation. Now, the transition from arithmetic to elementary algebra is the same sort of thing. For an image of the cart-wheel one substitutes the image of what may be named 'doing arithmetic'; it is a large, dynamic, virtual image that includes writing down, adding, multiplying, subtracting and dividing numbers in accord with the precepts of the homogeneous expansion. Not all of this image will be present at once, but any part of it can be present and, when one is on the alert, any part that happens to be relevant will pop into view. In this large and virtual image, then, there is to be grasped a new set of rules governing operations. The new rules will not be exactly the same as the old rules. They will be more symmetrical. They will be more exact. They will be more general. In brief, they will differ from the old much as the highly exact and symmetrical circle differs from the cart-wheel.

What are the new rules? At school the rules for fractions were generalized; rules for signs were introduced; rules for equations and for



indices were worked out. Their effect was to redefine the notions of addition, multiplication, powers, subtraction, division and roots; and the effect of the redefinitions of the operations was that numbers were generated, not merely by addition, but by any of the operations.

3.6 *Successive Higher Viewpoints*

The reader familiar with group theory will be aware that the definition of operations by rules and of numbers or, more generally, symbols by operations is a procedure that penetrates deeply into the nature of mathematics. But there is a further aspect to the matter, and it has to do with the gradual development by which one advances through intermediate stages from elementary to higher mathematics. The logical analyst can leap from the positive integers to group theory, but one cannot learn mathematics in that simple fashion. On the contrary, one has to perform, over and over, the same type of transition as occurs in advancing from arithmetic to elementary algebra.

At each stage of the process there exists a set of rules that govern operations which result in numbers. To each stage there corresponds a symbolic image of doing arithmetic, doing algebra, doing calculus. In each successive image there is the potentiality of grasping by insight a higher set of rules that will govern the operations and by them elicit the numbers or symbols of the next stage. Only in so far as a man makes his slow progress up that escalator does he become a technically competent mathematician. Without it, he may acquire a rough idea of what mathematics is about; but he will never be a master, perfectly aware of the precise meaning and the exact implications of every symbol and operation.

3.7 *The Significance of Symbolism*

The analysis also reveals the importance of an apt symbolism.

There is no doubt that, though symbols are signs chosen by convention, still some choices are highly fruitful while others are not. It is easy enough to take the square root of 1764. It is another matter to take the square root of MDCCLXIV. The development of the calculus is easily designated in using Leibniz's symbol, $dy\,dx$, for the differential coefficient, Newton's symbol, on the other hand, can be used only in a few cases and, what is worse, it does not suggest the theorems that can be established.

Why is this so? It is because mathematical operations are not merely

the logical expansion of conceptual premises. Image and question, insight and concepts, all combine. The function of the symbolism is to supply the relevant image, and the symbolism is apt inasmuch as its immanent patterns as well as the dynamic patterns of its manipulation run parallel to the rules and operations that have been grasped by insight and formulated in concepts.

The benefits of this parallelism are manifold. In the first place, the symbolism itself takes over a notable part of the solution of problems, for the symbols, complemented by habits that have become automatic, dictate what has to be done. Thus, a mathematician will work at a problem up to a point and then announce that the rest is mere routine. In the second place, the symbolism constitutes a heuristic technique; the mathematician is not content to seek his unknowns; he names them; he assigns them symbols; he writes down in equations all their properties; he knows how many equations he will need; and when he has reached that number, he can say that the rest of the solution is automatic. In the third place, the symbolism offers clues, hints, suggestions. Just as the definition of the circle was approached from the clue of the equality of the spokes, so generally insights do not come to us in their full stature; we begin from little hints, from suspicions, from possibilities; we try them out; if they lead nowhere, we drop them; if they promise success, we push them for all they are worth. But this can be done only if we chance upon the hints, the clues, the possibilities; and the effect of the apt symbolism is to reduce, if not entirely eliminate, this element of chance. Here, of course, the classical example is analytic geometry. To solve a problem by Euclidean methods, one has to stumble upon the correct construction. To solve a problem analytically, one has only to manipulate the symbols.

In the fourth place, there is the highly significant notion of invariance. An apt symbolism will endow the pattern of a mathematical expression with the totality of its meaning. Whether or not one uses the Latin, Greek or Hebrew alphabet, is a matter of no importance. The mathematical meaning of an expression resides in the distinction between constants and variables and in the signs or collocations that dictate operations of combining, multiplying, summing, differentiating, integrating, and so forth. It follows that, as long as the symbolic pattern of a mathematical expression is unchanged, its mathematical meaning is unchanged. Further, it follows that if a symbolic pattern is unchanged by any substitutions of a determinate group, then the mathematical meaning of the pattern is independent of the meaning of the substitutions.



In the fifth place, as has already been mentioned, the symbolism appropriate to any stage of mathematical development provides the image in which may be grasped by insight the rules for the next stage.

### 4. INVERSE INSIGHT

Besides direct insights, their clustering, and higher viewpoints, there exists the small but significant class of inverse insights. As direct, so also inverse insights presuppose a positive object that is presented by sense or represented by imagination. But while direct insight meets the spontaneous effort of intelligence to understand, inverse insight responds to a more subtle and critical attitude that distinguishes different degrees or levels or kinds of intelligibility. While direct insight grasps the point, or sees the solution, or comes to know the reason, inverse insight apprehends that in some fashion the point is that there is no point, or that the solution is to deny a solution, or that the reason is that the rationality of the real admits distinctions and qualifications. Finally, while the conceptual formulation of direct insight affirms a positive intelligibility though it may deny expected empirical elements, the conceptual formulation of an inverse insight affirms empirical elements only to deny an expected intelligibility.

Since the last phrase is crucial, let us attempt to elaborate it. By intelligibility is meant the content of a direct insight. It is the component that is absent from our knowledge when we do not understand and added to our knowledge inasmuch as we are understanding in the simple and straightforward manner described in the earlier sections of this chapter. Now such an intelligibility may be already reached or it may be merely expected. To deny intelligibility already reached is not the result of inverse insight; it is merely the correction of a previous direct insight, the acknowledgement of its shortcomings, the recognition that it leaves problems unsolved. But to deny an expected intelligibility is to run counter to the spontaneous anticipations of human intelligence; it is to find fault not with answers but with questions. In a demonstrative science it is to prove that a question of a given type cannot be answered. In an empirical science it is to put forward a successful hypothesis or theory that assumes that certain questions mistakenly are supposed to require an answer. Finally, the occurrence of an inverse insight is not established by the mere presence of negative concepts: thus,

'not-red', 'position without magnitude', 'non-occurrence' exclude respectively 'red', 'magnitude', 'occurrence'; but the latter terms refer to empirical components in our knowledge and not to the possibilities and necessities, the unifications and relations, that constitute the intelligibility known in direct insight.

While the general notion of inverse insight is fairly simple and obvious, I have been at some pains in presenting its characteristics because it is not too easy to set forth illustrations to the satisfaction of different groups of readers. Moreover, communication and discussion take place through concepts, but all insight lies behind the conceptual scene. Hence, while there is always the danger that a reader will attend to the concepts rather than the underlying insight, this danger is augmented considerably when the point to be grasped by insight is merely that there is no point. To make matters worse, inverse insights occur only in the context of far larger developments of human thought. A statement of their content has to call upon the later systems that positively exploited their negative contribution. The very success of such later systems tends to engender a routine that eliminates the more spontaneous anticipations of intelligence and then, to establish a key feature of an inverse insight, it may be necessary to appeal to the often ambiguous witness of history. In the midst of such complexity it very easily can happen that a reader's spontaneous expectation of an intelligibility to be reached should outweigh mere verbal admonitions to the contrary and, when that occurs, illustrations of inverse insight can become very obscure indeed. Accordingly, while there is nothing difficult about the examples to follow, I have thought it wise to indulge in a preliminary elucidation of the obvious.

As a first example of inverse insight we shall take what the ancients named incommensurable magnitudes and the moderns call irrational numbers. In both cases there is a positive object indicated by the terms 'magnitude', 'number'. In both cases there is a negative element indicated by the epithets, 'incommensurable', 'irrational'. Finally, in both cases the negation bears on the spontaneous anticipations of human intelligence. 'Incommensurable' denies the possibility of applying to certain magnitudes some type of measurement and Aristotle viewed this denial as *prima facie* a matter of high surprise. Even more emphatically 'irrational' denies a correspondence between certain numbers and human reason.

To indicate the relevant insight, let us ask why a surd is a surd.



Essentially the question is parallel to the earlier question, Why is a cartwheel round? But while the earlier answer revealed an intelligibility immanent in the wheel, the present answer consists in showing that a surd cannot possess the intelligibility one would expect it to have.

Thus, the square root of two is some magnitude greater than unity and less than two. One would expect it to be some improper fraction, say $m/n$, where $m$ and $n$ are positive integers and, by the removal of all common factors, $m$ may always be made prime to $n$. Moreover, were this expectation correct, then the diagonal and the side of a square would be respectively $m$ times and $n$ times some common unit of length. However, so far from being correct, the expectation leads to a contradiction. For if $\sqrt{2}=m/n$, then $2=m^2/n^2$. But if $m$ is prime to $n$, then $m^2$ is prime to $n^2$; and in that case $m^2/n^2$ cannot be equal to 2 or, indeed, to any greater integer. The argument is easily generalized and so it appears that a surd is a surd because it is not the rational fraction that intelligence anticipates it to be.

A second example of inverse insight is the non-countable multitude. There is a positive object, 'multitude'. There is a negative determination, 'non-countable'. Moreover, when 'countable' is taken so broadly that all integers, all rational numbers, even all real algebraic numbers* demonstrably are countable multitudes, when further it can be shown that to remove a countable multitude from a non-countable multitude leaves a non-countable multitude, one spontaneously anticipates that the numbers between zero and unity must be a countable multitude. In fact, it can be shown that the infinite decimals are a non-countable multitude, so that the algebraic fractions from zero to unity must be a negligible portion of the numbers in that interval.*

For a third example we turn to empirical science and consider the surprising part of Newton's first law of motion, namely, that a body continues in its existing state of uniform motion in a straight line unless that state is changed by external force.

In this statement and its context it is not too difficult to discern the three characteristics of the formulation of an inverse insight. For there is the positive object: a body continues to move at a uniform rate in a straight line. There is a negation: the continuance of the constant velocity depends not on the action of external force but on the absence

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of such action; for only as long as there is no acceleration, does the velocity remain constant; and the moment the sum of the external forces differs from zero, there arises an acceleration. Finally, this negation of external force runs counter to the spontaneous anticipations of human intelligence, for spontaneously one thinks of uniform motion not as of a state like rest but as of a change that requires an external cause.

However, some readers may wish to refine on the issue. They will agree that the necessity of an external cause had been stressed by the Aristotelian theory of celestial movements, of projectiles, and of motion in a vacuum. But they will add that the Aristotelian view had been contradicted at least from the time of John Philoponus. On this contrary view projectiles were kept in motion not by any external force but by some internal principle or power or property or quality or other immanent ground. Finally, they will ask whether it is quite certain that Newton did not appeal to some innate power of matter to account for the continuance of inertial states.

Now, clearly, Newtonian exegesis is not our present business. All we have to say is that inverse insight is not illustrated when explanation by external force is replaced by explanation in terms of some immanent power or property. For in that case there is merely the correction of an earlier direct insight by a later direct insight and, while the spontaneous anticipations of human intelligence are blocked in one direction, they are given an outlet in another.

Still for purposes of illustration it may be permissible to block this second outlet without reopening the first. No doubt, when an external mover or force is denied, one may spontaneously think that there must be some innate quality that provides the real explanation. But while the assertion of an external mover or force can be tested experimentally, the assertion of some innate quality, of some *vis materiae insita*, can hardly be regarded as a scientific statement. If one affirms that, when acceleration is zero, then the sum of the relevant external forces is also zero, one's affirmation admits the ordinary tests. But if one goes on to add that the innate qualities of matter render the action of external forces superfluous, one is very likely to be reminded that scientists do not appeal to occult causes.

Now if this remonstrance is regarded as peremptory, we arrive at an example of inverse insight. There is the positive object of inquiry: bodies continue in their existing states of uniform motion. There is the



negation: the continuance of uniform motion is not to be explained by any appeal to external forces. Finally, this negation is regarded as definitive for science, for science refuses to extrapolate from known laws to ulterior explanations in terms of vague qualities, properties, powers, and the like.

A fourth example of inverse insight may be derived from the basic postulate of the Special Theory of Relativity. The postulate itself is that the mathematical expression of physical principles and laws is invariant under inertial transformations. To reach our illustration we have only to grasp the concrete meaning of the postulate whenever it is invoked by a physicist engaged in understanding any set of physical data.

For then the positive object of inquiry consists in the data inasmuch as they are considered

(1) as referred to initial axes of co-ordinates, say $K$, and
(2) as referred to other axes, say $K'$, moving with a constant velocity relative to the axes, $K$.

The negative element in the conception of the positive object is indicated by the word 'invariant'. It means that the transformation from one set of axes to another does not lead to any modification in the form of the mathematical expression of the appropriate physical principles and laws. But when the form of the mathematical expression undergoes no change, there is no change in the intelligibility that is expressed mathematically. When there is no change in the intelligibility, there is no change in the act of understanding that grasps the intelligibility and expresses it mathematically. Accordingly, the concrete meaning of the postulate is that, though there is a difference in the spatio-temporal standpoint from which the data are considered, still there is no difference in the act of understanding the data, no difference in the general intelligibility grasped in the data, and no difference in the form of the mathematical expression of the intelligibility.

Finally, it is quite common for there to exist differences either in data or in spatio-temporal standpoint without any corresponding difference in the act of understanding. But in most of such cases there is no occasion for an inverse insight since, while the empirical difference is assigned no intelligible counterpart, still no one expects that really there must be an intelligible counterpart. Thus, there is a notable empirical difference between large and small circles, yet no one expects different definitions of large circles and of small circles or different theorems to

establish the different properties of large and small circles. However, while similar instances are very numerous, the invariance postulated by Special Relativity is not among them. For that invariance implies a drastic revision of ordinary notions of space and of time, and against any such revision the spontaneous anticipations of human intelligence vigorously rebel.

Hence, to recapitulate the main point, when the basic postulate of Special Relativity is interpreted concretely in terms of

(1) the data physicists consider,
(2) the insights they enjoy, and
(3) the form of the mathematical expression of the principles and laws reached by the insights,

there arises the following explanatory syllogism:

> When there is no difference in a physicist's insights, there should be no difference in the form of the mathematical expression of physical principles and laws.
>
> But when an inertial transformation occurs, there is no difference in a physicist's insights.
>
> Therefore, when an inertial transformation occurs, there should be no difference in the form of the mathematical expression of physical principles and laws.

The major premise postulates a correspondence between the insights of physicists and the form of the mathematical expression of physical principles and laws; in other words, it requires that the content of acts of understanding be reflected faithfully by the form of mathematical expressions. The minor premise contains our inverse insight: it denies a difference in insight that corresponds to the difference of an inertial transformation; in other words, it asserts for the whole of physics the defect of intelligibility in constant velocity that Newton asserted for mechanics by his first law of motion. The conclusion, finally, is true if the premises are true but, while the major premise may be regarded as a mere methodological rule, the minor premise is an assertion of empirical science and can be established only through the method of hypothesis and verification.

In conclusion, let us recall a point already mentioned. An inverse insight finds its expression only in some concomitant positive context. So the defect of intelligibility in constant velocity has been formulated in a whole series of different contexts. In the context of Eleatic philosophy Zeno's paradoxes led to a denial of the fact of motion. In the context of his philosophy of being Aristotle pronounced motion real yet regarded it as an incomplete entity, an infra-categorical object. In the context of mathematical mechanics Newton asserted a principle of inertia. In the context of Clerk-Maxwell's equations for the electromagnetic field, Lorentz worked out the conditions under which the equations would remain invariant under inertial transformations; Fitzgerald explained Lorentz's success by supposing that bodies contracted along the direction of motion; Einstein found a no less general explanation in problems of synchronization and raised the issue to the methodological level of the transformation properties of the mathematical expression of physical principles and laws; finally, Minkowski systematized Einstein's position by introducing the four-dimensional manifold. No doubt, it would be a mistake to suppose that the same inverse insight was operative from Zeno to Special Relativity. But throughout there is a denial of intelligibility to local motion and, while the successive contexts differ notably in content and in value, at least they point in the same direction and they illustrate the dependence of inverse insight on concomitant direct insights.



### 5. THE EMPIRICAL RESIDUE

If inverse insights are relatively rare, they are far from being unimportant. Not only do they eliminate mistaken questions but also they seem regularly to be connected with ideas or principles or methods or techniques of quite exceptional significance. From the oddities of the mathematical continuum through the notions of correlation and limit there arises the brilliance of continuous functions and of the infinitesimal calculus. Similarly the lack of intelligibility in constant velocity is linked with scientific achievements of the first order: the principle of inertia made it possible to conceive dynamics not as a theory of motions but as an enormously more compact and more powerful theory of accelerations; and the invariance of physical principles and laws under inertial transformations not only is an extremely neat idea but also has kept revealing its fruitfulness for the past fifty years.

To explore this significance, then, let us introduce the notion of an empirical residue that

(1) consists in positive empirical data,
(2) is to be denied any immanent intelligibility of its own, and

(3) is connected with some compensating higher intelligibility of notable importance.

In clarification of the first characteristic one may note that, inasmuch as a vacuum is merely an absence of data, it cannot be part of the empirical residue. In clarification of the second it is to be remembered that a denial of immanent intelligibility is not a denial of experience or description. Not only are elements in the empirical residue given positively but also they are pointed out, conceived, named, considered, discussed, and affirmed or denied. But though they are no less given than colour or sound or heat, though they may be thought about no less accurately and talked about no less fluently, still they are not objects of any direct insight and so they cannot be explained by transverse waves or longitudinal waves or molecular motion or any other theoretical construction that might be thought more apposite. Finally, in clarification of the third characteristic it is to be noted that inverse insight and the empirical residue are not exact correlatives. For inverse insight was not characterized by a connection with ideas, principles, methods, or techniques of exceptional significance. Again, the empirical residue has not been characterized by the spontaneity of the questions for intelligence that are to be met by a denial of intelligibility.

This difference not only makes the empirical residue a broader category than inverse insight but also renders a discussion of it more difficult. For a great part of the difficulty in discovering the further positive aspects of experience that are to be denied intelligibility is that no one supposes them to possess intelligibility.

Thus, particular places and particular times pertain to the empirical residue. They are positive aspects of experience. Each differs from every other. But because no one ever asks why one place is not another or why one time is not another, people are apt to be puzzled when the question is put, to imagine that something different from such obvious foolishness must be meant, and to experience a variety of fictitious difficulties before arriving at the simple conclusion that

(1) particular places and particular times differ as a matter of fact, and

(2) there is no immanent intelligibility to be grasped by direct insight into that fact.

For example, one will begin by saying that obviously the position, *A*, differs from the position, *B*, because of the distance, *AB*, that separates



them. But take three equidistant positions, *A*, *B*, *C*. Why are the distances, *AB*, *BC*, *CA*, different? One would be in a vicious circle if one doubled back and explained the difference of the distances by the difference of the positions. One cannot say that the distances differ in length for they are equal in length. But one may say that the distances differ because the directions differ. Still, why do the directions differ? And why are equal and parallel distances different distances? Now, perhaps, it will be urged that we are going too far, that some difference must be acknowledged as primitive, that everything cannot be explained. Quite so, but there is a corollary to be added. For what is primitive is not the content of some primitive insight but the content of some primitive experience to which no insight corresponds. Were it the content of some primitive insight, there would not be the conspicuous absence of a clear-headed explanation. But because the difference of particular places and the difference of particular times are given prior to any questioning and prior to any insight, because these given differences cannot be matched by any insights that explain why places differ and times differ, there has to be introduced the category of the empirical residue.

However, one may not surrender yet. For particular places and particular times can be united by reference frames; the frames can be employed to distinguish and designate every place and every time; and evidently such constructions are eminently intelligent and eminently intelligible. Now, no doubt, reference frames are objects of direct insight, but what is grasped by that insight is an ordering of differences that are not explained by the order but merely presupposed. So it is that different geometries grasped by different insights offer different intelligible orders for the differences in place or time that all equally presuppose and, quite correctly, none attempt to explain.

There is a further aspect to the matter. Because the differences of particular places and particular times involve no immanent intelligibility of their own, they do not involve any modification in the intelligibility of anything else. It is not mere difference of place but something different in the places that gives rise to different observations or different experimental results in different places. Similarly, it is not mere difference in time but something different at the time that gives rise to different observations and different experimental results at different times. Moreover, were that not so, every place and every time would have its own physics, its own chemistry, its own biology; and since a

science cannot be worked out instantaneously in a single place, there would be no physics, no chemistry, and no biology. Conversely, because the differences of particular places and particular times pertain to the empirical residue, there exists the powerful technique of scientific collaboration. Scientists of every place and every time can pool their results in a common fund, and there is no discrimination against any result merely because of the place or merely because of the time of its origin.

Even more fundamental than scientific collaboration is scientific generalization. When chemists have mastered all of the elements, their isotopes and their compounds, they may forget to be grateful that they do not have to discover different explanations for each of the hydrogen atoms which, it seems, make up about fifty-five per cent of the matter of our universe. But at least the fact that such a myriad of explanations is not needed is very relevant to our purpose. Every chemical element and every compound differs from every other kind of element or compound and all the differences have to be explained. Every hydrogen atom differs from every other hydrogen atom and no explanation is needed. Clearly, we have to do with another aspect of the empirical residue and, no less clearly, this aspect is coupled with the most powerful of all scientific techniques, generalization.

However, this issue has been booted about by philosophers ever since the Platonists explained the universality of mathematical and scientific knowledge by postulating eternal and immutable Forms or Ideas only to find themselves embarrassed by the fact that a single, eternal, immutable One could hardly ground the universal statement that one and one are two or, again, that a single, eternal, immutable Triangle would not suffice for theorems on triangles similar in all respects. So there arose, it seems, the philosophic problem of merely numerical difference and, connected with it, there have been formulated cognitional theories based on a doctrine of abstraction. Accordingly, we are constrained to say something on these issues and, lest we appear to be attempting to dilute water, we shall do so as briefly as possible.

The assertion, then, of merely numerical difference involves two elements. On the theoretical side it is the claim that, when any set of data has been explained completely, another set of data similar in all respects would not call for a different explanation. On the factual side it is the claim that, when any set of data has been explained completely, only an exhaustive tour of inspection could establish that there does not exist another set of data similar in all respects.



The basis of the theoretical contention is that, just as the same act of understanding is repeated when the same set of data is apprehended a second time, so also the same act of understanding is repeated when one apprehends a second set of data that is similar to a first in all respects. Thus, the physicist offers different explanations for 'red' and 'blue'; he offers different explanations for different shades of 'red'; and he would discern no sense in the proposal that he should try to find as many different explanations as there are different instances of exactly the same shade of exactly the same colour.

The factual contention is more complex. It is not an assertion that there exist different sets of data similar in all respects. It is not a denial of unique instances, i.e., of instances that are to be explained in a manner in which no other instance in the universe is to be explained. It is not even a denial that every individual in the universe is a unique instance. On the contrary, the relevant fact lies in the nature of the explanations that are applicable to our universe. It is to the effect that all such explanations are made up of general or universal elements and that, while these general or universal elements may be combined in such a manner that every individual is explained by a different combination of elements, still such a combination is an explanation of a singular combination of common properties and not an explanation of individuality. For if the individuality of the individual were explained, it would be meaningless to suppose that some other individual might be understood in exactly the same fashion. On the other hand, because the individuality of the individual is not explained, it is only an exhaustive tour of inspection that can settle whether or not there exists another individual similar in all respects. Hence, even if there were reached a single comprehensive theory of evolution that explained and explained differently every instance of life on this planet, still in strict logic we should have to inspect all other planets before we could be absolutely certain that in fact there did not exist another instance of evolution similar in all respects.

In brief, individuals differ, but the ultimate difference in our universe is a matter of fact to which there corresponds nothing to be grasped by direct insight. Moreover, as scientific collaboration rests on the empirically residual difference of particular places and of particular times, so scientific generalization rests on the empirically residual difference between individuals of the same class. Just what the lowest class is, has to be discovered by scientific advance in direct insight. Even if it

should prove that in some sense there are as many classes as individuals, still we can know at once that that sense is not that the individuality of individuals is understood but merely that singular combinations of universal explanatory elements may be set in correspondence with singular combinations of common properties or aspects in each individual. For the content grasped in insight can be embodied no less in imagination than in sense; and whether there is more than one instance in sense, can be settled only by an empirical tour of inspection.

Later we shall direct attention to further aspects of the empirical residue, for there exists a statistical method that rests on the empirically residual character of coincidental aggregates of events, and there is a dialectical method that is necessitated by the lack of intelligibility in man's unintelligent opinions, choices and conduct. But perhaps enough has been said for the general notion to be clear, and so we turn to the allied topic of abstraction.

Properly, then, abstraction is not a matter of apprehending a sensible or imaginative *Gestalt;* it is not a matter of employing common names just as it is not a matter of using other tools; finally, it is not even a matter of attending to one question at a time and, meanwhile, holding other questions in abeyance. Properly, to abstract is to grasp the essential and to disregard the incidental, to see what is significant and set aside the irrelevant, to recognize the important as important and the negligible as negligible. Moreover, when it is asked what is essential or significant or important and what is incidental, irrelevant, negligible, the answer must be twofold. For abstraction is the selectivity of intelligence, and intelligence may be considered either in some given stage of development or at the term of development when some science or group of sciences has been mastered completely.

Hence, relative to any given insight or cluster of insights the essential, significant, important consists

(1) in the set of aspects in the data necessary for the occurrence of the insight or insights, or

(2) in the set of related concepts necessary for the expression of the insight or insights.

On the other hand the incidental, irrelevant, negligible consists

(1) in other concomitant aspects of the data that do not fall under the insight or insights, or



(2) in the set of concepts that correspond to the merely concomitant aspects of the data.

Again, relative to the full development of a science or group of allied sciences, the essential, significant, important consists

(1) in the aspects of the data that are necessary for the occurrence of all insights in the appropriate range, or

(2) in the set of related concepts that express all the insights of the science or sciences.

On the other hand, the incidental, irrelevant, negligible consists in the empirical residue that, since it possesses no immanent intelligibility of its own, is left over without explanation even when a science or group of sciences reaches full development.

Finally, to conclude this chapter on the Elements of Insight, let us indicate briefly what is essential, significant, important in its contents and, on the other hand, what is incidental, irrelevant, negligible. What alone is essential is insight into insight. Hence, the incidental includes

(1) the particular insights chosen as examples,

(2) the formulation of these insights, and

(3) the images evoked by the formulation.

It follows that for the story of Archimedes the reader will profitably substitute some less resounding yet more helpful experience of his own. Instead of the definition of the circle he can take any other intelligently performed act of defining and ask why the performance is not safe, not accurate, not the accepted terminology, but a creative stroke of insight. Instead of the transition from elementary arithmetic to elementary algebra one may review the process from Euclidean to Riemannian geometry. Instead of asking why surds are surds, one can ask why transcendental numbers are transcendental. Similarly, one can ask whether the principle of inertia implies that Newton's laws are invariant under inertial transformations, what inspired Lorentz to suppose that the electromagnetic equations should be invariant under inertial transformations, whether an inverse insight accounts for the basic postulate of General Relativity, whether the differences of particular places or particular times are the same aspect of the empirical residue as the differences of completely similar hydrogen atoms. For just as in any

subject one comes to master the essentials by varying the incidentals, so one reaches familiarity with the notion of insight by modifying the illustrations and discovering for oneself and in one's own terms the point that another attempts to put in terms he happens to think will convey the idea to a probably non-existent average reader.

CHAPTER II

# HEURISTIC STRUCTURES OF EMPIRICAL METHOD

In the previous chapter insight was examined in a static fashion. It was related to inquiry, to images, to empirical data, and to different types of positive and negative explanatory concepts. But if a set of fundamental notions has been introduced, no effort has been made to capture the essential dynamism of human intelligence. Now a first move must be made in this direction and, as empirical science is conspicuously and methodically dynamic, it will be well to begin by outlining the similarities and dissimilarities of mathematical and scientific insights.

## 1.1 *Similarities of Mathematical and Scientific Insights*

Galileo's determination of the law of falling bodies not only is a model of scientific procedure but also offers the attraction of possessing many notable similarities to the already examined process from the image of a cart-wheel to the definition of the circle.

In the first place, the inquiry was restricted to the immanent intelligibility of a free fall. Just as we ruled out of consideration the purpose of cart-wheels, the materials from which they are made, the wheelwrights that make them, and the tools that wheelwrights use, so also Galileo was uninterested in the final cause of falling, he drew no distinction between the different materials that fall, he made no effort to determine what agencies produce a fall.

Secondly, just as we started from a clue, the equality of the spokes, so too Galileo supposed that some correlation was to be found between the measurable aspects of falling bodies. Indeed, he began by showing the error in the ancient, Aristotelian correlation that bodies fell according to their weight. Then he turned his attention to two measurable aspects immanent in every fall; the body traverses a determinate distance; it does so in a determinate interval of time. By a series of experiments he provided himself with the requisite data and obtained the desired measurements. Then he discovered that the measurements would satisfy a general rule; the distance traversed is proportional to

the time squared. It is a correlation that has been verified directly and indirectly for over four centuries.

Thirdly, once we had defined the circle, we found ourselves in a realm of the non-imaginable, of the merely supposed. Strangely, something similar happens when one formulates the law of falling bodies. It holds in a vacuum, and to realize a perfect vacuum is impossible. What can be established experimentally is that the more closely one approximates to the conditions of a vacuum, the more accurate the law of constant acceleration is found to be.

### 1.2 *Dissimilarities*

But besides similarities, there also are differences and these are perhaps more instructive. In reaching the definition of the circle, it was sufficient to take as our starting-point the mere image of a cart-wheel. There was no need for field-work. But to reach the law of falling bodies, Galileo had to experiment. Climbing the tower of Pisa and constructing inclined planes were an essential part of his job, for he was out to understand, not how bodies are imagined to fall, but how in fact they fall.

Secondly, the data that give rise to insight into roundness are continuous, but the data that give rise to insight into the law of falling bodies are discontinuous. One can imagine the whole cart-wheel or a whole loop of very fine wire. But no matter how many experiments one makes on falling bodies, all one can obtain is a series of separate points plotted on a distance-time graph. No doubt, it is possible to join the plotted points by a smooth curve, but the curve represents, not data that are known, but a presumption of what understanding will grasp.

Thirdly, the insight into the image of the wheel grasps necessity and impossibility: if the radii are equal, the curve must be round; if the radii drawn from the centre are unequal, the curve cannot be round. But the insight into the discontinuous series of points on the graph consists in a grasp, not of necessity or impossibility, but simply of possibility. The simplest smooth curve could represent the law of falling bodies. But any of a vast range of more elaborate curves could equally well pass through all the known points.

Fourthly, once one catches on to the law of the circle, the insight and consequent definition exert a backward influence upon imagination. The geometer imagines dots but thinks of points; he imagines fine threads, but thinks of lines. The thinking is exact and precise, and imagination does its best to keep pace. In like manner the empirical investigator will tend to endow his images with the closest possible approximation to the laws he conceives. But while his imagination will do its best, while his perceptions will be profoundly influenced by the habits of his imagination, none the less, the data that are available for the ideal observer make no effort towards such conformity. They go their own way with their unanalysed multiplicity and their refractoriness to measurements that are more than approximate.



Fifthly, as we have seen, higher viewpoints in mathematics are reached inasmuch as initial images yield insights, insights yield definitions and postulates, definitions and postulates guide symbolic operations, and symbolic operations provide a more general image in which the insights of the higher viewpoint are emergent. Now in empirical method, there is a similar circle but it follows a slightly different route. The operations that follow upon the formulation of laws are not merely symbolic. For the formulation expresses a grasp of possibility. It is a hypothesis. It provides a basis for deductions and calculations no less than mathematical premises. But it also provides a basis for further observations and experiments. It is such observation and experimentation, directed by a hypothesis, that sooner or later turns attention to data that initially were overlooked or neglected; it is attention to such further data that forces the revision of initial viewpoints and effects the development of empirical science.

The circuit, then, of mathematical development may be named immanent; it moves from images through insights and conceptions to the production of symbolic images whence higher insights arise. But the circuit of scientific development includes action upon external things; it moves from observation and experiment to tabulations and graphs, from these to insights and formulations, from formulations to forecasts, from forecasts to operations, in which it obtains fresh evidence either for the confirmation or for the revision of existing views.

## 2. CLASSICAL HEURISTIC STRUCTURES

In one respect this brief sketch must be completed at once. Quite airily, we have spoken of the initial clue. But just what is it? Where does it come from? Is it mere guess-work? One can be led on quite naturally to the definition of the circle, if one begins from a suspicion that a cart-wheel is round because its spokes are equal. Similarly, one

can proceed in intelligible fashion to the determination of the law of falling bodies, provided one presumes initially that the law will be a correlation of measurable aspects of a free fall. But this only makes the origin of the clue or hint or suggestion or presumption all the more significant.

### 2.1 *An Illustration from Algebra*

With another bow, then, to Descartes' insistence on understanding extremely simple things, let us examine the algebraist's peculiar habit of solving problems by announcing: Let $x$ be the required number.

Thus, suppose that the problem is to determine when first after three o'clock the minute hand exactly covers the hour hand. Then, one writes down: Let $x$ be the number of minutes after three o'clock. Secondly, one infers that while the minute hand moves over $x$ minutes, the hour hand moves over $x/12$ minutes. Thirdly, one observes that at three o'clock the hour hand has a 15 minute start. Hence,

$$x = \frac{x}{12} + 15 = 16\frac{4}{11}$$

The procedure consists in

(1) giving the unknown a name or symbol,
(2) inferring the properties and relations of the unknown,
(3) grasping the possibility of combining these properties and relations to form an equation, and
(4) solving the equation.*

### 2.2 *'Nature'*

Now let us generalize.

In every empirical inquiry there are knowns and unknowns. But the knowns are apprehended whether or not one understands; they are the data of sense. The unknowns, on the other hand, are what one will grasp by insight and formulate in conceptions and suppositions.

Accordingly, let us bestow a name upon the unknown. Rather, let us advert to the fact that already it has been named. For what is to be known by understanding these data is called their *nature*. Just as in algebra the unknown number is $x$, until one finds out what the number

* Because insight is into the presentations of sense or the representations of imagination, the third step in the solution of such problems is facilitated by drawing a diagram and marking all relevant quantities. In the present instance the equation becomes evident on inspection when one has marked the three distances, $x$, $x/12$ and 15.



is, so too in empirical inquiry, the unknown to be reached by insight is named 'the nature of . . .'. Once Galileo discovered his law, he knew that the nature of a free fall was a constant acceleration. But before he discovered the law, from the mere fact that he inquired, he knew that a free fall possessed a nature, though he did not know what that nature was.

The first step in the generalization is, then, that just as the mathematician begins by saying, Let the required number be $x$, so too the empirical inquirer begins by saying, Let the unknown be the nature of . . .

### 2.3 *Classification and Correlation*

Next, similars are similarly understood.

Hence, because individuality pertains to the empirical residue, one knows at once that the 'nature of . . .' will be universal, that when one understands these data, then one will understand similar data in exactly the same fashion.

Accordingly, just as the mathematician follows up his naming of the unknown as $x$ by writing down properties of $x$, so too the empirical inquirer follows up his declaration that he seeks the 'nature of . . .' by noting that that 'nature of . . .' must be the same for all similar sets of data.

But the similarities are of two kinds.

There are the similarities of things in their relations to us. Thus, they may be similar in colour or shape, similar in the sounds they emit, similar in taste or odour, similar in the tactile qualities of the hot and cold, wet and dry, heavy and light, rough and smooth, hard and soft.

There also are the similarities of things in their relations to one another. Thus, they may be found together or apart. They may increase or decrease concomitantly. They may have similar antecedents or consequents. They may be similar in their proportions to one another, and such proportions may form series of relationships, such as exist between the elements in the periodic table of chemistry or between the successive forms of life in the theory of evolution.

Now sensible similarities, which occur in the relations of things to our senses, may be known before the 'nature of . . .' has been discovered. They form the basis of preliminary classifications. They specify the 'nature of . . .', so that one states that one is seeking the nature of colour, the nature of heat, the nature of change, the nature of life.

On the other hand, similarities that reside in the relations of things to one another are the proximate materials of insight into nature. Hence, the empirical inquirer, to emphasize this fact, will say that his objective is not merely the 'nature of . . .' but more precisely, the unspecified correlation to be specified, the undetermined function to be determined.

The second step in the generalization is, then, that just as the mathematician states that he seeks an $x$ which has such and such properties, so too the empirical inquirer states that he seeks a 'nature of . . .' where the nature antecedently is specified by a classification based on sensible similarity and consequently will be known when some indeterminate function is determined.

The reader will observe that Galileo differed from his Aristotelian opponents by taking this second step. The Aristotelians were content to talk about the nature of light, the nature of heat, etc. Galileo inaugurated modern science by insisting that the nature of weight was not enough; from sensible similarity, which resides in the relations of things to our senses, one must proceed to relations that hold directly between things themselves.

### 2.4 *Differential Equations*

Now the correlations and functions that relate things directly to one another are determined empirically by measuring, plotting measurements on graphs, and grasping in the scattered points the possibility of a smooth curve, a law, a formulation. But our present concern is with the antecedent, heuristic clues. Accordingly, we recall that, besides individuality, the continuum also pertains to the empirical residue and, as well, that just as the universal is reached by abstracting from the individual, so also the techniques of the infinitesimal calculus deal with the intelligibility reached by abstracting from the non-countable infinity of the continuum.

The third step, then, in our generalization is the observation that, where the mathematician says, Let $x$ be the required number, the empirical inquirer can say, Let some indeterminate function, $f(x, y, z, \ldots)=0$, be the required function. Further, just as the mathematician reaches $x$ by making statements about it, so too the empirical inquirer can move towards the determination of his indeterminate function by writing down differential equations which it must satisfy.

This procedure is named by Lindsay and Margenau in their *Foundations of Physics*, the 'Method of Elementary Abstraction'. They illustrate it by examining the general features of a fluid in motion. Thus, if the fluid is continuous, then, at every point in the fluid there will be the velocity components, $u$, $v$, $w$, and a density, $r$. If the fluid is not vanishing into gas, then the excess rate of outflow over inflow with respect to any infinitesimal volume will equal the rate of decrease of density in that volume. Hence, there may be derived the equation:

$$\partial(ru)/\partial x+\partial(rv)/\partial y+\partial(rw)/\partial z=-\partial r/\partial t.$$

Further, if the motion is only in one direction, two of the terms on the left-hand side vanish. If the fluid is incompressible so that the density does not vary in time, the term on the right-hand side becomes zero. If the fluid is also homogeneous, so that the density does not vary in space, then the density, $r$, vanishes from the expressions on the left-hand side. Finally, if the velocity components, $u$, $v$, $w$, are equal to the first partial derivatives of some function of the co-ordinates, $x$, $y$, $z$, there arises Laplace's equation.

The foregoing equation of continuity can be combined with other equations based on similarly general considerations. Thus, by shifting from velocity and density to acceleration and pressure, three further differential equations can be obtained. By adding suitable assumptions and restrictions, there can be worked out the differential equation of a wave motion.*

What is happening? Consider the algebraic procedure that we are generalizing and observe the isomorphism. Where before we said, Let $x$ be the required number, now we say, Let the function, $f(x, y, z, t)=0$, be the required correlation. Where before we noted that, while the minute hand moves over $x$ minutes, the hour hand moves over $x/12$ minutes, now we work out a differential equation that expresses mathematically certain very general features of the data. Where before we appealed to the fact that at three o'clock the hour hand had a fifteen-minute start, now we turn our attention to the boundary conditions that restrict the range of functions satisfying the differential equation.

### 2.5 *Invariance*

Though a less inadequate account of the notion of invariance will be attempted in examining the notions of Space and Time in Chapter V,

* See Lindsay and Margenau, pp. 29 ff.

at least some mention of it should be made in the present outline of scientific clues and anticipations. Accordingly, we recall that the differences of particular places and particular times pertain to the empirical residue and, for that reason, not only are scientific discoveries independent of the place and time of their origin but also they can claim to be equally and uniformly valid irrespective of merely spatio-temporal differences. Hence, for example, the formulae for chemical compounds not only have the same intelligibility and meaning but also exactly the same symbolic representation no matter what the place or time. However, physical principles and laws are involved in a difficulty. For they regard motions of one kind or another; motions are changes in place and time; places and times lead to reference frames constructed to include and designate all points and instants relatively to a particular origin and orientation. It follows that if physical principles and laws refer to motions, they also refer to the particular origin and orientation of some particular reference frame and, unless a special effort is made, change in the choice of reference frame may result in change in the statement of the principle or law. On the other hand, when a special effort is made, the mathematical expression of physical principles and laws undergoes no change in form despite changes in spatio-temporal standpoint and then the mathematical expression is said to be invariant under some specified group of transformations.

Briefly, then, the meaning of invariance is that

(1) all scientists expect their correlations and laws to be independent of merely spatio-temporal differences,

(2) physicists are confronted with a special difficulty inasmuch as they have to use reference frames, and

(3) physicists surmount their peculiar difficulty by expressing their principles and laws in mathematical equations that remain invariant under transformations of frames of reference.

However, to determine under which group of transformations invariance is to be achieved, some further principle has to be invoked and, in fact, in different scientific theories different principles are invoked Of these the most general is the principle of equivalence which asserts that physical principles and laws are the same for all observers. Now at first sight this statement seems ambiguous. Does it mean that physical objects look the same from all observational standpoints? Or does it mean that physical principles and laws are simply and completely outside the range of seeing, hearing, touching, feeling, and all other direct and indirect acts of observing?



While some writers seem to favour the former view, there can be little doubt about Einstein's position. Moreover, that position follows quite plausibly from the premise that empirical science seeks not the relations of things to our senses but their relations to one another. For, as has been remarked, observations give way to measurements; measurements relate things to one another rather than to our senses; and it is only the more remote relations of measurements to one another that lead to empirical correlations, functions, laws. Now clearly if laws are reached by eliminating the relations of things to the senses of observers and by arriving at relations between the measured relations of things to one another, then there exists an extremely solid foundation for the affirmation that principles and laws are the same for all observers because they lie simply and completely outside the range of observational activities. It is, for example, not the appearance of colours but the general explanation in terms of wave-lengths of light that is exactly the same no matter what may be the state of observers' eyes, the lighting by which they see, or the speed with which they may happen to be in relative motion.

Hence, if physical principles and laws are independent of any movement of observers, they should be equally independent of any similar movement of reference frames. But observers may be moving with any linear or angular velocity provided the motion is continuous and provided it involves no excursions into the imaginary sections of a manifold constructed by introducing complex numbers. It follows that physical principles and laws should be independent of similar movements of reference frames. Accordingly, by the principle of equivalence the mathematical expression of physical principles and laws is to be expected to be invariant as long as transformation equations are continuous functions of real variables.

To implement this conclusion, which is no more than a general anticipation based on cognitional theory, two further steps are required. First, the broad invariance that we have described has to be conceived precisely in terms of tensors. Secondly, appropriate empirical hypotheses have to be formulated and verified. But by those steps there are reached the General Theory of Relativity and the Generalized Theory of Gravitation and incidentally it may not be amiss to note that our remote anticipation offers a simple explanation for certain aspects of

those theories. For what was anticipated was a non-relatedness of abstract laws to observers. It follows that the consequences of the anticipation should not be verified

(1) if the laws lose their abstract character through particularization,* or

(2) if investigation concentrates on the frequencies of concrete events accessible to observers as seems to be the case in Quantum Mechanics.

A less general anticipation of invariance is contained in the basic postulate of Special Relativity. Already in illustrating inverse insight we have had occasion to put this postulate in the form of an explanatory syllogism in which the major premise expressed an anticipation of invariance and the minor premise enounced the defect of intelligibility in inertial transformations. On the present analysis, then, the difference between the anticipations represented respectively by General and by Special Relativity is that, while both expect invariant mathematical expression to result from the abstractness of principles and laws, General Relativity implements this expectation by invoking a direct insight into the significance of measurements but Special Relativity implements it by invoking an inverse insight into the insignificance of constant velocity.

The exact nature of this difference may be clarified by two further remarks. On the one hand, it does not prevent Special Relativity from being regarded as a particular case of General Relativity, for General Relativity does not attribute any significance to constant velocity, and Special Relativity primarily regards laws reached by relating measurements to one another. On the other hand, the difference is a difference not merely in degree but also in kind, for the anticipations of General Relativity do not hold when the results of investigations include relations to observers, but the anticipations of Special Relativity do hold as long as the insignificance of constant velocity is extended to the whole of physics. So perhaps one may explain the fact that the anticipations of Special Relativity have been mated successfully with Quantum Mechanics.†

A third and still less general anticipation of invariance has been attributed retrospectively to Newtonian dynamics, and it is not difficult to

* See Lindsay and Margenau, p. 368.
† See Lindsay and Margenau, pp. 501 ff.



grasp in terms of insight the justice of this view. For, as has been noted, the defect in intelligibility known in inverse insight is formulated only by employing a positive context of concomitant direct insights. In particular, it has been remarked that the defect of intelligibility in constant velocity was expressed for mechanics by Newton in his first law of motion but for physics generally by Einstein in the basic postulate of Special Relativity. Accordingly, one can move backwards from Einstein to Newton if

(1) one holds fast to the defective intelligibility in constant velocity, and

(2) one changes the concomitant context of direct insights in terms of which the inverse insight regarding constant velocity is expressed.

Now the relevant differences in the concomitant context are threefold. First, Special Relativity regards all physical principles and laws, but Newtonian dynamics is concerned primarily with mechanics. Secondly, Special Relativity is primarily a field theory, that is, it is concerned not with the efficient, instrumental, material, or final causes of events, but with the intelligibility immanent in data; but Newtonian dynamics seems primarily a theory of efficient causes, of forces, their action, and the reaction evoked by action. Thirdly, Special Relativity is stated as a methodological doctrine that regards the mathematical expression of physical principles and laws, but Newtonian dynamics is stated as a doctrine about the objects subject to laws.

From these differences it follows that what Einstein stated for physics in terms of the transformation properties of the mathematical expression of principles and laws, Newton stated for mechanics in terms of the forces that move bodies. In both cases what is stated is a negation of intelligibility in constant velocity. But the Einsteinian context makes the statement an affirmation of invariance despite inertial transformations, while the Newtonian context makes the statement an affirmation of continued uniform motion in a straight line despite the absence of external forces. Finally, as the Einsteinian statement may be regarded as a methodological rule governing the expression of physical principles and laws, so the Newtonian statement may be regarded as a general boundary condition complementing the laws that equate

(1) force with change of momentum, and
(2) action with an equal and opposite reaction.

### 2.6 Summary

Our concern has been the methodical genesis of insight. Scientists achieve understanding, but they do so only at the end of an inquiry. Moreover, their inquiry is methodical, and method consists in ordering means to achieve an end. But how can means be ordered to an end when the end is knowledge and the knowledge is not yet acquired? The answer to this puzzle is the heuristic structure. Name the unknown. Work out its properties. Use the properties to direct, order, guide the inquiry.

In pre-scientific thought what is to be known inasmuch as understanding is achieved is named the 'nature of . . .'. Because similars are understood similarly, the 'nature of . . .' is expected to be the same for all similar data, and so it is specified as the nature of light, the nature of heat, and so forth, by constructing classifications based on sensible similarity.

Scientific thought involves a more exact anticipation. What is to be known inasmuch as data are understood is some correlation or function that states universally the relations of things not to our senses but to one another. Hence, the scientific anticipation is of some unspecified correlation to be specified, some indeterminate function to be determined; and now the task of specifying or determining is carried out by measuring, by tabulating measurements, by reaching an insight into the tabulated measurements, and by expressing that insight through some general correlation or function that, if verified, will define a limit on which converge the relations between all subsequent appropriate measurements.

This basic anticipation and procedure may be enriched in two further manners. First, functions are solutions of differential equations; but in many cases relevant differential equations can be deduced from very general considerations. Hence, the scientist may anticipate that the function, which is the object of his inquiry, will be one of the solutions of the relevant differential equations. Secondly, the functions that become known in the measure that understanding is achieved are, both in origin and in application, independent of the differences of particular places and particular times. In such a science as physics this anticipation of independence becomes formulated as the invariance of principles and laws under groups of transformations, and different grounds are invoked to determine which group of transformations is to leave the



mathematical expression of laws unchanged in form. So a direct insight into the significance of measurements yields the anticipations of General Relativity, an inverse insight into the insignificance of constant velocity yields the anticipations of Special Relativity, and a restriction of this inverse insight to the context of Newtonian dynamics yields the anticipations that sometimes are named Newtonian relativity.

Such in brief are the anticipations constitutive of classical heuristic structure. The structure is named classical because it is restricted to insights of a type most easily identified by mentioning the names of Galileo, Newton, Clerk-Maxwell, and Einstein. It is named heuristic because it anticipates insights of that type and, while prescinding from their as yet unknown contents, works out their general properties to give methodical guidance to investigations. It is named a structure because, though operative, it is not known explicitly until oversight of insight gives way to insight into insight.

In particular one should observe that classical heuristic structure has no suppositions except the minimal suppositions that insights of a certain type occur and that inquiry aiming at such insights may be not haphazard but methodical. Further, advertence to classical heuristic structure has no additional suppositions except the possibility of an insight that grasps the set of relations linking methodical inquiry with anticipated insights, data, similarities in data, measurements, curve-fitting, indeterminate functions, differential equations, the principle of inertia, Special Relativity, and General Relativity. If there has been communicated some grasp of such diverse objects within the unity of a single view, then there has been communicated an insight into the genesis of insight. No doubt, that is a very small thing. An insight is no more than an act of understanding. It may prove to be true or false or to hold some intermediate position of greater or less probability. Still it is solely the communication of that act of understanding that has been our aim and, if the reader has been concerned with anything else, he has done all that is necessary to miss the little we have had to offer in the present context.

A further observation is not without its importance. Precisely because our suppositions and our objective have been so restricted, our account of classical heuristic structure is essentially free from any opinion about corpuscles, waves, causality, mechanism, determinism, the uniformity of nature, truth, objectivity, appearance, reality. It follows immediately that if we venture to use the name, 'classical', we use it without being

involved in any of the extra-scientific views that historically have been associated with scientific discoveries and, to a greater or less extent, have influenced their interpretation. This point is, of course, of considerable importance at a time when a new statistical heuristic structure has grown enormously in prestige and it has become a matter of some obscurity whether the new approach conflicts with the assumptions of earlier science or merely with the extra-scientific opinions of earlier scientists. Finally, if we may close this section on a still more general note, it is not perhaps rash to claim that an analysis of scientific procedures in terms of insight is also new and that the value of such analysis cannot be tested except by working out its implications and confronting them, not with opinions on science based on other analyses, but solely with strictly scientific anticipations, procedures, and results.

### 3. CONCRETE INFERENCES FROM CLASSICAL LAWS

Before advancing to a consideration of statistical heuristic structure, it will be well to ask just how far the full realization of classical anticipations would bring the scientist towards an adequate understanding of data. Accordingly, we ask about the range of concrete inferences from classical laws and we do so all the more readily because discussions of this topic seem to have suffered from an oversight of insight.

For just as insight is a necessary intermediary between sets of measurements and the formulation of laws, so also it is needed in the reverse process that applies known laws to concrete situations. Hence, a concrete scientific inference has not two but three conditions: it supposes information on some concrete situation; it supposes knowledge of laws; and it supposes an insight into the given situation. For it is only by the insight that one can know

(1) which laws are to be selected for the inference,
(2) how the selected laws are to be combined to represent the spatial and dynamic configuration of the concrete situation, and
(3) what dimensions in the situation are to be measured to supply numerical values that particularize the selected and combined laws.

Further, such inferences can be carried out in two manners. While practical people wait for concrete situations to arise before attempting to work out their consequences, theoretical minds are given to anticipating ideal or typical cases and to determining how a deduction could be carried out in each case.



Now in these anticipatory concrete inferences a different type of insight comes into play. For in the practical inference the situation determines the relevant insight and the insight determines the selection, combination, and particularization of laws. But in the anticipatory inference insight is creative and constructive. It is not hampered by any given situation. Rather it tends to be a free exploration of the potentialities of known laws, and its principal fruit is the formulation of ideal or typical processes that are dominated throughout by human intelligence. For in such processes the basic situation is any situation that satisfies the requirements of the constructive insight and, provided the process is closed off against all extraneous influence, every antecedent and consequent situation must assume the dimensions determined by the successive stages of the imaginative model.

Moreover, it can happen that such ideal or typical processes can be verified in a sequence of concrete situations, and then three very notable consequences follow. In the first place, some insight or some set of unified insights can grasp not only the process as a whole but also every event in the whole. Secondly, this single insight or single unified set can be expressed in a corresponding combination of selected laws and any situation can be deduced from any other without any explicit consideration of intervening situations. Thirdly, when such processes exist and their laws are as yet unknown, their investigation enjoys a number of singular advantages. For the intelligible unity of the whole process implies

(1) that data on any situation are equivalent to data on the whole process,
(2) that if data are found to be significant in any situation, then similar data will be significant in every other situation, and
(3) that the accuracy of reports on any situation can be checked by inferences from reports on other situations.

Moreover, once initial difficulties are overcome and basic insights are reached, the investigation approaches a supreme moment when all data suddenly fall into a single perspective, sweeping yet accurate deductions become possible, and subsequent exact predictions regularly will prove to have been correct.

However, if the nature of statistical inquiry is to be understood, it is of considerable importance to grasp that a quite different type of process not only can be constructed but also probably can be verified.

Accordingly, let us divide ideally constructed processes into systematic and non-systematic. Let us define systematic processes by the already enumerated properties that, other things being equal,

(1) the whole of a systematic process and its every event possess but a single intelligibility that corresponds to a single insight or single set of unified insights,

(2) any situation can be deduced from any other without an explicit consideration of intervening situations, and

(3) the empirical investigation of such processes is marked not only by a notable facility in ascertaining and checking abundant and significant data but also by a supreme moment when all data fall into a single perspective, sweeping deductions become possible, and subsequent exact predictions regularly are fulfilled.

Now whenever a group or series is constructed on determinate principles, it is always possible to construct a different group or series by the simple expedient of violating the determinate principles. But the group of systematic processes is constructed on determinate principles. Therefore, by violating the principles one can construct other processes that are non-systematic.

It is to be noted that the construction of non-systematic processes rests on the same knowledge of laws and the same creative intelligence as the construction of systematic processes. Hence if one inclines to enlarge the group of systematic processes by postulating full knowledge of laws and an unlimited inventiveness, one must grant that the group of non-systematic processes also is constructed from an equally full knowledge of laws and an equally unlimited (though perhaps perverse) inventiveness. Finally, though we do not know all laws, none the less we can form the general notion of the systematic process; and similarly despite our ignorance of many laws we also can form the general notion of the non-systematic process.

For, in the first place, if non-systematic process is understood, the understanding will be multiple. There will be no single insight, or single set of unified insights, that masters at once the whole process and all its events. The only correct understanding will be either a set of different insights or else a set of different unified sets. In the former case the different insights will not be unified intelligibly and so they will not be related to one another in any orderly series or progression or grouping whatever. In the latter case the different sets of unified insights will



have no higher intelligible unity and so they will not be related to one another in any orderly series or progression or grouping whatever. Finally, let us say that a series, progression, grouping is orderly if the relations between the elements of the series, progression, grouping either

(1) can be grasped by an insight that can be expressed in general terms, or

(2) can be concluded from any single insight or any single set of unified insights.

Secondly, because different parts of the process are understood differently, there can be no single combination of selected laws that holds for the whole process. On the contrary, for every different insight or different set of unified insights there will be a different combination and perhaps even a different selection of laws. Again, just as the different insights or unified sets of insights, so the different selections and combinations will not satisfy any orderly series or progression or grouping whatever.

Thirdly, such non-systematic process may be deducible in all its events. Let us suppose

(1) the absence of extraneous interference,
(2) full information on some one situation,
(3) complete knowledge of all relevant laws,
(4) correct insights into the basic situation,
(5) sufficient skill in the manipulation of mathematical expressions,
(6) correct insights into deduced situations, and
(7) no restriction on the amount of time allowed for the deduction.

Then from the given situation the occurrence and the dimensions of the next significantly different situation can be deduced. Correct insights into the deduced data on this situation make it possible to deduce the occurrence and the dimensions of the third significantly different situation. Finally, since this procedure can be repeated indefinitely and since there are no restrictions on the amount of time to be devoted to the deduction, it makes no difference how many significantly different situations there are.

Fourthly, in a number of manners non-systematic process exhibits coincidental aggregates. For an aggregate is coincidental if

(1) the members of the aggregate have some unity based on spatial juxtaposition or temporal succession or both, and

(2) there is no corresponding unity on the level of insight and intelligible relation.

Now non-systematic process as a whole possesses a spatio-temporal unity but has no corresponding unity on the level of insight or intelligible relation.

Again, the several insights by which the several parts of non-systematic process are understood form another coincidental aggregate. For they are a multiplicity on the level of intelligibility but they possess some unity from the spatio-temporal unity of the process.

Similarly, the succession of different premises by which different stages of non-systematic process may be deduced are a third coincidental manifold. For they too are a multiplicity on the level of intelligibility but they possess some unity from the spatio-temporal unity of the process.

Further, the basic situation of non-systematic process must be a coincidental manifold. For it has unity by spatial juxtaposition; but it cannot be one on the level of insight and intelligible relation. If the basic situation were intelligibly one, then the deduction of the process from that intelligible unity would constitute an orderly grouping for the set of different insights and for the succession of different combinations of selected laws. But both the set of different insights and the succession of different combinations of selected laws are coincidental aggregates that cannot be unified by any orderly series or progression or grouping whatever. Therefore, the basic situation can be no more than a merely spatial unification of different intelligibilities that can be grasped only by a set of different and unrelated insights.

Similarly, if many different and unrelated insights are needed to understand the basic situation, the premises for a deduction from that situation cannot be a single, unified combination of selected laws. And since a coincidental aggregate of premises will yield a coincidental aggregate of conclusions, it follows that every deducible situation, provided it is a total situation, also will be a coincidental aggregate. Further, it follows that, when a non-systematic process happens to give rise to a systematic process (as in recent theories on the origin of planetary systems), then the total situation must divide into two parts of which one happens to fulfil the conditions of systematic process and the other fulfils the requirement of other things being equal.

Finally, there emerges the rule for constructing non-systematic processes. For a situation is 'random' if it is 'any whatever provided specified conditions of intelligibility are not fulfilled'. But non-systematic process results from any basic situation provided it lacks intelligible unity from a definitive viewpoint. Therefore, the rule for constructing non-systematic processes is to begin from any random basic situation.



Fifthly, if non-systematic processes exist, then the difficulty of investigating their nature increases with the number and diversity of their several distinct and unrelated intelligibilities. Data on one situation are not equivalent to data on the whole process but are relevant only to one of many parts of the whole. Again, the types of data significant in one part will not be significant in disparate parts, and so several different inquiries must be undertaken. Thirdly, reports on one situation ordinarily cannot be checked by comparing them with inferences from reports on other situations. Fourthly, there is no supreme moment when all data fall into a single perspective, for there is no single perspective to be had. Fifthly, even when the laws involved in the process are thoroughly understood, even when current and accurate reports from usually significant centres of information are available, still such slight differences in matters of fact can result in such large differences in the subsequent course of events that deductions have to be restricted to the short run and predictions have to be content with indicating probabilities. So, perhaps, it is that astronomers can publish the exact times of the eclipses of past and future centuries but meteorologists need a constant supply of fresh and accurate information to tell us about tomorrow's weather.

Let us now pause to take our bearings. We began by noting that concrete inferences from classical laws suppose not only knowledge of laws and information on some basic situation but also an insight that mediates between the situation and general knowledge. We went on to distinguish between practical insights that apply laws to given situations and constructive insights that invent typical or ideal processes. We have been engaged in explaining that, just as constructive insight can devise systematic processes with all their beautiful and convenient properties, so also it can devise non-systematic processes with a complete set of quite opposite properties. It remains that a few more general corollaries be added.

First, systematic process is monotonous, but non-systematic process can be the womb of novelty. For the possibility of leaping deductively from any situation of a systematic process to any other situation rests on

the fact that a systematic process is little more than a perpetual repetition of essentially the same story. On the other hand, the unfolding of a non-systematic process has to be followed through its sequence of situations. Significant changes occur and, as they occur, the relevant insights change. Hence, as will appear in Chapter IV, within a large non-systematic process there can be built a pyramid of schemes resting on schemes in a splendid ascent of novelty and creativeness.

Secondly, systematic process would seem to be reversible, that is, it would work equally well if, so to speak, the future were the past and the process ran backwards. For a systematic process is the expression of a single idea. Each successive situation is related to the next in accord with the dictates of the idea. Hence, to reverse the succession of dictates so that the process begins from a last situation and moves backwards to a first involves no new idea but merely a different and, it seems, equally workable application of the same idea. On the other hand, non-systematic process may easily be irreversible. For it is not the unfolding of some single idea, and successive situations are not related in accord with the dictates of any single insight or any single set of unified insights. What is in control is not intelligence but any random basic situation, and the resulting coincidental sequence of coincidental situations easily includes both the emergence and the destruction of systematic processes. Hence, to expect non-systematic process to be reversible is to expect destroyed systematic processes to re-emerge from their ruins; again, it is to expect that reversed systematic processes will resolve into their origins at the right moment and in the right manner though no provision is made for that resolution.

Thirdly, the distinction between systematic and non-systematic processes throws light on the precise meaning of closure. For there is an external closure that excludes outside interference. When it is applied to a systematic process, the whole course of events is mastered by intelligence with relative ease. But when it is applied to a non-systematic process, then it merely leaves internal factors all the freer to interfere with one another.

Fourthly, whether world process is systematic or non-systematic is a question to be settled by the empirical method of stating both hypotheses, working out as fully as one can the totality of their implications, and confronting the implications with the observable facts.

Fifthly, if world process proves to be non-systematic, then it contains coincidental aggregates and the word 'random' has an objective



meaning. In that case, there would be some interpretation of statistical science as the science of what exists. In other words, in that case it would be false to say that statistical science must be a mere cloak for ignorance. Moreover, even if world process proves to be systematic, still that will be true only on empirical grounds and *a posteriori*; it follows that it cannot be true *a priori* that statistical science cannot be the science of what exists. On the present showing, then, there can be no valid theoretical arguments that establish that statistical science in every possible meaning of the term must be a mere cloak for ignorance.

## 4. STATISTICAL HEURISTIC STRUCTURE

### 4.1 *Elementary Contrasts*

Classical and statistical investigations exhibit marked differences that provide a convenient starting-point for the present section.

In the first place, while classical investigation heads towards the determination of functions and their systematization, statistical investigation clings to concrete situations. Hence, while classical conclusions are concerned with what would be if other things were equal, statistical conclusions directly regard such aggregates of events as the sequences of occasions on which a coin is tossed or dice are cast, the sequences of situations created by the mobility of molecules in a gas, the sequences of generations in which babies are born, the young marry, and the old die.

Secondly, statistical inquiry attends not to theoretical processes but to palpable results. As Galileo sought the intelligibility immanent in a free fall, so Clerk-Maxwell sought the intelligibility immanent in the electromagnetic field. But in a statistical investigation such theoretical analyses and constructions are set aside. The movement of dice observes perfectly the laws of mechanics, but the laws of mechanics are not premises in the determination of the probability of casting a 'seven'. Doctors commonly succeed in diagnosing the causes of death, but such causes are not studied in fixing death rates. The statistical scientist seems content to define events and areas, to count the instances of each defined class within the defined area, and to offer some general but rather vague view of things as a whole.

Thirdly, statistical science is empirical, but it does not endeavour to measure and correlate the spatial, temporal, and other variables that so fascinate classical investigators. Its attention is directed to *frequencies* that are straightforward numerical answers to the straightforward question,

How often? Such frequencies may be *ideal* or *actual* but, while it is true that the ideal frequency or probability raises debatable issues, at least the *actual frequency* is a transparent report not of what should or might or will happen but of what in fact did happen. Such actual frequencies are *absolute* when they assign the actual number of events of a given kind within a given area during a given interval of time. However, since different areas commonly are not comparable, it is customary to proceed from absolute actual frequencies either to *rates*, say, per thousand of population or, when classes of events are alternative possibilities, to *relative actual frequencies* which are sets of proper fractions, say, $p/n$, $q/n$, $r/n$, . . . where $n=p+q+r+\ldots$.

Fourthly, behind the foregoing rather superficial differences, there is a profound difference in the mentality of classical and statistical inquirers. Had astronomers been content to regard the wandering of the planets as a merely random affair, the planetary system never would have been discovered. Had Joule been content to disregard small differences, the mechanical equivalent of heat would have remained unknown. But statistical inquirers make it their business to distinguish in their tables of frequencies between significant and merely random differences. Hence, while they go to great pains to arrive at exact numbers, they do not seem to attempt the obvious next step of exact explanation. As long as differences in frequency oscillate about some average, they are esteemed of no account; only when the average itself changes, is intellectual curiosity aroused and further inquiry deemed relevant.

### 4.2 *The Inverse Insight*

The existence of this radical difference in mentality demands an explanation, and the obvious explanation is the occurrence of something like an inverse insight. For an inverse insight has three characteristics: it supposes a positive object of inquiry; it denies intelligibility to the object; and the denial runs counter to spontaneous anticipations of intelligence. But the differences named random are matters of fact: they occur in frequencies determined by counting the events in a given class in a given area during a given interval of time. Further, random differences are denied intelligibility for, though statistical inquirers hardly would use such an expression, at least their deeds seem a sufficient witness to their thought. When differences are not random, further inquiry is in order; but when differences are random, not only is no inquiry attempted but also the very attempt would be pronounced



silly. Finally, this denial of intelligibility is in open conflict with the anticipations of classical investigation. For classical precept and example tirelessly inculcate the lesson that no difference is to be simply neglected; and while one may doubt that this classical attitude is more spontaneous than its opposite, at least one can speak of a devaluated inverse insight that divides classical and statistical anticipations.

Further, while this devaluated inverse insight bears on the frequencies of events, it does not follow necessarily that the defect of intelligibility resides in single events. Indeed, it seems quite possible to acknowledge random differences in frequencies and at the same time to maintain that single events are determinate, that they are not random, even that they are deducible. At least, the events must be determinate enough to be counted for, if they are not counted, there are no frequencies and so no random differences in frequencies. Again, one can acknowledge random differences in death rates without suggesting that single deaths were random or that doctors were unable to perform successful diagnoses. Finally, if single events need not be random, they may be deducible. For if it is possible to argue from effect to cause, from consequent to antecedent, it should be equally possible to move from cause to effect, from determining antecedent to determined consequent.

It seems, then, that if we are to discover a *fully general* account of the meaning of random differences, we must look not to single events but to events as members of a group. So the question becomes, How can there be a defect in intelligibility in a group of events if each event singly is quite determinate, if none are random, and if one by one all may be deduced?

Fortunately, if not accidentally, our previous discussion of concrete inferences from classical laws offers a ready answer to this question. For knowledge of laws can be applied

(1) to single events,
(2) to systematic processes, and
(3) to non-systematic processes.

Moreover, just as the assertion of random differences in frequencies need not imply that single events are indeterminate or random or that they are not deducible, so also in a non-systematic process each event may be determinate, none need be random and sometimes at least, if time were not money, all could be deduced. Again, just as the assertion of random differences springs from a devaluated inverse insight, so too

does the notion of a non-systematic process. For a non-systematic process is as positive an object of inquiry as any process, it is non-systematic inasmuch as it lacks the intelligibility that characterizes systematic process; and its properties are very surprising indeed when they are compared with what commonly Laplace is supposed to have meant when he claimed that any situation in world history could be deduced from any other.

The similarity of these two devaluated inverse insights provides an obvious clue and, to follow it up, let us consider the four statements·

(1) statistical inquiry is concerned with coincidental aggregates of events;

(2) statistical inquiry investigates what classical inquiry neglects;

(3) statistical inquiry finds an intelligibility in what classical inquiry neglects; and

(4) this intelligibility is denied when random differences are affirmed.

First, statistical inquiry is concerned with coincidental aggregates of events. For it is not concerned with the intelligibly grouped events of systematic process: there are no statistics on the phases of the moon or on the transit of Venus, and there are no random differences in ordinary astronomical tables. Again, it is not concerned with events taken singly. For each single event amounts to just one more or less in tables of frequencies and, in general, a difference of one more or one less may be regarded as random. Further, it is possible to discern random differences in some groups of events in which each event is determinate and deducible and no event is random. It remains, then, that the object of statistical inquiry is the coincidental aggregate of events, that is, the aggregate of events that has some unity by spatial juxtaposition or by temporal succession or by both but lacks unity on the level of insight and of intelligible relation. In other words, statistical inquiry is concerned with non-systematic process.

Secondly, statistical inquiry investigates what classical inquiry neglects For even if one grants that classical inquiry leads to the laws that explain every event, it remains that classical science rarely bothers to explain the single events of non-systematic process and, still less, does it offer any technique for the orderly study of groups of such events. Moreover, there are excellent reasons for this neglect The deduction of each of the events of a non-systematic process begins by demanding more abundant and more exact information than there is to be had. It proceeds through a sequence of stages determined by the coincidences of a random situation. It has to postulate unlimited time to be able to assert the possibility of completing the deduction. It would end up with a result that lacks generality for, while the result would hold for an exactly similar non-systematic process, it commonly would not provide a safe basis for an approximation to the course of another non-systematic process with a slightly different basic situation. Finally, it would be preposterous to attempt to deduce the course of events for every non-systematic process. Not only would the foregoing difficulties have to be surmounted an enormous number of times but this Herculean labour would seem to be to no purpose. How could non-systematic processes be classified? How could one list in an orderly fashion the totality of situations of all non-systematic processes? Yet without such a classification and such a list, how could one identify given situations with situations contained in the extremely long deductions of the extremely large set of non-systematic processes?



Thirdly, statistical inquiry finds an intelligibility in what classical inquiry neglects. So far we have been concerned to stress the defect of intelligibility in non-systematic process. But a mere defect in intelligibility is not the basis of a scientific method. There is needed a complementary direct insight that turns the tables on the defect. Just as scientific generalization exploits the fact that individuality pertains to an empirical residue, just as the real numbers, the theory of continuous functions, and the infinitesimal calculus exploit the defect of intelligibility in the continuum, just as scientific collaboration is possible because particular places and particular times pertain to the empirical residue, just as the principle of inertia and the basic postulate of Special Relativity rest on an empirically residual aspect of constant velocity, so also statistical science is the positive advance of intelligence through the gap in intelligibility in coincidental aggregates of events.

Accordingly, besides the devaluated inverse insight that has been our concern hitherto, there is to be acknowledged in statistical science another basic moment that is positive and creative. Aristotle was quite aware of what we have named non-systematic process, for he contended that the whole course of terrestrial events was just a series of accidents. But to this devaluated inverse insight he failed to add the further creative moment. Instead of discovering statistical method, he attempted to account for the manifest continuity of the terrestrial series of accidents

by invoking the continuous influence of the continuously rotating celestial spheres.

Fourthly, it is this further intelligibility that is denied when random differences are affirmed. For if the statistical investigator deals with non-systematic processes, he does not find the intelligibility of systematic process either in the differences he pronounces significant or in the differences he pronounces random. Again, to discover the intelligibility that statistical science finds in non-systematic process, we must look to the differences pronounced significant. It follows that differences in frequencies of events are random when they lack not only the intelligibility of systematic process but also the intelligibility of non-systematic process.

### 4.3 *The Meaning of Probability*

Still the reader will be more interested in hearing what this intelligibility is than in being told that it is lacking in random differences. Its name, then, is probability but to grasp the meaning of the name is to reach an explanatory definition. Let us begin from the definition and then try to understand it.

Consider a set of classes of events, $P, Q, R, \ldots$ and suppose that in a sequence of intervals or occasions events in each class occur respectively $p_1, q_1, r_1, \ldots p_2, q_2, r_2, \ldots p_i, q_i, r_i, \ldots$ times. Then the sequence of relative actual frequencies of the events will be the series of sets of proper fractions, $p_i/n_i, q_i/n_i, r_i/n_i, \ldots$ where $i=1, 2, 3, \ldots$ and in each case $n_i=p_i+q_i+r_i+\ldots$. Now if there exists a single set of constant proper fractions, say $p/n, q/n, r/n, \ldots$ such that the differences

$$\frac{p}{n}-\frac{p_i}{n_i}, \quad \frac{q}{n}-\frac{q_i}{n_i}, \quad \frac{r}{n}-\frac{r_i}{n_i}, \ldots$$

are always random, then the constant proper fractions will be the respective probabilities of the classes of events, the association of these probabilities with the classes of events defines a state, and the set of observed relative actual frequencies is a representative sample of the state.

The foregoing paragraph outlines a procedure in which the central moment is an insight. By that insight the inquirer abstracts from the randomness in frequencies to discover regularities that are expressed in constant proper fractions named probabilities. There results the solution of two outstanding methodological problems. Because the probabilities are to hold universally, there is solved the problem of reaching general knowledge of events in non-systematic processes. Because states are defined by the association of classes of events with corresponding probabilities, there is by-passed the problem of distinguishing and listing non-systematic processes. However, both the probabilities and the states they define are merely the fruits of insight. They are hypothetical entities whose existence has to be verified and, in fact, becomes verified in the measure that subsequent frequencies of events conform to probable expectations. In turn, this need of verification provides a simple formulation for the notion of a representative sample. For a set of relative actual frequencies is a representative sample if the probabilities to which they lead prove to be correct. On the other hand, a set of relative actual frequencies is not a representative sample if the probabilities to which they lead run counter to the facts. It follows that the basic practical problem of statistical inquiry is the selection of representative samples and, indeed, that its solution must depend not merely on a full theoretical development of statistical method but also on the general knowledge of individual investigators and on their insights into whatever specific issues they happen to be investigating.



Such, then, is the general context, but our concern must centre on the insight by which intelligence leaps from frequencies to probabilities and, by the same stroke, abstracts from the randomness in frequencies. Now an insight is neither a definition nor a postulate nor an argument but a preconceptual event. Hence our aim must be to encourage in readers the conscious occurrence of the intellectual events that make it possible to know what happens when probability is grasped. First, then, we shall consider an easier insight that bears some general resemblance to insights into probability. Secondly, we shall consider an insight that occurs when a particular case of probability is understood. Thirdly, we shall move towards the general heuristic structure within which the notion of probability is developed and methods of determining its precise content are perfected.

In the first place, the mathematical notion of limit bears a general resemblance to the notion of probability. Accordingly, let us consider the simple sum,

$$S=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots \qquad [\text{to } n \text{ terms}]$$
$$=1-\frac{1}{2^n}$$

where, as $n$ increases, $S$ differs from unity by an ever smaller fraction and so, by assigning $n$ ever larger values, the difference between the sum, $S$, and unity can be made as small as one pleases. In the limit then, when the number of terms in the series is infinite, the sum, $S$, is unity. However, one cannot write out an infinite number of terms; one cannot even conceive each of an infinite number of terms. Moreover, while it is contradictory to suppose that an unending series is ended, still one can understand the principle on which each fraction in the series is constructed, one can tell whether or not any fraction belongs to the series, one can conceive as many of the fractions as one pleases, and one can grasp that the more terms there are to the series, the nearer the sum is to unity. Finally, there is no contradiction in thinking or speaking of *all* the terms in the series, and one can see that there is no point in bothering about explicit conception of the remainder because it contains nothing that is not already understood. Now advertence to this absence of further intelligibility in the remainder is the abstractive aspect of the insight that claims the whole series to be understood sufficiently in its content and in its properties for it to be summed and for the sum to be equated with unity.

But, like a mathematical limit, a probability is a number. Like a limit, a probability is a number that cannot be reached from the data of a problem without the intervention of an insight. Again, just as the limit we considered lay beyond more terms than can be conceived, so a probability lies concealed within the random oscillations of relative actual frequencies. Finally, just as intelligence can reach a limit by grasping that there is nothing further to be understood in the unconceived infinite remainder of further terms, so also intelligence can reach probabilities by abstracting from the random oscillations of relative actual frequencies to discover a set of universally valid constants.

In the second place, to move closer to our quarry, let us analyse the tossing of a coin in the hope of generating the insight that pronounces the probability of 'heads' to be one-half. The result, then, of a toss is either of the alternatives, 'heads' or 'tails'. In any given instance the result might have been different if

(1) the initial position of the coin had been different, or
(2) different linear and angular momenta had been imparted to it, or
(3) the motion had been arrested at a different point.



Let us name these three the determinants of the result and direct our attention to the set of possible combinations of determinants.

First, the set is very large. For any of a very large group of initial positions can be combined with any of a very large group of initial linear and angular momenta; and any of these combinations can be combined with any of a very large group of points of arrested movement.

Secondly, the set of possible combinations divides into two exactly equal parts. For whenever 'heads' results, 'tails' would have resulted if the coin had been turned over and exactly the same toss and catch had been executed. Similarly, whenever 'tails' results, 'heads' would have resulted if the coin had been turned over and exactly the same toss and catch had been executed.

Thirdly, every sequence of actual combinations is a random selection from the set of possible combinations. It is a selection inasmuch as it need not include all possible combinations. It is a random selection inasmuch as it may be any whatever provided specified conditions of intelligibility are not fulfilled. Now intelligibility is to be excluded not from single tosses but from the sequence of tosses as a sequence. It is not to be excluded from single tosses for there is no reason to suppose that tossing a coin involves a suspension of the laws of mechanics or of any similar science. It is to be excluded from the sequence as a sequence for we have every reason to assert that a sequence of tosses is not a systematic process. Hence, every sequence of actual combinations of determinants is a coincidental aggregate. It will possess the unity of a temporal succession. But while any single combination may be deducible from prior events, any sequence of combinations is deducible only from some prior coincidental aggregate; for the sequence cannot be orderly in the sense that there is some insight or some set of unified insights that can be expressed in general terms and can determine the exact content of the sequence.

Now the relative actual frequency of 'heads' is the fraction obtained by dividing the number of times 'heads' occurs on any given succession of tosses by the number of tosses in that succession. Clearly, this fraction can and often will differ from one-half. For the result of each toss is settled by the actual combination of determinants, and that combination may be any combination whatever. However, differences between relative actual frequencies and one-half must be a coincidental aggregate. For if they were not, they would form an orderly series; if

the differences formed an orderly series, the results would have to form an orderly series; if the results formed an orderly series, the sequence of combinations of determinants would form an orderly series. *Ex hypothesi*, this conclusion is false, therefore, the supposition was false. Moreover, relative actual frequencies cannot help oscillating about one-half. For the set of possible combinations divides into two exactly equal parts; and every sequence of actual combinations is a random selection from the set of possible combinations. Now in a random selection of a sequence the sequence is stripped of all order, all regularity, all law; hence, while it can and will include runs of 'heads' and runs of 'tails', it cannot possibly stick to one alternative to the exclusion of the other, and so relative actual frequency is bound to oscillate about one-half.

It has been shown that the relative actual frequencies of 'heads'

(1) can and often do differ from one-half, but
(2) only at random, and
(3) in a manner that yields an oscillation about one-half as a centre.

Intelligence, then, can grasp a regularity in the frequencies by abstracting from their random features and by settling on the centre about which they oscillate. That abstractive grasp of intelligibility is the insight that is expressed by saying that the probability of 'heads' is one-half.

However, it is only in games of chance that there can be discerned an antecedent symmetry in the set of possible combinations of determinants of events. In other instances probabilities have to be reached *a posteriori* and, to reach them, a statistical heuristic structure has to be developed. To this issue we turn in the next subsection not, indeed, in the hope of determining what precisely probability must be in all cases but rather with the intention of grasping the underlying anticipations that inform statistical inquiry and are to be expected gradually to mount through trial and error, through theoretical discoveries and developing techniques, to some rounded methodological position such as already is enjoyed in classical investigations. In other words, besides the methodical genesis of scientific insights, there is the genesis of scientific method itself and, when a satisfactory account of the former is still a matter of obscure debates, a study of human understanding can draw no less profit from a consideration of the latter.



### 4-4 *Analogy in Heuristic Structure*

The present subsection is a protracted analogy. Under ten successive headings we shall recall distinctive features of classical heuristic structure, note their reason or ground, and in each case proceed to an analogous feature in a statistical heuristic structure.

First, then, there is the unspecified heuristic concept. For the goal of every inquiry is an act of understanding, and the basic device of methodical inquiry is to name the unknown that will become known when the anticipated act of understanding occurs. Hence, just as the classical inquirer seeks to know the 'nature of . . .' so the statistical inquirer will seek to know the 'state of . . .'.

Secondly, there is a specification of the heuristic concept by pre-scientific description. For all empirical inquiry presupposes some object that already is given but as yet is not understood; and every such object possesses its pre-scientific description that provides an initial specification for the heuristic concept. Hence, just as classical inquiry comes to know natures by understanding 'data of different kinds', so statistical inquiry comes to know states by understanding 'ordinary and exceptional, normal and abnormal runs of events'.

Thirdly, linking the open heuristic concept with the pre-scientifically described object there is the heuristic theorem. Because similars are understood similarly, natures are linked with data classified by sensible similarity. So we speak of the nature of colour or the nature of sound. Similarly, because a notable regularity is compatible with random differences in runs of events, states are linked with runs that despite occasional lapses are ordinary or normal or, again, with runs that are pronounced exceptional or abnormal though they contain a few ordinary or normal elements. So we speak of the state of a person's health, brokers speak of the state of the market, and the President of the United States discourses on the state of the nation.

Fourthly, to effect a transformation of pre-scientific anticipations and descriptions, there has to be formulated an ideal of scientific explanation. Hence, just as the classical inquirer places knowledge of nature in the discovery and verification of determinate functional relations, so the statistical inquirer places knowledge of states in the association of sets of classes of events with corresponding sets of probabilities. In other words, just as the mysterious nature of gravity turns out to be for the scientist merely a constant acceleration, so the mysterious state of so-

and-so's health turns out to be for the scientist a schedule of probabilities attached to a schedule of classes of events.

Fifthly, from the formulation of the precise scientific objective there follows the displacement of pre-scientific by scientific description. Thus, to determine functional relations measurement is added to observation and mere sensible similarity gives way to similarities of conjunction and separation, of proportion and concomitant variation. In like manner to determine sets of probabilities the adjectives, ordinary and exceptional, normal and abnormal, are replaced by actual counting of events and the consequent tabulation of rates of relative actual frequencies. Moreover, to justify this numerical accuracy, exact classifications are borrowed from classical science and every resource is employed to delimit, as far as possible, internally homogeneous volume-intervals of events.

Sixthly, just as classical inquiry derives a general view of its possibilities from the mathematical investigation of functions and of spatio-temporal relations, so statistical inquiry finds similar guidance and orientation in the calculus of probabilities.

Seventhly, just as classical inquiry evolves practical techniques of curve-fitting to aid the transition from measurements to functional relations, so statistical inquiry develops similar techniques to aid the transition from relative actual frequencies to probabilities.

Eighthly, just as classical inquiry proceeds not only from below upwards from measurements through curve-fitting but also from above downwards from differential equations to their solutions, so also a comparable department of statistical inquiry has discovered that the solution of operator equations yields eigenfunctions and eigenvalues that serve both to select classes of events and to determine the respective probabilities of the selected classes.

Ninthly, just as classical discovery is a leap of constructive intelligence that goes beyond ascertained measurements to posit a functional relation on which the relations between all appropriate subsequent measurements should converge as on a limit, so also statistical discovery (as distinct from statistical information) is a leap of constructive intelligence that goes beyond ascertained relative actual frequencies to assign probabilities where differences between probabilities and relative actual frequencies

(1) should always be a coincidental aggregate, and



(2) in each case should be eliminable by extending the investigation of that case.

Hence, just as classical laws are universal and constant while measurements are particular and subject to the variations introduced by extraneous influences, so statistical states are universal and constant though relative actual frequencies are particular and subject to random differences.

However, while both types of discovery are universal and so abstract, still they involve different types of abstraction. In both classical and statistical constructs there is abstraction from the empirically residual aspects of individuality, of the continuum, of particular places and times, and of constant velocity. But classical laws, at least in the determination of each law, also abstract from coincidental aggregates inasmuch as they demand the qualification, 'other things being equal'. On the other hand, statistical states express an intelligibility immanent in coincidental aggregates and, to reach this intelligibility, they abstract from the random differences in relative actual frequencies.

Tenthly, no less than the classical law, the statistical state has to be verified. For knowledge of states is derived from particular frequencies by a leap of constructive intelligence. That leap is neither the recognition of a fact nor the grasp of a necessity but simply an insight into possibility. The known frequencies are satisfied by the supposition of a state that universally is manifested by events of determinate classes occurring with determinate probabilities. But further investigation can compromise this result in a variety of manners. It may reveal an unsatisfactory classification of events, an underestimation of the complexity of the sequence of situations, a failure to reach representative samples. Then relative actual frequencies have to be ascertained on a more exact or broader basis, and the constructive leap has to be repeated in a new manner.

Still, though both classical and statistical hypotheses need verification, verification has not the same meaning in both cases. Because the relations between measurements converge on the functional relations that express classical laws, it is possible to substitute the numerical values determined by the measurements for the variables that are functionally related by the laws In contrast, because relative actual frequencies differ at random from probabilities, it is not possible to

deduce the probabilities from any fully determinate mathematical formula by substituting for the variables of the formula the fractions that correspond to relative actual frequencies.

The converse to this difference in the meaning of verification appears in the difference between classical and statistical predictions. Classical predictions can be exact within assignable limits, because relations between measurements converge on the functional relations that formulate classical laws. But because relative actual frequencies differ at random from probabilities, statistical predictions primarily regard the probabilities of events and only secondarily determine the corresponding frequencies that differ at random from the probabilities. Hence, even when numbers are very great and probabilities high, as in the kinetic theory of gases, the possibility of exceptions has to be acknowledged; and when predictions rest on a statistical axiomatic structure, as in quantum mechanics, the structure itself seems to involve a principle of indeterminacy or uncertainty.

### 4.5 *Some Further Questions*

Possible further questions abound. But as the shrewd reader will have surmised, our purpose has been not to work out definitive foundations for statistical science but to grasp in some fashion the statistical heuristic structure that not only tackles specific problems but also develops its own methods as it goes along and thereby sets up an exigence for a succession of new and better foundations.

I shall be asked whether probable events sooner or later occur. From the viewpoint of empirical inquiry the answer seems to be affirmative. If events are probable, they do not diverge systematically from their probabilities. But if they occur neither sooner nor later, then there is empirical evidence for the intervention of some systematic factor.

However, if with the mathematicians one envisages an infinity of occasions, then the qualifying phrase, 'neither sooner nor later', admits so broad a meaning that empirical evidence for a systematic factor never can be reached. A common solution to this antinomy is to say that very small probabilities are to be neglected and this, I believe, can be defended by granting mathematical and denying empirical existence to the assumed infinity of occasions. Logically, however, such a solution implies that mathematical and scientific probability are distinct notions with different implications or, perhaps, it invites



the development of a mathematical theory in which the field of occasions gradually expands.

Again, I may be asked for the operational meaning of the highly theoretical coincidental aggregate. The answer is that the appropriate operation occurs on the methodological level. Either a range of observations is to be subsumed under classical heuristic structure or it is to be subsumed under statistical heuristic structure. On the former hypothesis it will be possible to discover some orderly series, progression, or grouping. On the latter hypothesis no such series, progression, or grouping exists. Both hypotheses can be formulated; their implications are to be worked out; and the facts are to decide which hypothesis is, if not ultimate truth, at least the best available opinion at the given stage of scientific development.

Finally, if probabilities must be verified, it also is true that there is a probability of verifications. But it is of no little importance to grasp that this second probability shares the name but not the nature of the first. For the first probability, apart from random differences, corresponds to the relative actual frequency of events. It is the regularity in the frequencies and it is to be known by a leap of constructive intelligence that grasps the regularity by abstracting from the randomness. In contrast, the second probability is not some fraction that, apart from random differences, corresponds to the relative actual frequency of verifications. A preponderance of favourable tests does not make a conclusion almost certain; indeed, a very few contrary tests suffice to make it highly improbable. More fundamentally, the second probability is not known by a leap of constructive intelligence that abstracts from random differences, for such leaps never yield anything but hypotheses. As will appear in Chapters IX and X, the second probability is known through acts of reflective understanding and judgment; it means that an affirmation or negation leads towards the unconditioned; and it is estimated, not by counting verifications and abstracting from random differences, but by criticizing verifications and by taking everything relevant into account.

For these reasons, then, we distinguish sharply between 'probably occurring' and 'probably true'. For the same reasons we refuse to identify 'certainty' in the sense of unit probability with 'certainty' in the sense of 'certainly verified'. It follows that we find it meaningless to represent by a fraction the probability of a verification. Similarly, we find it fallacious to argue that probable events are not certain events

because probable judgments are not certain judgments. Indeed, that fallacy would wreck our analysis. Not only are there two meanings to probability and two meanings to certainty but also there are two manners in which some events of non-systematic process can be investigated. Classical procedures would yield *particular*, probably verified, conclusions about single events assigned a *unit* probability, where statistical procedures would yield *general*, probably verified, conclusions about events as members of coincidental aggregates by assigning them *fractional* probabilities.

Before closing it may be well to add a word on the use of the terms, 'classical' and 'statistical'. In contemporary physics it is customary to oppose 'classical' to 'quantum' and 'statistical' to 'mechanical'. So there arises the familiar division of classical mechanics (Newton), classical statistics (Boltzmann), quantum mechanics (Schrödinger, Heisenberg), and quantum statistics (Bose-Einstein, Fermi-Dirac). Clearly, however, the present study of heuristic structures demands not a fourfold but a twofold division. Either intelligence anticipates the discovery of functional relations on which relations between measurements will converge, or else it anticipates the discovery of probabilities from which relative actual frequencies may diverge though only at random. The latter alternative has a fairly clear claim to the name, 'statistical'. The former alternative is not limited to Newtonian mechanics and, in the opinion of many, does not regard quantum mechanics. It is a mode of inquiry common to Galileo, Newton, Clerk-Maxwell, and Einstein; it is as familiar to the chemist as to the physicist; it long was considered the unique mode of scientific investigation; it has been the principal source of the high repute of science. In such a work as the present no one, I trust, will be misled if so classical a procedure is named 'classical'.

### 5. SURVEY

Perhaps enough progress has been made for the rather novel orientation of this inquiry to come into better focus. We began from the description of a discovery to proceed to distinguish insights, their cumulation to higher viewpoints, and the significance of grasping that at times the point is that there is no point. In the present chapter we have moved not forward and outward to conclusions about objects but rather backward and inward to the subject's anticipations of insights that have not occurred and to the methodical exploitation of such anticipations. In that inward movement the reader can foresee the direction in which



the whole work will advance. For our goal is not any scientific object, any universal and necessary truth, any primary propositions. Our goal is the concrete, individual, existing subject that intelligently generates and critically evaluates and progressively revises every scientific object, every incautious statement, every rigorously logical resting place that offers prematurely a home for the restless dynamism of human understanding. Our ambition is to reach neither the known nor the knowable but the knower. Chapter I spoke of the insights he seeks. Chapter II has introduced the heuristic structures that inform his seeking. Chapters III to V will consolidate this position. Chapters VI and VII will turn to the activities of more or less intelligent common sense. Chapter VIII will bring science and common sense together. Chapters IX and X will tackle the problems of critical judgment and, incidentally, will explain to impatient readers what they have been about while we in the first eight chapters were attempting to communicate to them the necessary prior insights. Finally, Chapters XI to XVII will endeavour to grasp within a single view how the totality of views on knowledge, objectivity, and reality proceed from the empirical, intellectual, and rational consciousness of the concrete subject.

CHAPTER III

# THE CANONS OF EMPIRICAL METHOD

An examination of insight not only reveals the heuristic structures involved in empirical inquiry but also explains the rules or canons that govern the fruitful unfolding of the anticipations of intelligence.

Six canons will be presented, namely,

(1) selection,
(2) operations,
(3) relevance,
(4) parsimony,
(5) complete explanation,
(6) statistical residues.

There is a canon of selection, for the empirical inquirer is confined to insights into the data of sensible experience. There is a canon of operations, for he aims at an accumulation of such insights, and the accumulation is reached, not in the mathematical circuit through insights, formulations, and symbolic images, but in the fuller circuit that adds observations, experiments, and practical applications. There is a canon of relevance, for pure science aims immediately at reaching the immanent intelligibility of data and leaves to applied science the categories of final, material, instrumental, and efficient causality. There is a canon of parsimony, for the empirical investigator may add to the data of experience only the laws verified in the data; in other words, he is not free to form hypotheses in the style of Descartes' vortices; but he must content himself with the laws and systems of laws, exemplified by Newton's theory of universal gravitation, and characterized generally by their verifiability. There is a canon of complete explanation: ultimately science must account for all data, and the account must be scientific; specifically the old philosophic opinion that extension is a real and objective primary quality cannot dispense one from the task of determining empirically the correct geometry of experienced extensions and durations. Finally, there is a canon of statistical residues; though all data must be explained, one must not jump to the conclusion that all will be explained by laws of the classical type; there exist statistical residues and their explanation is through statistical laws.

Before undertaking a fuller account of these canons, it may not be amiss to recall our viewpoint and purpose. The reader must not expect





us to retail the history of the development of empirical method, nor look for descriptive accounts of what scientists do, nor anticipate an argument based on the authority of great names in science, nor hope for a summary of directives, precepts, and recipes to guide him in the practice of scientific investigation. Our aim still is an insight into the nature of insight. Our presumption is that empirical investigators are intelligent. Our supposition will be that the reader is already sufficiently familiar with scientific history and procedure, authoritative pronouncements and practical directives. Our single purpose is to reveal the intelligible unity that underlies and accounts for the diverse and apparently disconnected rules of empirical method. Our concern is not what is done, or how it is done, but why. And our interest in seeking the reason why is not to extend methodology but to unify it, not to unify it that methodology may be improved but to unify it in the hope of exhibiting still more clearly and convincingly the fact and the nature of insight.

## 1. THE CANON OF SELECTION

First, there is a canon of selection.

If a correlation or hypothesis or law or probability expectation or theory or system pertains to empirical science, then

(1) it involves sensible consequences,
(2) such consequences can be produced or, at least, observed.

Inversely, empirical method prescinds from all questions and answers that do not involve distinctive, sensible consequences; and it discards all that involve such consequences logically yet fail to be confirmed by the results of observation or experiment.

The necessity of some canon of selection is obvious. Possible correlations, hypotheses, laws, probability expectations, theories, and systems form an indefinitely large group. They can be set up at will by the simple process of definition and postulation. But there is no reason why the empirical inquirer should investigate all the trees in this endless forest of possible thoughts, and so he needs some canon of selection.

The neatness of the canon of selection is no less clear. Not merely does it exclude at a stroke all the correlations and theories that cannot be relevant to empirical inquiry because they possess no sensible consequences. Also it operates progressively and cumulatively by discarding all the correlations and theories that possess sensible consequences by

logical implication but have been tried and found wanting. Finally, the canon of selection has its positive aspect; besides ruling the irrelevant out of consideration, it directs the scientist's efforts to the issues that he can settle by the decisive evidence of observation and experiment.

However, the neatness and simplicity of the canon of selection can prove a trap for the unwary. If the canon demands sensible consequences, still it is satisfied when those consequences are so slight that only an expert equipped with elaborate apparatus can detect them. If the sensible consequences must be involved by the correlation or law or expectation, still grasping that implication may suppose a profound mastery of a field, a capacity to follow recondite and intricate mathematical operations, and the audacity necessary to form new, primitive concepts and to follow long chains of abstract reasoning. Hence, besides the hodmen of science that gather the facts, there are also the architects of theories and systems. If no theory and no system pertains to empirical science unless it involves distinctive, sensible consequences, still an appropriate division of labour may well result in some empirical inquirers devoting most of their time and energy to the development of concepts and postulates, theorems and corollaries. Finally, as the canon of selection is not to be misinterpreted as a mere charter for obtuseness, still less is it to be taken as a mere excuse for logical fallacy. Questions that do not satisfy the canon of selection do not arise within the confines of empirical science, but it does not follow immediately that they do not arise at all. Issues that cannot be settled by observation or experiment cannot be settled by empirical method, but it does not follow immediately that they cannot be settled at all.

### 1.1 *The Restriction to Sensible Data*

Two further points call for consideration.

As we have formulated it, the canon of selection demands sensible consequences. But it may be urged that empirical method, at least in its essential features, should be applicable to the data of consciousness no less than to the data of sense. Now, on this matter a great deal might be said, but the present is not the time for it. We have followed the common view that empirical science is concerned with sensibly verifiable laws and expectations. If it is true that essentially the same method could be applied to the data of consciousness, then respect for ordinary usage would require that a method, which only in its essentials is the same, be named a generalized empirical method.



### 1.2 *What Are Sensible Data?*

A more urgent issue is raised by the question, What are sensible data?

A datum of sense may be defined as the content of an act of seeing, hearing, touching, tasting, smelling. But the difficulty with that definition is that such contents do not occur in a cognitional vacuum. They emerge within a context that is determined by interests and preoccupations. Nor is this true merely of ordinary perceptions, of the milkmaid who laughed at Thales for falling into the well. It is more conspicuously true of the scientific Thales, so interested in the stars, that he did not advert to the well. Accordingly, it would be a mistake to suppose that scientific observation is some mere passivity to sense impressions. It occurs within its own dynamic context and the problem is to distinguish that cognitional orientation from the orientation of concrete living.

To be alive, then, is to be a more or less autonomous centre of activity. It is to deal with a succession of changing situations; it is to do so promptly, efficaciously, economically; it is to attend continuously to the present, to learn perpetually from the past, to anticipate constantly the future. Thus, the flow of sensations, as completed by memories and prolonged by imaginative acts of anticipation, becomes the flow of perceptions. It is of the latter, perceptual flow that we are conscious. It is only when the perceptual flow goes wrong that the mere sensation bursts into consciousness as, for example, in the experience of trying to go down another step when already one has reached the floor.

Now what differentiates the perceptual flow in one man from that of another, is found in the pattern of interests and objectives, desires and fears, that emphasize elements and aspects of sensible presentations, enrich them with the individual's associations and memories, and project them into future courses of possible, fruitful activity. In some such fashion, it would seem, must be explained the differences in the perceptions of men and women, of people in different occupations, different climates, different stages in human history.

Hence, to become a scientific observer is, not to put an end to perception, but to bring the raw materials of one's sensations within a new context. The interests and hopes, desires and fears, of ordinary living have to slip into a background. In their place, the detached and disinterested exigences of inquiring intelligence have to enter and assume control. Memories will continue to enrich sensations, but they will be

memories of scientific significance. Imagination will continue to prolong the present by anticipating the future, but anticipations with a practical moment will give way to anticipations that bear on a scientific issue. Just as the woodsman, the craftsman, the artist, the expert in any field acquires a spontaneous perceptiveness lacking in other men, so too does the scientific observer.

Still there is a difference and to it the scientist alludes when he insists that scientific observation is a matter of seeing just what there is to be seen, hearing exactly whatever sounds are sounded, and so forth. This claim cannot, I think, be taken literally, for the impartial and accurate observer, no less than anyone else, is under the dominance of a guiding orientation. Still, the claim does possess its elements of truth, for the guiding orientation of the scientist is the orientation of inquiring intelligence, the orientation that of its nature is a pure, detached, disinterested desire simply to know. For there is an intellectual desire, an Eros of the mind. Without it, there would arise no questioning, no inquiry, no wonder. Without it, there would be no real meaning for such phrases as scientific disinterestedness, scientific detachment, scientific impartiality. Inasmuch as this intellectual drive is dominant, inasmuch as the reinforcing or inhibiting tendencies of other drives are successfully excluded, in that measure the scientific observer becomes an incarnation of inquiring intelligence and his percepts move into coincidence with what are named the data of sense. Accordingly, it is not by sinking into some inert passivity but by positive effort and rigorous training that a man becomes a master of the difficult art of scientific observation.

### 2. THE CANON OF OPERATIONS

Secondly, there is a canon of operations.

Just as inquiry into the data of sense yields insights that are formulated in classical and statistical laws, so inversely, the laws provide premises and rules for the guidance of human activity upon sensible objects. Such activity, in its turn, brings about sensible change to bring to light fresh data, raise new questions, stimulate further insights, and so generate the revision or confirmation of existing laws and in due course the discovery of new laws.

In the first instance, then, the canon of operations is a principle of cumulative expansion. Laws guide activities, which bring forth new laws, which guide further activities, and so forth indefinitely.



Secondly, the canon of operations is a principle of construction. Man knows best what man makes for himself and so we began our study of insight by examining that elementary artefact, the cart-wheel. But the development of science is followed by a technological expansion, by a vast increase of the things that man can make for himself and so can understand adequately because he has made them. Moreover, the more refined and resourceful technology becomes, the greater the frequency of the artificial synthesis of natural products. Thus, Nature itself becomes understood in the same fashion as man's own artefacts.

Thirdly, the canon of operations is a principle of analysis. Clearly, man can analyse the objects that he himself can construct. But it is no less true that he can also analyse objects which, as yet, he cannot manage to construct. For analysis is a mental construction and, where operational control fails, theoretical knowledge can step in to account for the failure of control, to identify the uncontrolled factors, to determine and measure their activity and influence, to discount their perturbing effect, and so to extrapolate to the law that would hold did they not interfere.

Fourthly, the canon of operations is a principle of cumulative verification. For laws guide operations successfully in the measure that they are correct. Hence, in so far as laws and their implications in a vast variety of situations are repeatedly found successful guides of operations, their initial verification is cumulatively confirmed.

Fifthly, the canon of operations provides a test of the impartiality and accuracy of observations. I do not mean that it makes intellectual detachment and disinterestedness superfluous for, as is clear, the power of the totalitarian state can both corrupt the judge and pack the jury. But, when a general conspiracy is absent, when ordinary good will can be presupposed, then the canon of operations, sooner or later, will exhibit on a grand scale in conspicuous failures even slight mistakes and oversights in observation.

Sixthly, the canon of operations is a principle of systematization. Insights yield simple laws, but simple laws are applicable only in pure cases. The law of a free fall holds in a vacuum. But operations do not occur in a vacuum. Hence, one is driven to determine the law of air resistance and the laws of friction. Similarly, Boyle's law has to be complemented with Charles' and Gay-Lussac's, and all three need to be corrected by Van der Waals' formula. Thus, the canon of operations is a perpetual recall from the abstract realm of laws to the complexity

of the concrete and so to the necessity of ever more laws. Nor is this all. A mere congeries of laws will not suffice. For if one is to operate upon the concrete, one must be able to employ at once several laws. To employ several laws at once, one must know the relations of each law to all the others. But to know many laws, not as a mere congeries of distinct empirical generalizations, but in the network of interrelations of each to all the others, is to reach a system.

Seventhly, the canon of operations is a source of higher viewpoints. Already attention has been drawn* to the difference between the circuit of the mathematician and the circuit of the empirical scientist. The mathematician mounts to higher viewpoints inasmuch as the symbolic representation of his previous terms and relations supplies the image in which insight grasps the rules of a more comprehensive systematization. But the empirical scientist advances to higher viewpoints, not solely by the construction of symbolic images, but more fundamentally by the expansiveness, the constructiveness, the analyses, the constant checking, and the systematizing tendencies of the canon of operations. In virtue of that canon, fresh data are ever being brought to light to force upon scientific consciousness the inadequacies of existing hypotheses and theories, to provide the evidence for their revision and, in the limit, when minor corrections no longer are capable of meeting the issue, to demand the radical transformation of concepts and postulates that is named a higher viewpoint.

### 3. THE CANON OF RELEVANCE

Thirdly, there is a canon of relevance.

The canon of selection and the canon of operations might be regarded as obverse and reverse of the same coin. Both are concerned with the elementary fact that the empirical inquirer is out to understand, not what he may imagine, but what he actually sees. The canon of relevance, on the other hand, aims at stating the type of understanding proper to empirical science.

Now it would be a mistake to say that the empirical scientist has no use whatever for final, material, instrumental, or efficient causes. Inasmuch as he praises the value and utility of science, he speaks of final causes. Inasmuch as he places that value and utility in the technological transformation of raw materials, he knows and acknowledges material and instrumental causes. Inasmuch as he accepts and acts upon the canon

* Chapter II, §1.2.



of operations, he is an efficient cause engaged in testing his knowledge by its consequences.

However, it is also clear that such types of causality lie not in the core but on the periphery of empirical science. They are the concern, not of pure, but only of applied science. They have to do with the use to which science may be put rather than with the inner constituents of science itself.

The canon of relevance regards such inner constituents. It states that empirical inquiry primarily aims at reaching the intelligibility immanent in the immediate data of sense. Once that intelligibility is reached, one can go on to ask about the value or utility of such knowledge, about the tools that can be fashioned under its guidance, about the transformations of materials man can effect with such tools. But the first step, on which all others rest, is to grasp the intelligibility immanent in the immediate data of sense.

What precisely does the canon mean?

First, it presupposes that the same data can provide a starting-point for different types of insight.

Secondly, it observes that questions about final, material, instrumental, and efficient causality automatically head one away from the data in hand. If I ask about the end of the cart-wheel, I turn to carts and carting and soon find myself involved in the economics of transportation. If I ask about the wood or iron of the cart-wheel, the issue is shortly transposed to forestry and mining. If I ask about the wheelwright's tools, I am led on to discuss technology. If I inquire into the wheelwright himself, I am confronted with the sociology of the division of labour and with the psychology of the motivation of craftsmen.

Thirdly, it also observes that there is a further type of insight that arises immediately from the data. Such is the grasp that precedes and grounds the definition of the circle. Such was Galileo's insight formulated in the law of falling bodies. Such was Kepler's insight formulated in the laws of planetary motion. Such was Newton's insight formulated in the theory of universal gravitation. Such has been the point in the now well established technique of measuring and correlating measurements. Such is the goal of the classical heuristic structure that seeks to determine some unknown function by working out the differential equations, of which the unknown function will be a solution, and by imposing by postulation such principles as invariance and equivalence.

Fourthly, it notes that this intelligibility, immanent in the immediate

data of sense, resides in the relations of things, not to our senses, but to one another. Thus, mechanics studies the relations of masses, not to our senses, but to one another. Physics studies the relations of types of energy, not to our senses, but to one another. Chemistry defines its elements, not by their relations to our senses, but by their places in the pattern of relationships named the periodic table. Biology has become an explanatory science by viewing all living forms as related to one another in that complex and comprehensive fashion that is summarily denoted by the single word, evolution.

Fifthly, it notes that this intelligibility is hypothetical. It does not impose itself upon us, as does the multiplication table or the binomial theorem. It announces itself as a possibility, as what could be the relevant correlation or function or law. Now the necessary must be, but the possible, though it can be, may, in fact, be or not be. Hence, empirical science rests upon two distinct grounds. As insight grasping possibility, it is science. As verification selecting the possibilities that in fact are realized, it is empirical.

There is, then, an intelligibility immanent in the immediate data of sense; it resides in the relations of things, not to our senses, but to one another; it consists not in an absolute necessity, but in a realized possibility.

Ought there not to be introduced a technical term to denote this type of intelligibility? The trouble is that the appropriate technical term has long existed but also has long been misunderstood. For the intelligibility that is neither final nor material nor instrumental nor efficient causality is, of course, formal causality. But when one speaks of formal causality, some people are bound to assume that one means something connected with formal logic, others are bound to assume that one means merely the heuristic notion of the 'nature of . . .', the 'such as to . . .', the 'sort of thing that . . .'. If both of these misinterpretations are excluded, what we have called the intelligibility immanent in sensible data and residing in the relations of things to one another, might be named more briefly formal causality or rather, perhaps, a species of formal causality.

## 4. THE CANON OF PARSIMONY

Fourthly, there is a canon of parsimony.

It is at once obvious and difficult. It is obvious inasmuch as it forbids the empirical scientist to affirm what, as an empirical scientist, he does not know. It is difficult inasmuch as knowing exactly what one knows and what one does not know has been reputed, since the days of Socrates, a rare achievement. None the less, some account of this fundamental canon must be attempted at once, even though its full meaning and implications can come to light only later.



On the previous analysis, then, empirical method involves four distinct elements, namely

(1) the observation of data,
(2) insight into data,
(3) the formulation of the insight or set of insights, and
(4) the verification of the formulation.

Now, the empirical investigator cannot be said to know what is not verified and he cannot be said to be able to know the unverifiable. Because, then, verification is essential to his method, the canon of parsimony in its most elementary form excludes from scientific affirmation all statements that are unverified and, still more so, all that are unverifiable.

### 4.1 *Classical Laws*

Secondly, verification is of formulations, and formulations state

(1) the relations of things to our senses, and
(2) the relations of things to one another.

It follows that formulations contain two types of terms which may be named respectively, experiential conjugates and pure or explanatory conjugates.

Experiential conjugates are correlatives whose meaning is expressed, at least in the last analysis, by appealing to the content of some human experience.

Thus, 'colours' will be experiential conjugates when defined by appealing to visual experience; 'sounds' when defined by appealing to auditory experiences; 'heat' when defined by appealing to tactile experience; 'force' when defined by appealing to an experience of effort, resistance, or pressure.

It is clear enough that experiential conjugates satisfy the canon of parsimony. The fundamental set of such terms is verified, not only by scientists, but also by the secular experience of humanity. Scientists add further terms in virtue of their specific preoccupation but as long as

these terms satisfy the definition of the experiential conjugates, they will be in principle verifiable.

Pure (or explanatory) conjugates, on the other hand, are correlatives defined implicitly by empirically established correlations, functions, laws, theories, systems.

Thus, masses might be defined as the correlatives implicit in Newton's law of inverse squares. Then, there would be a pattern of relationships constituted by the verified equation; the pattern of relationships would fix the meaning of the pair of coefficients, $m_1$, $m_2$; and the meaning so determined would be the meaning of the name, mass. In like manner, heat might be defined implicitly by the first law of thermodynamics and the electric and magnetic field intensities, $E$ and $H$, might be regarded as vector quantities defined by Maxwell's equations for the electromagnetic field.*

Now, such pure conjugates satisfy the canon of parsimony. For the equations are or can be established empirically. And by definition pure conjugates mean no more than necessarily is implicit in the meaning of such verified equations.

There is, however, a difference between the mode of verifying pure conjugates and the mode of verifying experiential conjugates. For the experiential conjugate is either a content of experience, such as seeing red or touching extension, or else a correlative to such a content, for instance, red as seen or extension as touched, or finally, a derivative of such correlatives, as would be the red that could be seen or the extension that could be touched On the other hand, the pure conjugate has its verification, not in contents of experience nor in their actual or potential correlatives, but only in combinations of such contents and correlatives. I see, for instance, a series of extensions and alongside each I see a yard-stick; from the series of combinations I obtain a series of measurements; from another series of combinations I obtain another series of measurements; from the correlation of the two series, together with the leap of insight, I am led to posit as probably realized some continuous function; pure conjugates are the minimal correlatives implicit in such functions, and their verification finds its ground, not in experience as such, but only in the combination of combinations, etc., etc., etc., of experiences.

As the reader will have noted, the definitions of pure and experiential conjugates drop all mention of things whether related to one another

* See, on this point, Lindsay and Margenau, p. 320.



or to our senses. The reason for this omission is that the notion of the 'things' is highly ambiguous and, as yet, we are unprepared to apply the canon of parsimony to it.* However, though the notion of thing has been omitted, the point of the distinction between the relations of things to one another and to our senses remains. For in every experience one may distinguish between content and act, between the seen and the seeing, the heard and the hearing, the tasted and the tasting, and so forth Let us represent, then, any series of experiences by the series of pairs, $AA'$, $BB'$, $CC'$, . . . , where the unprimed letters denote contents and primed letters denote the corresponding acts. Now correlations may be reached by combining the unprimed components, $A$, $B$, $C$, . . . , or by combining the primed components $A'$, $B'$, $C'$, . . . , or by combining both primed and unprimed components. In the first case, one will deal with the relations of contents to one another and one will prescind from the corresponding acts; and in this fashion, without any mention of things, one deals with what hitherto has been named the relations of things to one another. In the second case, one will prescind from contents and correlate acts, to obtain a psychological or cognitional theory. In the third case, one will be employing experiential conjugates and further information will be needed to settle whether one is working towards the goal of natural science or of cognitional theory.

Further, as this analysis reveals, there are only three basic alternatives. Either one's terms are experiential conjugates or else they are pure conjugates based on combining contents alone or finally they are a special case of pure conjugates based on combining acts alone. Still, theoretical analysis is one thing, and concrete practice is another. Thus, one would be inclined to say that physicists move easily and unconsciously back and forth between the use of experiential and pure conjugates. When they are called upon to define their terms, commonly they will suppose that definition comes at the beginning and so offer definitions of experiential conjugates. On the other hand, methodologists and theorists of empirical science will be puzzled by the multiplicity of definitions available in a mature science and so they tend to disagree with one another. Thus E. Cassirer in his well-known *Substance and Function* emphasizes the relational and serial aspect of scientific terms. V. Lenzen in his *Nature of Physical Theory* emphasizes the genetic process that begins from experiential contents of force, heat, extension, duration,

* See Chapter VIII.

etc., to move through a process of redefinition towards terms implicitly defined by empirically established principles and laws. Finally, Lindsay and Margenau in their *Foundations of Physics*, while they are more concerned with ideas than concepts, may be said to exhibit a preference for terms implicitly defined by equations.

For our purpose it would seem to be sufficient to reveal the materials which scientists and theorists of science employ in different manners and to show that these materials, despite incidental variations, satisfy the canon of parsimony

### 4.2 *Statistical Laws*

However, besides classical laws, there also are statistical laws; and since the latter as well as the former are verifiable, it would seem that, besides pure and experiential conjugates, one must also recognize events. When the demonstrator in a lecture room propounds a law of nature and proceeds to illustrate it by an experiment, he does not inform his class that the law will be refuted if the experiment does not work. On the contrary, he points out that the law retains its validity even if it happens that the experiment is a failure. And members of the class may add interest to the proceedings by determining the statistical law of the demonstrator's successes. The law of nature, then, is one thing. The event of its illustration is another. And such events are subject to laws of a different type which is named statistical.

What, then, is an event? The simplest answer is to say that it is a primitive notion too simple and obvious to be explained. Still, all primitive notions, however simple and obvious, are related to other equally primitive notions, and the set may be fixed by offering the data in which insight may grasp the relations.

Let us begin, then, by formulating our answer Events stand to conjugates as questions for reflection stand to questions for intelligence.

What is meant by a conjugate has been explained

Moreover, knowledge of conjugates results from a process of inquiry, of asking questions; and the relevant questions all have the peculiarity that none of them can be answered appropriately by simply saying either 'Yes' or 'No'. Thus, when one asks what is the 'nature of . . .', 'the sort of thing that . . .', the 'such as to . . .', the correlation to be specified, the indeterminate function to be determined, it is always meaningless to answer 'Yes' or 'No'. One is called upon to state the nature, specify the correlation, determine the function, and that can be done only by achieving the insights that ground the formulation, first, of experiential and, later, of pure conjugates.



But for every answer to a question for intelligence, there is a corresponding question for reflection; and all questions for reflection have the peculiarity that they can be answered appropriately simply by saying either 'Yes' or 'No'. If I ask what a body is, I can also ask whether there are bodies. If I ask how bodies fall, I can also ask whether bodies fall. If I ask how bodies would fall in a vacuum, I can also ask whether any bodies ever fall in a vacuum. Generally, the enunciation of every law can be followed by the question for reflection that asks whether the law is verified, and the definition of every term can be followed by the question for reflection whether the defined exists or occurs. Inversely, whenever one asserts verification or existence or occurrence, one may be asked what is verified, what exists, what occurs.

Thus, questions for intelligence and questions for reflection are universally concomitant and complementary.

There is a parallel concomitance and complementarity between conjugates and events. Without events, conjugates can be neither discovered nor verified. Without conjugates, events can be neither distinguished nor related. Such, I submit, is the elementary scheme in which insight can grasp what is meant by the otherwise puzzling name, event.

Now formulations that concern events satisfy the canon of parsimony. For probability expectations or statistical laws are formulations that answer the question for intelligence, How often? They concern events, for the frequency they assign is a frequency of events. Finally, the frequency assigned by a statistical law is verifiable; for the assigned frequency is an ideal frequency; it is distinct from the actual frequencies that can diverge from it in non-systematic fashion; and it can be verified by appealing to those actual frequencies.

At this point our account of the canon of parsimony must be brought to a close. As the reader will have observed, attention has been confined to the positive aspects of the canon, to the experiential conjugates, the pure conjugates, and the events that are the terms of verifiable formulations. Whether things and their existence satisfy the canon, is a further issue on which we have not touched. On the other hand, the negative or exclusive aspects of the canon, though they constitute its chief significance and utility, are too numerous to be mentioned and can best be dealt with incidentally when occasion arises.

### 5. THE CANON OF COMPLETE EXPLANATION

Fifthly, there is a canon of complete explanation.

The goal of empirical method is commonly stated to be the complete explanation of all phenomena or data.

In a sense, perseverance in the pursuit of this goal is assured by the canon of selection especially when it is implemented by the canon of operations. Any particular investigator may overlook or ignore certain data. But his oversight or disregard will normally be corrected by other investigators substantiating their hypotheses and refuting those of their predecessors by appealing to hitherto neglected facts.

None the less, a separate enunciation of this canon is relevant particularly at the present time when a mistaken twist given to scientific method at the Renaissance finally is being overcome.

Where we distinguished between experiential and pure conjugates, Galileo distinguished between secondary and primary qualities. Secondary qualities were merely subjective appearances that arise in an animal's senses as a result of the action of other primary qualities, such appearances were illustrated by colour as seen, sounds as heard, heat as felt, tickling as experienced, and the like. Primary qualities, on the other hand, were the mathematical dimensions of the real and objective, of matter in motion. Hence, while we would place scientific progress in the movement from experiential to pure conjugates, Galileo placed it in the reduction of the merely apparent secondary qualities to their real and objective source in primary qualities.

The crucial difference between the two positions regards space and time. For Galileo, they were primary qualities, for there would be extension and duration if there were matter and motion whether or not any animals with their sensitive experiences existed. For us, on the other hand, there is to be drawn the same distinction between extension and duration as experiential and as pure conjugates as there is to be drawn between the two formulations of colours or sound or heat or electric phenomena.

As experiential conjugates, extensions and durations are defined as correlatives to certain familiar elements within our experience.

As pure conjugates, extension and duration are defined implicitly by the postulate that the principles and laws of physics are invariant under inertial or, generally, under continuous transformations.

Thus, on our analysis, the space-time of Relativity stands to the extensions and durations of experience in exactly the same relations as wave-lengths of light stand to experiences of colour, as longitudinal waves in air stand to experience of sound, as the type of energy defined by the first law of thermodynamics stands to experiences of heat, etc.



Moreover, in our analysis, this conclusion rests upon the canon of complete explanation. All data are to be explained. The explanation of data consists in a process from experiential conjugates towards pure conjugates. Therefore, from extensions and durations as experienced, there must be a process to extensions and durations as implicitly defined by empirically established laws.

Further, as extension and duration, so also local movement has a preliminary definition in terms of experiential conjugates and an explanatory definition in terms of pure conjugates. It was obvious and excusable for Galileo and Kepler and Newton to conceive local movement in the two steps of determining a path or trajectory and then correlating points on the path with instants of time. After all, when a man crosses the street, we see at once the whole distance that he traverses but we apprehend the duration of his movement as concomitant with the duration of our watching. None the less, this account of local movement can be no more than preliminary for, throughout, it is in terms of movement as related to us, as in terms of experiential conjugates. What movement is, when movements are defined in terms of their relations to one another, is another question. The answer to it will depend upon the answer that determines extensions and durations as pure conjugates; and so it is that Relativity mechanics conceives a velocity, not as a function of three dimensions with time as a parameter, but as a function of four dimensions, of which three are spatial and the fourth temporal.

If we add the canon of parsimony to the canon of complete explanation, more fundamental objections to the Galilean theory of scientific explanation come to light.

Both experiential and pure conjugates are verifiable, and in so far as either are verified, they possess an equal claim upon reasonable affirmation. It follows that Galileo's repudiation of secondary qualities as mere appearance is a rejection of the verifiable as mere appearance.

Inversely, Galileo did not base his affirmation of the reality and objectivity of primary qualities upon a claim that these qualities, as he conceived them, were verifiable or verified. Accordingly, his affirmation was extra-scientific. It did not satisfy the canon of parsimony; and today, if anyone were to try to bring the Galilean position into line

with that canon, first of all he would have to settle an account with Einstein who has made various proposals regarding the space-time of physics and has some grounds for supposing his line of thought verifiable and, to some extent, verified.

## 6. THE CANON OF STATISTICAL RESIDUES

Sixthly, there is a canon of statistical residues. It presupposes the existence of inquiry of the classical type and from that premise it concludes to the existence of residues that call for statistical inquiry.

### 6.1 *The General Argument*

The basic distinction is between abstract system and particular cases. Both are objects of insight. But the particular case is the typical instance, presented by sense or imagination, and understood by insight into the presentation. In contrast, the abstract system is neither sensible nor imaginable; it is a conceptual object constituted by terms and relations that, at least in the last resort, are defined implicitly.

Particular cases are relevant both to the genesis and to the application of abstract system. For the formulation of system comes at the end of a cumulative series of insights into different particular cases. Again, once abstract system is formulated, it can be applied to concrete situations only in so far as there occur insights into the situations as sensibly given; for without such insights there cannot be selected the relevant laws of abstract system, there cannot be determined the mode in which the laws combine in the concrete situation, and there cannot be substituted numerical values for the variables and undetermined constants of the general formulae.

Now let us suppose full knowledge of all classical principles and laws. Then we suppose full knowledge of abstract system: for principles and laws are relations; such relations necessarily involve the terms that they define implicitly; and abstract system consists in terms implicitly defined by the relations expressed in verified principles and laws.

However, if this full knowledge of abstract system is to be applied to the concrete universe, there will be needed a manifold of insights into particular cases. For, as was noted above, abstract system is applied to concrete situations only and inasmuch as insight into the situations selects the relevant laws, determines the mode of their combination, and substitutes numerical values for the variables and undetermined constants of the laws.



Still, the manifold of particular cases is enormous, and so there arises the question whether it can be cast into some ordered sequence. If it can, then knowledge of the sequence and of a few strategically chosen particular cases would suffice to transform mastery of abstract system into a scientific understanding of the universe. But if it cannot, if the manifold of particular cases does not form any kind of ordered sequence, then abstract system can be applied only to a limited range of particular cases, and new methods must be found if we are to reach an understanding of the concrete universe as a whole.

In fact, it can be shown that there do exist recurrent particular cases. For example, our planetary system is periodic; it is an individual set of masses; most of them are visible; and a relatively small number of concrete insights makes it possible to determine an indefinite sequence of particular cases.

On the other hand, while such schemes of recurrence are many not only in number but also in kind, still each presupposes materials in a suitable constellation that the scheme did not bring about, and each survives only as long as extraneous disrupting factors do not intervene. The periodicity of the planetary system does not account for its origin and cannot guarantee its survival.

Moreover, there does not seem to exist any universal scheme that controls the emergence and survival of the schemes that we know. Accordingly, in the last analysis we are driven to accept the second alternative. There does not exist a single ordered sequence that embraces the totality of particular cases through which abstract system might be applied to the concrete universe. In other words, though all events are linked to one another by law, still the laws reveal only the abstract component in concrete relations; the further concrete component, though mastered by insight into particular cases, is involved in the empirical residue from which systematizing intelligence abstracts; it does not admit general treatment along classical lines; it is a residue, left over after classical method has been applied, and it calls for the implementation of statistical method.

Such is the general argument, and a more detailed account of its meaning now has to be attempted.

### 6.2 *The Notion of Abstraction*

A first task is to clarify the notion of abstraction. On a simple and common view, the abstract is an impoverished replica of the concrete.

'Red' means what is common to all instances of 'red'. 'Man' means what is common to all instances of 'man'. That is all there is to it.

Now with this view of abstraction, one can admit classical laws and one can admit statistical laws but one will be at a loss to determine some coherent manner in which both classical and statistical laws can be acknowledged. This may be shown as follows:

Let $A, B, C,$ denote sensible data, and let $a, a', a'', \ldots; b, b', b'', \ldots; c, c', c'', \ldots$; denote the totality of their impoverished replicas. Then, there is no aspect of sensible data without its impoverished replica; inversely, the totality of sensible data can be constructed out of the totality of impoverished replicas.

Hence, if one admits some classical laws, one admits that some impoverished replicas are related systematically. Moreover, if one admits the classical laws as objective, there must be systematic relations not only between the impoverished replicas but also between the concrete aspects of sensible data to which they correspond. It follows that the classical laws can be objective only if they hold in the concrete. Finally, it will be only by denying the canon of complete explanation of all data, that one can admit systematic relations between some impoverished replicas and deny systematic relations between others. It will follow that the only laws will be classical laws, and that statistical laws cannot be more than a cloak for ignorance.

Inversely, if one admits some statistical laws, then one denies systematic relations between some impoverished replicas. If the statistical laws are objective, there cannot be systematic relations between the corresponding aspects of sensible data. At least in those cases, classical laws are excluded. Moreover, to show that classical laws are not merely the macroscopic illusion resulting from a multitude of microscopic, random occurrences, a correct theory of the abstract is needed; and in the present hypothesis, that correct theory is lacking.

What is, then, the correct theory?

So far from being a mere impoverishment of the data of sense, abstraction in all its essential moments is enriching. Its first moment is an enriching anticipation of an intelligibility to be added to sensible presentations; there is something to be known by insight. Its second moment is the erection of heuristic structures and the attainment of insight to reveal in the data what is variously named as the significant, the relevant, the important, the essential, the idea, the form. Its third moment is the formulation of the intelligibility that insight has revealed. Only in this third moment does there appear the negative aspect of abstraction, namely, the omission of the insignificant, the irrelevant, the negligible, the incidental, the merely empirical residue. Moreover, this omission is neither absolute nor definitive. For the empirical residue possesses the universal property of being what intelligence abstracts from. Such a universal property provides the basis for a second set of heuristic procedures that take their stand on the simple premise that the non-systematic cannot be systematized.



Now our whole effort has been to draw attention to the fact of insight, to the enriching moments on which abstraction follows. Accordingly, it is in this sense that we affirm classical laws to be abstract. So far from being an impoverishment of sensible data, abstraction is an enrichment that goes beyond them. Because abstraction goes beyond the sensible field, the frontiers of the abstract are not coterminous with the frontiers of the experienced. Hence, full and exact knowledge of the systems to be reached by abstraction by no means denies the existence of an empirical residue that is non-systematic. Again, just as in abstraction we prescind from the empirical residue, so when we come to the concrete applications of abstract principles and laws, we are forced to take into account the non-systematic conditions under which the systematic has its concrete realization

### 6.3 *The Abstractness of Classical Laws*

In the second place, it may be well to recall that classical laws are abstract

(1) in their heuristic anticipation,
(2) in the experimental techniques of their discovery,
(3) in their formulation, and
(4) in their verification.

They are abstract in their heuristic anticipation. For that anticipation rests on the detached and disinterested drive of inquiry, and it consists in a pure desire to understand. Hence, the canon of relevance demands that one seek the immanent intelligibility of the data; the canon of parsimony demands that one add to the data no more than the formulation of what is grasped by understanding and verified, and the canon of complete explanation demands that this parsimonious addition of intelligibility be effected for all data. Moreover, this anticipated enrichment is seen to be universal: the nature to be known will be the same

for all data that are not significantly different, and the correlation to be specified is reached only if it holds for all parallel instances.

Secondly, classical laws are abstract in the experimental techniques of their discovery. For the experimenter makes no pretence to deal with concrete situations in their native complexity; on the contrary, he aims overtly at reducing that complexity to a minimum and so he does all he can to bring the concrete into some approximation to an ideal, typical, definable conjunction of materials and agents. Accordingly, as he begins with an effort to secure materials from which all impurities have been removed, so he ends with an argument that rests on their theoretical definitions. As he begins by requiring instruments constructed in accord with accurate specifications, so he ends by interpreting their performance on the basis of their ideal, often schematic, structure. He measures, but he does so many times, and his accepted result is just the probable mean of actual results. He reaches a conclusion with which others agree, but the agreement makes allowance for the intrusion of extraneous factors and it acknowledges no more than a limited number of significant decimal places. At every turn it seems apparent that the concern of experiment is to determine, not the particular observable qualities of the particular materials with which one deals, but a theoretical correlation between definable and abstract entities.

Thirdly, classical laws are abstract in their formulation. As laws, they are correlations linking correlatives, and the correlatives are never the unique data of some particular time and place. Indeed, they are not even generalized data, but generalized combinations of combinations of combinations of data. Nor may one suppose that the data, taken in these serial combinations, uniquely determine what the law must be. For the discontinuous set of observations, represented, say, by points on a graph, can be satisfied by any number of laws, of which the scientist chooses the one that, all things considered, he reputes to be the simplest. Enriching abstraction is still at work.

Fourthly, classical laws are abstract in their verification. For verification is reached, not by appealing to this or that isolated instance, but by securing as large and various a range of instances as both direct and indirect procedures make possible. It follows that what is verified is, not this or that particular proposition, but the general, abstract formulation that alone admits the large and various range of applications. Again, to repeat the argument from another viewpoint, what is verified is what can be refuted or revised. What can be refuted or revised, is the general, abstract formulation. And so what is verified is the general, abstract formulation.



### 6.4 *Systematic Unification and Imaginative Synthesis*

In the third place, an objection must be met. Taken singly, classical laws are abstract. But what is true of single laws, need not be true of the totality of laws. The single laws are abstract because they do not cover the totality of aspects of the data. But the totality of laws would cover that totality of aspects, and so the totality would be not abstract but concrete.

Now this objection may be merely a reversion to the assumption that abstraction yields merely an impoverished replica of sensible data. In that case, it has been met already. For the totality of aspects of data explained by the totality of classical laws will not include the aspects that we have named an empirical residue.* Even when all classical laws are known, individuality and continuity, particular place and particular time, will not be explained but abstracted from.

However, the objection may be advanced by those that grant abstraction to be not impoverishing but enriching. They will point out that the canon of operations forces empirical inquiry to go beyond the mere aggregation of isolated laws to the development of systems. It is not enough to know the law of falling bodies, the law of air resistance, the law of friction. One also has to know how to apply these laws simultaneously if one is to solve practical problems. Hence, the discovery of laws has to be accompanied by the discovery of correlations between laws and, no less, of correlations between the correlations. There exists, then, a movement towards the systematic unification of classical laws and, as this unification is prompted by concrete problems, one may expect that, when all laws are known exactly and completely, there also will be known a systematic unification commensurate with world process in its concrete, historical unfolding.

This consideration is, I think, impressive. But, strangely enough, world process in its concrete historical unfolding rather conspicuously makes a large and generous use of the statistical techniques of large numbers and long intervals of time; it exhibits not a rigid but a fluid stability; it brings forth novelty and development; it makes false starts and suffers break-downs. It would seem, then, that an understanding of the concrete unfolding of the world process will not be based exclu-

* See Chapter I, § 5.

sively on classical laws, however exactly and completely known, but in a fundamental manner will appeal to statistical laws.

Accordingly, the facts force us to a closer scrutiny of the argument from the systematic unification of laws, and the scrutiny brings to light an underlying ambiguity. It is one thing to attain a systematic unification; it is another to reach an imaginative synthesis. Thus, Riemannian geometry is a systematic unification, for it provides a single set of principles and techniques for dealing with *n*-dimensional manifolds of various curvatures. But Riemannian geometry is not an imaginative synthesis for we cannot imagine more than three dimensions and we normally imagine only flat surfaces. Again, Ptolemy and Copernicus possessed imaginative syntheses of celestial movements, but the laws of those movements were discovered by Galileo and Kepler, and the systematic unification of the laws was the achievement of Newtonian mechanics. To offer another example, nineteenth-century physicists made a notable series of efforts to construct an imaginable model of the aether.* But the fruit of their labours was a systematic set of equations verifiable in pointer readings. Today one may prefer Einstein, who clings to determinist views, or one may join the majority, who regard Quantum Mechanics as satisfactory. But neither alternative offers an imaginative synthesis. For Einstein offers a set of differential equations for a four-dimensional, curved manifold, and Quantum Mechanics as it originated by giving up the attempt to carry through N. Bohr's model of the atom, so now it refuses to portray the objective process that leads up to observables.

There is, then, a difference between systematic unification and imaginative synthesis. Systematic unification is effected in the logical or conceptual order. It is attained when the totality of laws is reduced to minimum sets of defined terms and postulates, so that any law can be related to any other, and any aggregate of laws can be intelligibly combined and simultaneously employed. On the other hand, an imaginative synthesis is secured when images, informed by insight, are altered in accord with known laws. In this fashion one may imagine the sun, the planets, and their satellites in appropriate collocations and in order stand their imagined movements in accord with mechanical laws. Clearly, such imaginative synthesis goes beyond the abstract content of the laws and supposes that certain bodies exist in certain relative positions with velocities less than the velocity of escape. One has passed from the tasks of pure science; one has introduced the suppositions and the facts that pertain to applied science. Now the ultimate attainment of a systematic unification of classical laws will not include particular matters of fact, and so that ultimate attainment cannot include an imaginative synthesis.

* See E. T. Whittaker, *A History of the Theories of Aether and Electricity*, Dublin University Press, Longmans, London, 1911.



As systematic unification does not include imaginative synthesis, so it does not even guarantee its possibility. It is true enough that images are necessary for the emergence of insights, but the images may be not representative but symbolic, not pictures of the visible universe but mathematical notations on pieces of paper. Even if one supposed that, just as the image of the cart-wheel approximates the definition of the circle, so some representative image approximated every classical law, none the less, it would not follow that the aggregate of approximate images might somehow coalesce into a composite picture that approximated to the systematic unification of all laws.

The objection, then, breaks down on two points. In itself, it is inconclusive. Knowledge of all classical laws would be an understanding of the concrete only if it included a vast imaginative synthesis. It is true that empirical inquiry heads for a systematic unification of its laws. But there is no evidence that such a systematic unification ensures the possibility of any imaginative synthesis. Moreover, if the totality of classical laws provided an understanding of the concrete, statistical laws would be superfluous. But the conspicuous use of statistical techniques in world process shows that statistical laws are not superfluous in an understanding of our universe.

### 6.5 *The Existence of Statistical Residues*

In the fourth place, an attempt must be made to indicate more precisely both the indeterminacy of abstract classical laws and the nature of the consequent statistical residues. Hence, it will be argued

(1) that classical laws hold in concrete instances only inasmuch as conditions are fulfilled,

(2) that the conditions to be fulfilled form diverging series, and

(3) that in the general case the patterns of such diverging series are a non-systematic aggregate.

6.5.1 *Classical Laws Conditional.* First, it is possible to apply classical laws to concrete situations and thereby reach conditioned predictions.

For example, if two motor cars are headed for the same spot, if their distances from the spot and their speeds are equal, then they will collide, provided they do not alter their directions or speeds, and provided that no obstacles force them to do so.

Similarly, in the general case, an event, $Z$, can be concluded from prior circumstances, $Y$, provided some $P, Q, R, \ldots$ continue to occur and provided some $U, V, W, \ldots$ do not intervene.

Secondly, the necessity of positing conditions is universal. For the link between the antecedent circumstances and the consequent event rests on abstract classical laws. Just as the discovery of such laws rests on an experimental exclusion of extraneous factors, just as their verification stands despite contrary instances in which extraneous factors are not excluded, so when one returns from the abstract to concrete applications, the possible existence of extraneous factors has to be taken into account.

Thirdly, when the deduced or predicted event is fully determinate, then the conditions must be fulfilled right up to the occurrence of the event.

To return to the example of the two motor cars, it is one thing to infer or predict a collision, and it is quite another to infer or predict that a first contact will be between a very small area, $P$, on one car and a similar very small area, $Q$, on the other. If the cars are travelling at sixty miles an hour and at the present instant they are just one inch apart, one might say that a collision is inevitable. No matter what happens in the remaining fraction of a second, there will be some impact. But under the same assumptions one cannot offer to drop all provisos and yet predict a first contact between specified small areas. For in the last fraction of a second there could occur some alteration of the speed or direction or swaying of either car; and that alteration would upset the prediction.

6.5.2 *The Diverging Series of Conditions.* Next, in the general case, conditions form a diverging series.

For in the general case, any event, $Z$, is deducible from antecedent circumstances, $Y$, provided some $P, Q, R, \ldots$ continue to occur and provided some $U, V, W, \ldots$ do not intervene.

It follows that the occurrence of the $P, Q, R, \ldots$ and the non-occurrence of the $U, V, W, \ldots$ are similarly deducible.

It follows further that the occurrence of, say, $P$, is conditioned by the occurrences, $A, B, C, \ldots$ and the non-occurrences, $G, H, I, \ldots$. Similarly, there will be series of positive and negative conditions for $Q, R, \ldots$ and for $U, V, W, \ldots$. Similarly, each term in these series will have its series of positive and negative conditions, and so forth.



Such, then, is the diverging series of conditions. Any event, $Z$, will occur on the fulfilment of a set of conditions. Each condition in the set will be fulfilled on the fulfilment of its additional set of conditions. Since there are no unconditioned events, there are no unconditioned fulfilments of conditions. Since there are no unconditioned fulfilments of conditions, the diverging series has as many removes as one cares to explore. Finally, since each event ordinarily has several conditions, the series ordinarily diverges.

Certain further properties of the diverging series of conditions may be noted immediately.

Just as the series diverges when one goes back from an event, $Z$, to its antecedents, so it converges when one advances from the antecedents to the event. Accordingly, if one were to suppose that the concrete pattern of the diverging series had been worked out to some $n$th remove and if one ascertained the fulfilment of all the conditions at that remove, then one's enormous labour would yield no more than the deduction of the event, $Z$, and the intervening occurrences and the non-occurrences. So far from promising the deduction of all world situations from a single situation, this structure offers no more than the deduction of a converging series of events from as large a set of initial observations as one pleases.

Moreover, the conditions of any event, $Z$, at any $n$th remove, are scattered in space and time. They are scattered in space, inasmuch as the occurrences and non-occurrences conditioning the event, $Z$, whether directly or indirectly, proximately or remotely, may be found in any direction and at any distance from the event, $Z$. They are scattered in time, inasmuch as the influence from the condition to the conditioned is propagated with a finite velocity and, in different cases, traverses either equal distances with unequal speeds or unequal distances with equal speeds. Evidently, this scattering of the conditions makes it imperative to know beforehand the aggregate of concrete patterns of diverging series of conditions for events of all kinds; otherwise, one would not know which observations to make and it would be only by luck that one hit upon those that were relevant.

6.5.3 *The Non-systematic Aggregate of Diverging Series.* It was shown in Chapter II that coincidental aggregates can be investigated with scientific generality only by statistical methods. But statistical methods reveal states and probabilities. They tell us nothing about concrete patterns of diverging series of conditions for particular determinate events. It follows that if such concrete patterns are to be investigated with scientific generality then they must not be coincidental aggregates.

However, in the general case, concrete patterns of diverging series of conditions are coincidental aggregates. For any event, say $Z$, occurs if positive conditions, $P$, $Q$, $R$, . . . , occur and negative conditions, $U$, $V$, $W$, . . . , do not occur. What is true of $Z$ is true of all its conditions. Nor, in the general case, can anything beyond the fulfilment of these conditions be required. On the other hand, to demand that the diverging series of conditions be not a coincidental aggregate is to add to the conditions necessary for the occurrence of $Z$; and to introduce such an addition is to depart from the general case and set up a particular case.

Further, even when particular cases exist, they cannot be explained completely along classical lines. For there exists a particular case if there exists an orderly sequence of sets of events such that, other things being equal, the events, $P_i$, result from the events, $P_{i-1}$, for all positive integral values of $i$ from 2 to $n$, where either $n$ is as great a positive integer as one cares to assign or else there is a final set of events, $P_n$, that is similar in all respects to an initial set, $P_1$. Clearly, the diverging series of conditions is brought to heel by any such scheme of perpetual continuity or of perpetual recurrence. Still such a scheme holds only under the proviso that other things are equal and the introduction of defensive mechanisms cannot eliminate the proviso since the mechanisms themselves will depend on classical laws. Moreover, as schemes cannot guarantee their own survival, so they cannot explain their own origin. For if there is a first instance of a set of events, $P_i$, then there is no prior instance in the sequence or circle to account for the first instance; and if there is no first instance, then the origin of the sequence or circle, so far from being explained, is merely denied.

Still it may be urged that, perhaps, world process as a whole is systematic and so, perhaps, the total concrete pattern of diverging series of conditions is in fact orderly. But, in the first place, this is merely a hypothesis. In the second place, it is an extremely doubtful hypothesis, for world process as a whole seems marked by the characteristically statistical devices of large numbers and long intervals of time. Finally, while this doubtful hypothesis implies that statistical method is ultimately mistaken, there is no difficulty in framing opposite hypotheses of equal value which, if true, would imply that ultimately classical method is mistaken.



In the present subsection (§ 6.5) we set out to indicate an exact meaning for both the indeterminacy of classical laws and the consequent canon of statistical residues. It has been argued that classical laws are indeterminate because they are abstract and so can become determinate premises for the deduction of determinate events only if sets of positive and negative conditions are fulfilled. Moreover, from this indeterminacy of the abstract there follows a canon of statistical residues because in the general case such sets of conditions are coincidental aggregates and coincidental aggregates can be investigated with scientific generality only by statistical methods.

In conclusion, two points may be noted. The root fallacy in determinist opposition to the objectivity of statistical knowledge is an oversight of insight. The determinist begins by overlooking the fact that a concrete inference from classical laws supposes an insight that mediates between the abstract laws and the concrete situation; and once that oversight occurs there is precluded a discovery of the difference between systematic processes and coincidental aggregates.

Secondly, our analysis prescinds from all questions regarding the intellectual capacity of Laplace's demon and other non-human beings. Clearly such issues have no bearing on the nature of empirical science or, indeed, of human understanding. Finally, this restriction seems contained in our definition of an orderly sequence; for a sequence is orderly if it can be mastered by an insight that can be expressed in general terms and, it would seem, only human insights can be so expressed.

## 6.6 *The General Character of Statistical Theories*

The statistical heuristic structure, worked out in Chapter II (§ 4.4) may now be determined more fully in the light of the six canons of empirical method.

6.6.1 *Events.* First, then, statistical theories deal with events. For it is the event, the occurrence, the actual happening that cannot be settled by classical laws without the introduction of a concrete, non-systematic manifold of further determinations.

6.6.2 *Not Processes.* Secondly, statistical theories will not analyse processes. For the processes that lead up to events fall under the patterns of diverging series of conditions. Such patterns form a non-systematic aggregate, and the non-systematic as such is not open to investigation.

6.6.3 *Observable Events.* The distinction between process and events raises a further question. For a process seems to be simply a continuum of events. On what principle, then, are some events in the continuum selected by a statistical theory? And on what ground are the rest of the events placed beyond the field of statistical knowledge?

Clearly, the selection is effected by the possibility of observation and, in this respect, there is no difference between classical and statistical theory. A continuum of accurate measurements just cannot be achieved.

The difference arises in the meaning that may be assigned to continuous functions. Because classical theory can envisage concrete process, its continuous functions can be taken to refer to a continuum of events. Because statistical theory, in so far as we provide a meaning for it, prescinds from process, its continuous functions express merely the continuity of the ideal norm from which any observable events diverge non-systematically.

6.6.4 *Foundations.* The foregoing distinction between classical and statistical theories reaches back into the obscure region named foundations. A set of logical or mathematical premises acquires actual objective reference only through a philosophy or a verified scientific theory (see pp. 304–15). If the scientific theory is classical, the reference may be to concrete process. But if the scientific theory is statistical, then the acquired objective reference can be only to isolated events and their probabilities. Note, however, in qualification that 'classical' for us is stripped of its associations with empiricist philosophy, while 'probability' stands within an open heuristic structure and receives its properties from the developing structure.

6.6.5 *Use of Classical Concepts.* Scientifically significant statistical theory will define events by introducing the pure conjugates of classical laws.

For events must be defined if they are to be assigned any frequency but unity. In other words, only the defined type of event is not occurring always and everywhere.



The definition of events must be sought in conjugates. For the event corresponds to the 'Yes' in answer to a question for reflection, and the question for reflection has its content from an answer to a question for intelligence. By the canon of parsimony, verifiable answers to questions for intelligence are in terms of experiential or of pure conjugates.

But statistical investigations in terms of experiential conjugates contain no promise of scientific significance. For experience is within the reach of everyone, but a significant contribution to science rests upon knowledge of previous achievement. Such knowledge in one way or another involves pure conjugates, and so pure conjugates will be used in defining the events of scientifically significant statistical laws. Hence, Quantum Mechanics defines its observables by appealing to classical physics, which developed the notions of Cartesian co-ordinate, linear and angular momentum, energy, and so forth.

6.6.6 *Images and Parsimony.* The canon of parsimony excludes any problem concerning the picture of objects too small to be sensed. For the image as image can be verified only by the occurrence of the corresponding sensation. Thus, the visual image of a small ball can be verified only by seeing a small ball, and the visual image of a wave can be verified only by seeing a wave. When the sensations neither occur nor can occur, all that can be verified are certain equations and the terms implicitly defined by such equations.

It is to be noted that this conclusion rests on a divergence from Galilean assumptions. For on those assumptions, secondary qualities such as colour, sound, heat, and the like, are merely apparent; they are to be attributed not to objects but to our subjectivity. On the other hand, the mathematical dimensions of matter in motion are constitutive of the real and objective, so to deny them is to eliminate the object. Hence, on the Galilean view, electrons cannot be red or green or blue, hard or soft, hot or cold, but they must have dimensions either of little balls or of waves or of some other compatible set of primary qualities.

6.6.7 *A Principle of Uncertainty.* An axiomatic structure for statistical laws will involve an uncertainty principle.

For the concrete includes a non-systematic component, and so the concrete cannot be deduced in its full determinacy from any set of systematic premises.

But an axiomatic structure is a set of systematic premises Its implications reach to the concrete, for they regard statistical laws that deal with events, and events are always fully concrete.

Therefore, the axiomatic structure for statistical laws must have some means of cutting short its implications before the full determinations of the concrete are reached. And any such means falls under the general case of an uncertainty principle.

On this analysis, then, indeterminacy is a general characteristic of statistical investigations. So, prior to the measured uncertainty of Heisenberg's equation, there was the unmeasured uncertainty inherent in classical statistics in which predictions were unique but, none the less, were not expected to be correct in every case.*

Nor is this generality surprising. It runs parallel to the possibility of deducing Heisenberg's principle from a general axiomatic structure It follows from the fact that the deduction of conclusions supposes systematic relations so that, if some relations are not systematic, the field of possible conclusions must be restricted.

### 6.7 *Indeterminacy and the Non-systematic*

The foregoing account of the general character of statistical investigations must not be mistaken for a description of Quantum Mechanics The canon of statistical residues is methodological. Its generality is not that of recent physics but of statistical method. Its basis lies not in the conclusions of subatomic investigation but in the analysis of the cognitional process that begins from data and inquiry, proceeds through insight and formulation, and recommences when experiments yield significantly new data. Its technical terms are derived not from the usage that scientists have found suitable for their purposes but from the exigences of a quite different study Accordingly, as already we have had occasion to insist, only a further critical and creative effort can bring our conclusions into contact with the diverse interpretations of the results of contemporary physics.

For the canon of statistical residues involves three elements, and all three can be stated only in cognitional terms. The first element is the indeterminacy of the abstract: classical laws can be applied to concrete situations only by adding further determinations derived from the situations. The second element is the non-systematic character of the further determinations. It does not mean that the further determinations are not related to one another by law; it means that the law is only an abstract part in a concrete relation of determinate numbers, magnitudes, relative positions, etc. It does not mean that these concrete relations cannot be mastered by insight into relevant presentations; it means that the concrete insight has a fuller object than the abstract formulation. It does not mean that no attempt can be made at a conceptual account of the concrete relations; it means that such a conceptual account bogs down in an unmanageable infinity of cases It does not mean that concrete relations are never recurrent or that accurate prediction is never possible, it means that schemes of recurrence do not fall under some overarching scheme, that they are merely instances in which law triumphs over the empirical residue, that such triumphs of law do not occur in accord with some further classical law. The third element, finally, is the inverse insight: if the intelligibility of abstract system is not to be had, still generality is not to be renounced; for there is the generality of the ideal frequency of events; and from such an ideal frequency the non-systematic cannot diverge in any systematic fashion.

* See Lindsay and Margenau, p. 398



Not only is the canon of statistical residues methodological but also it stands in a context of other canons that involve a transposition of current issues. A canon of relevance has fixed attention on what insight adds to data. A canon of parsimony has restricted scientific affirmation to defined types of verifiable propositions. A canon of complete explanation has placed space and time in much the same position as sensible qualities. Within that context there is no need to attempt to exorcize the images of the older determinists with the images of the new indeterminists. It is true enough that data are hazy, that measurements are not perfectly accurate, that measuring can distort the measured object But those truths miss the methodological point. One can affirm them yet continue to misconceive classical laws. The law of falling bodies is not a statement of what would happen in a perfect vacuum; it is the statement of an element in an abstract system, and the complete system can be applied to any particular case. Again, Einstein's differential equations are not statements about positions and velocities in defiance of Heisenberg's principle, they are statements of the abstractness and so invariance of classical laws The proper answer to the old determinism is an affirmation, not of an indeterminism on the same imaginative level, but of the indeterminacy of the abstract.

Finally, may we claim that this transposition hits the mark? Between

indeterminism and probability the only apparent link is a common lack of precision and definiteness. But the indeterminacy of the abstract brings to light the non-systematic character of the concrete. And the essence of probability is that it sets an ideal norm from which actual frequencies can diverge but not systematically.

CHAPTER IV

# THE COMPLEMENTARITY OF CLASSICAL AND STATISTICAL INVESTIGATIONS

A review of the main points that have been made will prove, perhaps, the most expeditious introduction to the problem of the present chapter.

Our study of human intelligence began from an account of the psychological aspects of insight. It turned to geometrical definitions as products of insight and thence to the redefinitions that result from higher viewpoints. The argument then twisted to the queer type of insight that grasps that the understanding of given data or of the answer to a given question consists in understanding that there is nothing to be understood. Finally, there was effected a generalization that acknowledged in all data an empirical residue from which intelligence always abstracts.

The second chapter switched to insights in the field of empirical science. After a brief contrast between mathematical and scientific developments of understanding, attention centred on the origin of the clues that form the first moment of insight. It was seen that, by inquiring, intelligence anticipates the act of understanding for which it strives. The content of that anticipated act can be designated heuristically. The properties of the anticipated and designated content constitute the clues intelligence employs to guide itself towards discovery. Finally, since there are not only direct insights that understand what is to be understood but also the queer type of insights that understand that there is nothing to be understood, heuristic structures fall into two groups, namely, the classical and the statistical. A classical heuristic structure is intelligent anticipation of the systematic-and-abstract on which the concrete converges. A statistical heuristic structure is intelligent anticipation of the systematic-and-abstract setting a boundary or norm from which the concrete cannot systematically diverge.

Of themselves, heuristic structures are empty. They anticipate a form that is to be filled. Now just as the form can be anticipated in its general properties, so also can the process of filling be anticipated in its general properties. There exist, then, canons of empirical method. If insight is

to be into data, there is a canon of selection. If insights into data accumulate in a circuit of presentations, insights, formulations, experiments, new presentations, there is a canon of operations. If applied science involves insights into materials, purposes, agents, and tools, then pure science, as prior to applied, will be concerned solely with the immanent intelligibility of data and so will be subject to a canon of relevance. If pure science goes beyond the data inasmuch as it grasps their immanent intelligibility, still it adds to the data no more than that intelligible content; there results a canon of parsimony, which excludes any affirmation that goes beyond what can be verified in the data. If some data are to be understood, then all are to be understood; the scientific goal is the understanding of all phenomena, and so scientific method is subject to a canon of complete explanation; it follows that no exception is to be made for experienced extensions or for experienced durations; and this conclusion implies a shift from a Galilean to an Einsteinian viewpoint. Finally, though all data are to be explained, it remains that certain aspects of all data are explained in the queer fashion already noticed. There exist statistical residues, for the totality of the systematic is abstract, the abstract is applied to the concrete only by the addition of further determinations and, from the nature of the case, the further determinations cannot be systematically related to one another.

Now this bare enumeration of the points that have been made in our first three chapters confronts us with a problem. Both the heuristic structures of science and the canons of empirical method involve a duality. Besides grasping the intelligibility immanent in data in a positive fashion, human intelligence also grasps a domination of the concrete by the abstract-and-systematic. However, though one admits this duality as a fact, one still may ask whether it is ultimate, whether classical and statistical inquiries are isolated or related procedures, whether they lead to isolated or related results. An answer to these questions is sought in the present chapter, and it falls into three parts.

First, it will be advanced that classical and statistical investigations are complementary as types of knowing. In their heuristic anticipations, in their procedures, in their formulations, in their differences of abstractness, in their verification, and in their domains of data, each will be shown to complement and to be complemented by the other.

Secondly, besides the complementarity in knowing, there is a complementarity in the to-be-known. Whether one likes it or not, heuristic structures and canons of method constitute an *a priori*. They settle in



advance the general determinations, not merely of the activities of knowing, but also of the content to be known. Just as Aristotle's notions on science and method resulted in his cosmic hierarchy, just as the Galilean reduction of secondary to primary qualities necessitated a mechanist determinism, so too our simultaneous affirmation of both classical and statistical investigations involves a world view. What is that view?

Thirdly, there is a clarification that results from contrast. Accordingly, after endeavouring to determine the world view, to which one commits oneself by accepting the heuristic structures and the canons of empirical method, there are set forth its differences from the world views of Aristotle, Galileo, Darwin, and contemporary indeterminists.

## 1. COMPLEMENTARITY IN THE KNOWING

### 1.1 *Complementary Heuristic Structures*

First, the heuristic anticipations of classical and of statistical procedures are complementary. For the systematic and the non-systematic are the contradictory alternatives of a dichotomy. Inquiry of the classical type is an anticipation of the systematic. Inquiry of the statistical type is an anticipation of the non-systematic. Now the relations between data must be either systematic or non-systematic. It follows that in any given case either the classical or the statistical anticipation must be correct.

Two corollaries follow.

The first is the openness of empirical method. The mere fact of inquiry is itself a presupposition, for it implies that there is something to be known by understanding the data. Still this presupposition is inevitable, for it marks the difference between the scientific and the non-scientific attitudes to experience. Moreover, this presupposition is minimal. For it does not determine *a priori* whether any selected range of data is to be reduced to system in the classical fashion or, on the other hand, is to be accounted for by showing how the concrete diverges non-systematically from systematic expectations.

The second corollary is the relevance of empirical method. For empirical method is a matter of trial and error, and the only way to settle whether a given aggregate of observations is or is not reducible to system is to formulate both hypotheses, work out their implications, and test the implications against observed results.

### 1.2 *Complementary Procedures*

Next, classical and statistical investigations are complementary procedures For they separate systematically and non-systematically related data, and the isolation of either type is a step towards the determination of the other.

With such separation everyone is familiar when it is effected physically by experimentation. As has been seen, the aim of the experimenter is to isolate a definable conjunction of elements and to exhibit their operations as they occur when uninfluenced by extraneous factors.

Again, physical separation is not always possible, and then one attempts to do by thought what one cannot achieve by deed. In this fashion, as soon as a science has made some progress, it invokes its known laws in seeking the determination of the unknown. Thus, once Boyle's law is known, one assumes it in determining Charles' law, once both are known, one assumes both in determining Gay-Lussac's law. Similarly, in all departments, known laws are employed to guide experiment, to eliminate the consideration of what already has been explained, and to provide premises for the interpretation of observed results.

Moreover, such separation, whether physical or mental, is not confined to classical laws. All laws belong to a single complementary field. For this reason it has been possible to invoke the laws of probable errors and thereby to eliminate a non-systematic component in observations and measurements. In like manner, Mendel's statistical laws of macroscopic, genetic characters led to the postulation of microscopic entities named genes; to each gene was assigned, on the classical model, a single, determinate effect and manifestation; genes with incompatible effects were classified as dominant and recessive; and so statistical combinations of classically conceived genes became the explanation of non-systematic, macroscopic phenomena.

The reader may be surprised that we lump together the laws of probable errors and the Mendelian laws of heredity. But from our viewpoint they belong together. In both cases a component in the data is brought under law. In both cases the discovery of the law grounds a mental separation of the component, subject to known law, from other components still to be determined. In both cases this mental separation opens the way to the determination of further laws. In both cases, finally, it is the discovery of a statistical law that grounds the mental separation and that can lead to the discovery no less of classical than of statistical laws.



This complementarity of classical and statistical procedures has an important corollary. For the experimental, physical exclusion of extraneous factors is not always possible. When it is not, there exists the alternative of discovering the law of the extraneous factor and then allowing for its influence in interpreting one's result. Now the corollary, to which we would draw attention, is that statistical laws can be employed in this fashion to the determination of classical laws. For knowledge of statistical laws enables one to separate mentally the non-systematic component in the data and so it leaves one free to investigate the remaining systematic component.

It will be asked, then, whether the statistical investigations of Quantum Mechanics may be expected to prepare the way for a later resurgence of classical thought in the field of subatomic physics.

This question is, I think, ambiguous. One may mean a return to the former type of classical thought with its imaginable models, its belief in the universal possibility of imaginative synthesis, its affirmation of a mechanist determinism, and its concept of explanation as the reduction of secondary to primary qualities. On the other hand, it is possible to speak of 'classical' thought in a transposed and analogous sense. In that case, one would grant to imagination a notable heuristic value, for images supply the materials for insights; but, at the same time, one would deny to unverified and unverifiable images any representative value; classical laws would be conceived as abstract, the abstraction would be conceived as enriching, and so full knowledge of classical laws would not preclude the existence of statistical residues.

Once this distinction is drawn, our answer to the foregoing question becomes obvious. In the light of the canons of complete explanation, of parsimony, and of statistical residues, we cannot expect any return to the older type of classical thought. Again, in the same light, we must expect Quantum Mechanics, if interpreted statistically, to open the way to a new development of 'classical' thought in a transposed and analogous sense. Indeed, Pauli's exclusion principle provides a premise for the determination of the states of electrons in atoms; and while changes of these states seem to occur statistically, still the series of states is as regular and systematic as the periodic table of chemical elements.* In like manner, one might note classical tendencies in the discovery of new

* See Lindsay and Margenau, pp. 488 f.

subatomic entities over and above the more familiar electrons, protons and neutrons.

### 1.3 *Complementary Formulations*

Thirdly, classical and statistical formulations are complementary. For classical formulations regard conjugates, which are verified only in events. And statistical formulations regard events, which are defined only by conjugates.

The dependence of classical upon statistical formulation comes to light, when one probes into the meaning of the classical proviso, 'other things being equal'. What are the other things? In what does their equality consist? These questions cannot be given an answer that is both detailed and systematic. For the proviso, which limits classical laws, is effectively any relevant pattern of a diverging series of conditions. Such series vary with circumstances, and the aggregate of patterns of such series is both enormous and non-systematic. In other words, classical laws tell what would happen if conditions were fulfilled; statistical laws tell how often conditions are fulfilled, and so the phrase, other things being equal, amounts to a vague reference to the statistical residues, which are the province of the complementary statistical laws.

The inverse dependence of statistical upon classical formulations comes to light, when one asks which statistical investigations possess scientific significance. Thus, anyone would acknowledge a difference in such significance between determining the frequency of red hair in trombone players and, on the other hand, measuring the intensity of line spectra. In either case one arrives at a number that may be regarded as an actual frequency, but it is not apparent that in both cases one has an equal chance of contributing to the advance of science. For the advance of science is secured by operating in the light of present knowledge and towards the solution of well-formulated problems. As soon as any department of science has passed beyond its initial stages, it begins to desert the expressions of ordinary language and to invent technical terms of its own. Such technical terms have their origin in the correlations that have been found significant; they are or, in some fashion, they depend upon what we have named pure conjugates. Accordingly, inasmuch as the statistical investigator proceeds in the light of acquired knowledge and towards the solution of well-formulated problems, he will be led to define events by appealing, directly or indirectly, to the pure conjugates that are implicit in classical laws.



However, the reader may ask whether this view can be regarded as definitive. It is true enough that the scientific classifications and definitions of the present are dependent on the discovery and formulation of classical laws. But may one not expect that a fuller development of statistical inquiry will result in the implicit definition of technical terms by statistical and not classical laws?

While there seem to be those that would answer this question affirmatively, I cannot see my way to agreeing with them. My reason runs as follows. The answer 'Yes' to a question for reflection obtains a determinate meaning only by reverting from the 'Yes' to the question and to its origin in the descriptive or explanatory answer to a question for intelligence. Now the event, the happening, the occurring, corresponds to the bare 'Yes'. To say what happens, what occurs, one must raise a question that cannot be answered by a 'Yes' or a 'No'. One must appeal either to the experiential conjugates of description or to the pure conjugates of explanation. On this showing, then, one cannot expect events to generate their own definitions any more than one can expect 'Yes' or 'No' to settle what is affirmed or denied. Finally, if events cannot generate their own definitions, then, frequencies of events cannot do so; for there seems no reason to expect that different types of events must have different numerical frequencies or, indeed, that the numerical frequencies could serve to specify the kinds of events to which one wishes to refer.

There is, then, a complementarity of classical and statistical formulations. For if statistical formulations are to be significant contributions to the advance of science, they will appeal to the experiential and pure conjugates of classical classifications and definitions. Inversely, the conjugates of classical formulations are verifiable only in statistically occurring events and their immanence in statistical residues is revealed by the proviso, 'other things being equal'.

It may not be out of place to conclude this subsection by clarifying a slight puzzle. It is true enough that statistical laws also are immanent in statistical residues, and so hold under the general proviso, 'other things being equal.' If '*P* follows *Q*' has the probability, $p\ q$, still there are conditions for the occurrence of the occasion, *Q*, and it is only when those conditions are fulfilled that the probability, $p\ q$, is verifiable. The frequency of such fulfilment might be indicated by saying that '*Q* follows *R*' has the probability, $q/r$, so that one statistical law would be dependent on another. Still this interdependence of statistical laws, while

true enough, is beside our present point. It in no way invalidates the significant contention that the dependence of classical upon statistical formulations is revealed by the proviso, 'other things being equal.'

### 1.4 *Complementary Modes of Abstraction*

Fourthly, there is a complementarity in modes of abstraction.

Classical heuristic procedure rests on the assumption that to some extent the relations between data are systematic, and it devotes its efforts to determine just what those systematic relations are.

Statistical heuristic procedure rests on an assumption of non-systematic relations and it aims at determining an ideal frequency from which actual frequencies may diverge but only non-systematically.

In both cases the result obtained is abstract. For the classical law represents the systematic and prescinds from the non-systematic. On the other hand, the statistical law represents, not the actual frequency of actual events, but the ideal frequency from which actual frequencies diverge.

Now while both types of law are abstract, still their modes of abstraction differ. The classical law is concerned simply with the systematic, it disregards the non-systematic. The statistical law, on the contrary, assumes the non-systematic as a premise. By itself, of course, such a premise could yield no conclusions such as the abstract, ideal, universal frequencies named probabilities. What concerns the statistical inquirer is, then, neither the purely systematic, nor the purely non-systematic, but the systematic as setting ideal limits from which the non-systematic cannot diverge systematically.

Clearly, these two modes of abstraction are complementary. In its first movement, inquiry aims at determining the systematic component in data; in its second movement, inquiry turns to the more concrete task of determining the manner in which the systematic component in data moderates the non-systematic. The complete view results only from the combination of the two movements, and so the two are complementary.

There is another aspect to this complementarity. The systematic relations, with which classical inquiry is concerned, mainly are the relations of things, not to our senses, but to one another. In so far as the relations of things to one another are considered in the abstract and so as independent of their relations to our senses, there arises a principle of equivalence for all senses since all equally are abstracted from. On the other hand, once one moves from classical to statistical inquiry, the foregoing interpretation of the principle of equivalence vanishes. It is true that, just as probability theory is not to be denied the convenience of continuous functions, so there is no *a priori* reason to deprive it of the advantage of full invariance. However, as will be recalled (see p. 98), statistical theory deals with events selected from processes by the possibility of accurate measurement, and the continuity of its functions seems to refer, not to the continuity of concrete process but, so to speak, to the permanent availability of the ideal norms from which events diverge in non-systematic fashion. As such norms, so their invariance lies outside the field of explanatory relations.



### 1.5 *Complementarity in Verification*

Fifthly, classical and statistical laws are complementary in their verification. This may be stated roughly by saying that classical laws determine what would happen if conditions were fulfilled, while statistical laws determine how often one may expect conditions to be fulfilled. However, a fuller account of this complementarity may be given by showing how the determination of either classical or statistical laws leaves room for the determination of the other.

Thus, if one were to suppose exact and complete knowledge of all classical laws, one would not preclude the possibility of the verification of statistical laws. For a set of classical laws, say *P*, would be exact and complete, if there were no possibility of replacing them by some different set, say *Q*. Now, there would be no possibility of replacing *P* by *Q*, if there were no systematic divergence between the data and the set of laws, *P*; for the sets, *P* and *Q*, differ as laws and so differ systematically; and so the verification of the set, *Q*, in place of the set, *P*, supposes a systematic divergence between the set, *P*, and the data. Finally, though there is no systematic divergence between the set, *P*, and the data, there can be a non-systematic divergence that would provide the field for the investigation and verification of statistical laws.

Again, as has been seen (see pp. 86–97), exact and complete knowledge of classical laws not merely can leave room for possible statistical investigation but also must do so. For such exact and complete knowledge would embrace all the systematic relations between determinate data; none the less, such knowledge would be abstract and so in need of further determinations to be applied to concrete instances;

it follows that the further determinations cannot be systematically related to one another; and so there must be a field for statistical laws.

Finally, statistical investigations in their turn have no genuine tendency to totalitarian aspirations. For besides statistical predictions, there exist the fully accurate predictions that are exemplified by astronomy and that rest on the existence of schemes of recurrence. Moreover, the intelligent manner of making these predictions is to analyse the schemes into their component classical laws. Copernicus corrected Ptolemy's imaginative scheme; Kepler corrected the circles of Copernicus; but it was Newton who worked out the underlying laws and Laplace who revealed the periodicity of the planetary system. From that discovery of laws the great movement of thought, named modern science, received its most powerful confirmation. It did so because it ended, at least for two centuries, the more common human tendency to speak, not of precise laws, but of the common run of events or the ordinary course of Nature. At the present moment, the profound significance of statistical laws is coming to light. But if this new movement is not to degenerate into the old talk about what commonly happens, it must retain its contact with the empirically established precision of classical formulations. For statistical laws are of no greater scientific significance than the definitions of the events whose frequencies they determine; unless these definitions are determined scientifically, statistical thought lapses into pre-scientific insignificance.

### 1.6 *Complementarity in Data Explained*

Sixthly, classical and statistical laws are complementary in their domains of data. By this is meant, not that some data are explained by classical laws and other data by statistical laws, but rather that certain aspects of all data receive the classical type of explanation while other aspects of the same data are explained along statistical lines.

As has been seen,* the classical heuristic assumption is that similars are similarly understood. Consequently, preliminary classifications are based on similarity to sense. However, the scientist is interested in the relations of things, not to our senses, but to one another. Accordingly, the preliminary classifications are superseded by the emergence and development of technical terms that are derived, not from sensible similarity, but from similarities of constant or regularly varying proportion; and in the limit there are reached what we have named pure

* Cf. Chapter II, § 2.3.



conjugates, that is, terms implicitly defined by the empirically established correlations in which they occur.

Still to account for data as similar is not to account for data in all their aspects. Each datum is just this instance of the given. It emerges within a continuous manifold. It is in a particular place and at a particular time. It occurs rarely or frequently. Now these aspects of all data are disregarded in explanations of the classical type. The law of the lever tells us nothing about the frequency of levers, about the places where they are to be found, about the times at which they function. Hence, explanations of the classical type have to be complemented by explanations of a further, different type.

Nor is it difficult to see, at least in some general fashion, that statistical laws can provide the complementary explanation. For the general form of the statistical law is that on $p$ occurrences of the occasion, $P$, there tend to be $q$ occurrences of the event, $Q$. Now the occasion, $P$, is itself an event or a combination of events. In either case it will possess its probability. In like manner, the occasions on which $P$ is probable, will have their probability, and so there arises an indefinite regress of probabilities from events of the type, $Q$. More generally, for events of any type, $X$, there are corresponding indefinite regresses of probabilities.

Now, it is not immediately apparent that such regresses can be combined into a single view. But it suffices for present purposes to remark that, were such a combination possible, one would be on the way to attaining a statistical explanation of data in their numbers and in their spatio-temporal distribution. To invoke only the simplest considerations, low probabilities are offset by large numbers of occasions, so that what is probable only once on a million occasions, is to be expected a million times on a million million occasions. In like manner, the rarity of occasions is offset by long intervals of time, so that if occasions arise only once in a million years, still they arise a thousand times in a thousand million years. At once there emerges the explanatory significance of statistical laws. Why are there in the world of our experience such vast numbers and such enormous intervals of time? Because probabilities are low, numbers have to be large; because occasions are rare, time intervals have to be long.

By itself, this is a very modest conclusion. Still, though the achievement is quite negligible, the potentialities are extremely significant. Statistical laws possess a capacity to generate explanation. Their heur-

istic assumption is simply that the non-systematic cannot diverge systematically from the systematic. But this incapacity for systematic divergence, when combined with large numbers and long intervals of time, is equivalent to a positive tendency, to an intelligible order, to an effective thrust, that is no less explanatory than the rigorous conclusions based on classical laws. In other words, probability is one thing, and chance is another. Probability is an ideal norm that, for all its ideality, is concretely successful in the long run. Chance is merely the non-systematic divergence of actual frequencies from the ideal frequencies named probabilities. Chance explains nothing. It pertains irretrievably to the merely empirical residue, to the aspects of data from which intelligence always abstracts. But probability is an intelligibility; it is, as it were, rescued from the merely empirical residue by the roundabout device in which inquiring intelligence sets up the heuristic anticipations of the statistical type of investigation.

### 1.7 *Summary*

We have been considering the complementarity of classical and statistical investigations as forms of knowing. We have found such complementarity to exist at each of the stages or components of the process of inquiry. There is the classical heuristic anticipation of the systematic; there is the complementary statistical heuristic anticipation of the non-systematic. Next, to determine either a classical or a statistical law is to prepare the way for the determination of further laws of either type; for both classical and statistical laws pertain to a single complementary field, and to know either is to effect a mental separation between types of data that have been accounted for and types that still remain to be explained. Thirdly, there is a complementarity of formulations; the experiential and pure conjugates of classical laws can be verified only in events; the events occur only if other things are equal; and the failure to specify the other things amounts to an unconscious acknowledgement of the non-systematic aggregate of patterns of diverging series of conditions. Inversely, as conjugates are verified only in events, so events are defined only by conjugates, and statistical laws of events can possess scientific significance only in the measure that they employ definitions generated by classical procedures. Fourthly, there is a complementarity in modes of abstraction; classical laws regard the systematic in abstraction from the non-systematic, the relations of things to one another in abstraction from their relations to our senses; but statistical laws consider the systematic as setting bounds to the non-systematic and they are confined to the observable events that include a relation to our senses. Fifthly, the two types of law are complementary in their verification: exact and complete knowledge of classical laws cannot successfully invade the field of statistical laws; and statistical investigations are confronted with regular recurrences that admit explanations of the classical type. Finally, there is complementarity in the aspects of data explained by the different types of laws; data as similar are explained on classical lines; but their numbers and their distributions become intelligible only by some synthesis of statistical considerations.



## 2. COMPLEMENTARITY IN THE KNOWN

Just as the first part of this chapter was devoted to exhibiting the complementarity of classical and of statistical investigations from the viewpoint of knowing, so now the second part is to be directed to the determination of the corresponding complementarity from the viewpoint of what is to be known. For knowing and known, if they are not an identity, at least stand in some correspondence and, as the known is reached only through knowing, structural features of the one are bound to be reflected in the other. Aristotle's world view stemmed from his distinction between the necessary laws of the heavenly bodies and the contingent laws of things on this earth. Mechanist determinism had its scientific basis in the Galilean concept of explanation as the reduction of secondary to primary qualities. In similar fashion some parallel implication cannot be avoided by any fully conscious methodology and so, if we are not to play the ostrich, we must face the question, what world view is involved by our affirmation of both classical and statistical laws.

### 2.1 *General Characteristics of the View*

Certain general characteristics of our position may be indicated immediately.

In the first place, it will be concerned with the intelligibility immanent in the universe of our experience. For it will be a conclusion from the structure of empirical method and, by the canon of relevance, empirical method is confined to determining such immanent intelligibility. Hence, we shall have nothing to say in this chapter about the end or purpose of this universe, about the materials from which it was

fashioned, about the principal or instrumental agents responsible for it. Our efforts will be limited to determining the immanent design or order characteristic of a universe in which both classical and statistical laws obtain.

In the second place, our account of this design or order will be generic. A specific account would have to draw upon the content of the empirical sciences. It would have to appeal, not to classical and statistical laws in general, but to the precise laws that can be empirically established. Our account, on the other hand, will rest not on the results of scientific investigations but simply and solely upon the dynamic structure of inquiring intelligence. Accordingly, if in the course of the exposition any particular scientific conclusions are invoked, their function will be not determinative but merely illustrative. Just as mechanist determinism has been a world view that is independent of the precise content of classical laws, so too our objective is a similarly generic structure that is compatible not only with present classical and statistical laws but also with their future revisions.

In the third place, our account of the design or order of this universe will be relatively invariant. The content of the natural sciences is a variable. There has been the science of the Renaissance. There has been the science of the Enlightenment. There is the science of today. There will be the successive stages of scientific development in the future. But knitting together these diverse manifestations of scientific thought, generating each in turn only to bring forth the revision and transformation of each, there is the underlying invariant that loosely may be named scientific method and more precisely, I think, would be designated as the dynamic structure of inquiring intelligence. For, as has been seen, it is the desire to understand that results both in the heuristic structure of classical procedure and in the complementary structure of statistical investigation; and it is the nature of insight that accounts for the six canons of selection, operations, relevance, parsimony, complete explanation, and statistical residues, in accord with which the heuristic structures generate the series of scientific theories and systems. Now our premise is to be, not the variable contents of the sciences, but the invariant forms governing scientific investigation. It follows that the design of the universe, to which we shall conclude, will enjoy the invariance of the premise which we shall invoke.

Still, I have said that our account will be only relatively invariant, and the reason for this restriction is plain enough. For our appeal will be, not to the structure of the human mind itself, but only to our account of that structure. Just as the natural sciences are subject to revision, so too one may expect our account of inquiring intelligence to be subjected to rearrangements, modifications, and improvements. In the measure that such changes will affect the premises of the present argument, in the same measure they will also affect the conclusions. Accordingly, the world view to be presented will be invariant, inasmuch as it will be independent of changes in the content of the natural sciences; but it will be only relatively invariant, for it cannot be independent of revisions of our analysis of empirical method.



In the fourth place, our account of a world view within the limits of empirical science will not be complete in this chapter. In treating the canon of parsimony, we postponed the question of the validity of the notion of the thing. In a later chapter, that question will have to be met, and then a further complement to the present account will be added.

In the fifth place, our account will not claim to be deductive. Perhaps one might argue in strictly deductive fashion from the complementary structure of the knowing to the corresponding complementarity of the known. But, if that procedure is possible, it also requires an elaboration that for present purposes would be excessive. Accordingly, our appeal will be to insight. We shall begin from the problem of showing how both classical and statistical laws can coalesce into a single, unified intelligibility commensurate with the universe of our experience. Against this problem we shall set our clue, namely, the scheme of recurrence. On the one hand, the world of our experience is full of continuities, oscillations, rhythms, routines, alternations, circulations, regularities. On the other hand, the scheme of recurrence not only squares with this broad fact but also is related intimately both to classical and to statistical laws. For the notion of the scheme emerges in the very formulation of the canons of empirical method. Abstractly, the scheme itself is a combination of classical laws. Concretely, schemes begin, continue, and cease to function in accord with statistical probabilities. Such is our clue, our incipient insight. To develop it we shall consider

(1) the notion of a conditioned series of schemes of recurrence,
(2) the probability of a single scheme,
(3) the emergent probability of a series of schemes, and
(4) the consequent characteristics of a world order.

2.2 *Schemes of Recurrence*

The notion of the scheme of recurrence arose when it was noted that the diverging series of positive conditions for an event might coil around in a circle. In that case, a series of events, *A*, *B*, *C*, . . . would be so related that the fulfilment of the conditions for each would be the occurrence of the others. Schematically, then, the scheme might be represented by the series of conditionals, If *A* occurs, *B* will occur, if *B* occurs, *C* will occur, if *C* occurs, . . . *A* will recur. Such a circular arrangement may involve any number of terms, the possibility of alternative routes, and in general, any degree of complexity.

Two instances of greater complexity may be noted. On the one hand, a scheme might consist of a set of almost complete circular arrangements, of which none could function alone yet all would function if conjoined in an interdependent combination. On the other hand, schemes might be complemented by defensive circles, so that if some event, *F*, tended to upset the scheme, there would be some such sequence of conditions as, If *F* occurs, then *G* occurs; if *G* occurs, then *H* occurs; if *H* occurs, then *F* is eliminated.

In illustration of schemes of recurrence the reader may think of the planetary system, of the circulation of water over the surface of the earth, of the nitrogen cycle familiar to biologists, of the routines of animal life, of the repetitive, economic rhythms of production and exchange. In illustration of schemes with defensive circles, one may advert to generalized equilibria. Just as a chain reaction is a cumulative series of changes terminating in an explosive difference, so a generalized equilibrium is such a combination of defensive circles that any change, within a limited range, is offset by opposite changes that tend to restore the initial situation. Thus, health in a plant or animal is a generalized equilibrium; again, the balance of various forms of plant and animal life within an environment is a generalized equilibrium; again, economic process was conceived by the older economists as a generalized equilibrium.

However, we are concerned, not with single schemes, but with a conditioned series of schemes. Let us say that the schemes, *P*, *Q*, *R*, . . . form a conditioned series, if all prior members of the series must be functioning actually for any later member to become a concrete possibility. Then, the scheme, *P*, can function though neither *Q* nor *R* exists; the scheme, *Q*, can function, though *R* does not yet exist, but *Q* cannot function unless *P* is already functioning; and *R* cannot function unless *Q* is already functioning.



Thus, by way of a simple illustration, one may advert to the dietary schemes of animals. All carnivorous animals cannot live off other carnivorous animals. Hence, a carnivorous, dietary scheme supposes another herbivorous, dietary scheme but, inversely, there could be herbivorous animals without any carnivorous animals. Again, plants cannot in general live off animals, the scheme of their nourishment involves chemical processes; and that scheme can function apart from the existence of any animals. Finally, chemical cycles are not independent of physical laws yet, inversely, the laws of physics can be combined into schemes of recurrence that are independent of chemical processes.

Such in briefest outline is the notion of the conditioned series of schemes of recurrence. Let us seek a slight increase in precision by drawing a threefold distinction between

(1) the possible seriation,
(2) the probable seriation, and
(3) the actual seriation.

The actual seriation is unique. It consists of the schemes that actually were, are, or will be functioning in our universe along with precise specifications of their places, their durations, and their relations to one another.

The probable seriation differs from the actual. For the actual diverges non-systematically from probability expectations. The actual is the factual, but the probable is ideal. Hence, while the actual seriation has the uniqueness of the matter of fact, the probable seriation has to exhibit the cumulative ramifications of probable alternatives. Accordingly, the probable seriation is not a single series but a manifold of series. At each stage of world process there is a set of probable next stages, of which some are more probable than others. The actual seriation includes only the stages that occur. The probable seriation includes all that would occur without systematic divergence from the probabilities.

The possible seriation is still more remote from actuality. It includes all the schemes of recurrence that could be devised from the classical laws of our universe. It orders them in a conditioned series that ramifies not only along the lines of probable alternatives but also along lines of mere possibility or negligible probability. It is equally relevant to our

universe and to any other universe subject to the same classical laws, no matter what its initial numbers, diversities, and distribution of elements

Of the three seriations, then, the possible exhibits the greatest complexity and variety. It depends solely on a consideration of classical laws. It suffers from the indeterminacy of the abstract, and so exhibits the process of any universe with laws similar to ours. The probable seriation depends on statistical as well as classical laws, and, indeed, on the statistical laws that arise from the initial or basic situation of our world Still, if it is not as abstract as the possible seriation, none the less, it is ideal. For each moment of world history, it assigns a most probable future course. But it also assigns a series of less probable courses, and it has to acknowledge that any of these may prove to be the fact Finally, the actual seriation is unique, but it purchases its uniqueness by going beyond the field of all laws, classical and statistical, and entering the field of observation, in which alone non-systematic divergences from probability are determinate.

### 2.3 *The Probability of Schemes*

Our outline of the notion of a conditioned series of schemes of recurrence supposes that one can attribute a probability to the emergence and to the survival of a scheme of recurrence However, our account of probability has been in terms of the frequency, not of schemes, but of events. Have schemes any probability? If they have, is there a distinct probability for their emergence and another for their survival? Such questions must be met.

Consider a set of events of the types, $A, B, C, \ldots$ and a world situation in which they possess respectively the probabilities, $p, q, r, \ldots$ Then by a general rule of probability theory, the probability of the occurrence of all the events in the set will be the product, $pqr, \ldots$, of their respective probabilities.

Now let us add a further assumption. Let us suppose that the set of events, $A, B, C,$ . satisfies a conditioned scheme of recurrence, say $K$, in a world situation in which the scheme, $K$, is not functioning but in virtue of the fulfilment of prior conditions, could begun to functi n Then, if $A$ were to occur, $B$ would occur If $B$ were to occur, $C$ wo ld occur. If $C$ were to occur, . $A$ would occur. In brief, if any of the events in the set were to occur, then, other things being equal, the rest of the events in the set would follow.

In this case, we may suppose that the probabilities of the single



events are respectively the same as before, but we cannot suppose that the probability of the combination of all events in the set is the same as before. As is easily to be seen, the concrete possibility of a scheme beginning to function shifts the probability of the combination from the product, $pqr, \ldots$, to the sum, $p+q+r+ \ldots$. For, in virtue of the scheme, it now is true that $A$ and $B$ and $C$ and . . . will occur, if either $A$ or $B$ or $C$ or . . . occurs; and by a general rule of probability theory, the probability of a set of alternatives is equal to the sum of the probabilities of the alternatives.

Now a sum of a set of proper fractions, $p, q, r, \ldots$ is always greater than the product of the same fractions. But a probability is a proper fraction. It follows that, when the prior conditions for the functioning of a scheme of recurrence are satisfied, then the probability of the combination of events, constitutive of the scheme, leaps from a product of fractions to a sum of fractions.

There exists, then, a probability of emergence for a scheme of recurrence. That probability consists in the sum of the respective probabilities of all the events included in the scheme, and it arises as soon as the prior conditions for the functioning of the scheme are satisfied.

There also exists a probability for the survival of schemes that have begun to function. For, of itself, a scheme tends to assure its own perpetuity. The positive conditions for the occurrence of its component events reside in the occurrence of those events. Even negative conditions, within limited ranges, can be provided for by the development of defensive circles. None the less, the perpetuity of a scheme is not necessary. Just as classical laws are subject to the proviso, other things being equal, so also are the schemes constituted by combinations of classical laws; and whether or not other things will continue to be equal, is a question that admits an answer only in terms of statistical laws Accordingly, the probability of the survival of a scheme of recurrence is the probability of the non-occurrence of any of the events that would disrupt the scheme.

### 2 4 *Emergent Probability*

There have been formulated the notion of a conditioned series of schemes of recurrence and, as well, the general sense in which one can speak of the probability of the emergence and the survival of single schemes. From these considerations there now comes to light the notion of an emergent probability. For the actual functioning of earlier

schemes in the series fulfils the conditions for the possibility of the functioning of later schemes. As such conditions are fulfilled, the probability of the combination of the component events in a scheme jumps from a product of a set of proper fractions to the sum of those proper fractions. But, what is probable, sooner or later occurs. When it occurs, a probability of emergence is replaced by a probability of survival; and as long as the scheme survives, it is in its turn fulfilling conditions for the possibility of still later schemes in the series.

Such is the general notion of emergent probability. It results from the combination of the conditioned series of schemes with their respective probabilities of emergence and survival. While by itself it is extremely jejune, it possesses rather remarkable potentialities of explanation. These must now be indicated in outline, and so we attempt brief considerations of the significance for emergent probability of spatial distribution, absolute numbers, long intervals of time, selection, stability, and development.

The notion of a conditioned series of schemes involves spatial concentrations. For each later set of schemes becomes possible in the places where earlier schemes are already functioning. Accordingly, the most elementary schemes, which are earliest in the series, can occur anywhere in the initial distribution of materials. But the second batch can occur only where the first have in fact occurred, the third can occur only where the second have in fact occurred, and so on. Moreover, since the realization of the schemes is in accord with the probabilities, which may be low, one cannot expect all possibilities to be actuated. Hence, elementary schemes will not be as frequent as they could be, to narrow the possible basis for schemes at the second remove. These will not be as frequent as they could be, to narrow again the possible basis for schemes at the third remove, and so forth. It follows that, however widespread the realization of elementary schemes, there will be a succession of constrictions of the volumes of space in which later schemes can be found. Similarly, it follows that the points, so to speak, of greatest and least constriction occur where the probabilities of emergence of the next set of schemes are respectively the lowest and the highest. Finally, it follows that, since the latest schemes in the series have the greatest number of conditions to be fulfilled, their occurrence will be limited to a relatively small number of places.

Secondly, there is the significance of absolute numbers. For large numbers offset low probabilities. What occurs once in a million occasions, is to be expected a million times on a million million occasions. Now the minimum probability pertains to the latest schemes in the series, for their emergence supposes the emergence of all earlier schemes. It follows that the lower the probability of the last schemes of the conditioned series, the greater must be the initial absolute numbers in which elementary schemes can be realized. In brief, the size of a universe is inversely proportionate to the probability of its ultimate schemes of recurrence.



Thirdly, there is the significance of long intervals of time. No matter how great the universe and how widespread the functioning of elementary schemes, there is an increasing concentration of the spatial volumes in which later schemes can be realized. Sooner or later, the initial benefit of large numbers is lost by the successive narrowing of the basis for further developments. But at this point long intervals of time become significant. Just as a million million simultaneous possibilities yield a million probable realizations, whose probability is one in a million, so also a million million successive possibilities yield a million probable realizations under the same expectation.

Fourthly, there is a selective significance attached to the distinction between probabilities of emergence and probabilities of survival. If both are low, the occurrence of the scheme will be both rare and fleeting. If both are high, the occurrences will be both common and enduring. If the probability of emergence is low and that of survival is high, the scheme is to be expected to be rare but enduring. Finally, in the opposite case, the expectation is that the scheme will be common but fleeting.

Fifthly, this selectivity has its significance for stability. The functioning of later schemes depends upon the functioning of earlier schemes, so that if the earlier collapse, then the later will collapse as well. It follows that the line of maximum stability would be of common and enduring schemes while the line of minimum stability would be of rare and fleeting schemes.

Sixthly, no less than stability, the possibility of development must be considered. Unfortunately, these two can conflict. Schemes with high probabilities of survival tend to imprison materials in their own routines. They provide a highly stable basis for later schemes, but they also tend to prevent later schemes from emerging. A solution to this problem would be for the earlier conditioning schemes to have a high probability of emergence but a low probability of survival. They would

form a floating population, on which later schemes could successively depend. Because their probability of survival is low, they would readily surrender materials to give later schemes the opportunity to emerge. Because their probability of emergence was high, they would readily be available to fulfil the conditions for the functioning of later schemes.

Needless to say, the foregoing considerations are extremely rudimentary. They are limited to the emergent probability of any conditioned series of schemes of recurrence. They make no effort towards developing that notion in the direction of its application to the conditions of the emergence and survival of modes of living. However, while absolutely such a fuller exposition would be desirable, still it has no place in a merely generic account of world order. For the premise of a generic account is, not the content of the natural sciences, but the possibility and validity of their assumptions and method.

The point we are endeavouring to make, within the limits of our narrow premise, is that the notion of emergent probability is explanatory. Intelligent inquiry aims at insight. But classical laws alone offer no insight into numbers, distributions, concentrations, time intervals, selectivity, uncertain stability, or development. On the contrary, they abstract from the instance, the place, the time, and the concrete conditions of actual functioning. Again, statistical laws, as a mere aggregate, affirm in various cases the ideal frequency of the occurrence of events. They make no pretence of explaining why there are so many kinds of events or why each kind has the frequency attributed to it. To reach explanation on this level, it is necessary to effect the concrete synthesis of classical laws into a conditioned series of schemes of recurrence, to establish that such schemes, as combinations of events, acquire first a probability of emergence and then a probability of survival through the realization of the conditioned series, and finally to grasp that, if such a series of schemes is being realized in accord with probabilities, then there is available a general principle that promises answers to questions about the reason for numbers and distributions, concentrations and time intervals, selectivity and uncertain stability, development and breakdowns. To work out the answers pertains to the natural sciences. To grasp that emergent probability is an explanatory idea, is to know what is meant when our objective was characterized as a generic, relatively invariant, and incomplete account of the immanent intelligibility, the order, the design of the universe of our experience.



### 2.5 *Consequences of Emergent Probability*

There remains the task of working out the generic properties of a world process in which the order or design is constituted by emergent probability. This we shall attempt in two main steps. First, we shall summarize the essentials of the notion of emergent probability. Secondly, we shall enumerate the consequences of that notion to be verified in world process.

The essentials of the notion of emergent probability may be indicated in the following series of assertions:

1. An event is what is to be known by answering 'Yes' to such questions as: Did it happen? Is it occurring? Will it occur?
2. World process is a spatio-temporal manifold of events. In other words, there are many events and each has its place and time.
3. Events are of kinds. Not every event is a new species, else there could be neither classical nor statistical laws.
4. Events are recurrent. There are many events of each kind, and all are not at the same time.
5. There are regularly recurrent events. This regularity is understood, inasmuch as combinations of classical laws yield schemes of recurrence. Schemes are circular relationships between events of kinds, such that if the events occur once in virtue of the circular relationships then, other things being equal, they keep on recurring indefinitely.
6. Schemes can be arranged in a conditioned series, such that the earlier can function without the emergence of the later, but the later cannot emerge or function unless the earlier already are functioning.
7. Combinations of events possess a probability, and that probability jumps, first when a scheme becomes concretely possible in virtue of the fulfilment of its prior conditions, and secondly when the scheme begins actually to function.
8. The actual frequencies of events of each kind in each place and at each time do not diverge systematically from their probabilities. However, actual frequencies may diverge non-systematically from probabilities, and that non-systematic divergence is chance. Accordingly, probability and chance are distinct and are not to be confused.
9. Emergent probability is the successive realization in accord with

successive schedules of probability of a conditioned series of schemes of recurrence.

The consequent properties of a world process, in which the design is emergent probability, run as follows:

1. There is a succession of world situations. Each is characterized (1) by the schemes of recurrence actually functioning, (2) by the further schemes that now have become concretely possible, and (3) by the current schedule of probabilities of survival for existing schemes and of probabilities of emergence for concretely possible schemes.
2. World process is open. It is a succession of probable realizations of possibilities. Hence, it does not run along the iron rails laid down by determinists nor, on the other hand, is it a non-intelligible morass of merely chance events.
3. World process is increasingly systematic. For it is the successive realization of a conditioned series of schemes of recurrence, and the further the series of schemes is realized, the greater the systematization to which events are subjected.
4. The increasingly systematic character of world process can be assured. No matter how slight the probability of the realization of the most developed and most conditioned schemes, the emergence of those schemes can be assured by sufficiently increasing absolute numbers and sufficiently prolonging intervals of time. For actual frequencies do not diverge systematically from probabilities; but the greater the numbers and the longer the time intervals, the clearer the need for a systematic intervention to prevent the probable from occurring.
5. The significance of the initial or basic world situation is limited to the possibilities it contains and to the probabilities it assigns its possibilities. By the initial world situation is meant the situation that is first in time; by the basic world situation is meant the partial prolongation through time of initial conditions, such as arises, for instance, in certain contemporary hypotheses of continuous creation.

   In either case, what is significant resides in possibilities and their probabilities, for, in all its stages, world process is the probable realization of possibilities. While the determinist would desire full information, exact to the *n*th decimal place, on his initial or basic situation, the advocate of emergent probability is quite satisfied with any initial situation in which the most elementary schemes can emerge and probably will emerge in sufficient numbers to sustain the subsequent structure.



6. World process admits enormous differentiation. It envisages the totality of possibilities defined by classical laws. It realizes these possibilities in accord with its successive schedules of probabilities. And, given sufficient numbers and sufficient time, even slight probabilities become assured.
7. World process admits break-downs. For no scheme has more than a probability of survival, so that there is for every scheme some probability of a break-down; and since earlier schemes condition later schemes, a break-down of the former entails the break-down of the latter.
8. World process includes blind alleys. For schemes with a high probability of survival have some probability of emergence. In so far as they emerge, they tend to bind within their routines the materials for the possibility of later schemes and so to block the way to full development.
9. The later a scheme is in the conditioned series, the narrower is its distribution. For actual realization is less frequent than its concrete possibility; and each later set of schemes is concretely possible only where earlier, conditioning schemes are functioning.
10. The narrower the basis for the emergence of each later set of schemes, the greater the need to invoke long intervals of time. For in this case, the alternative of large numbers is excluded.
11. The greater the probabilities of blind alleys and break-downs, the greater must be the initial absolute numbers, if the realization of the whole series of schemes is to be assured. For in this case the device of long time intervals might not be efficacious. Blind alleys with their inert routines could last for extremely long periods and, when they suffered break-downs, they might result in another blind alley. Again, a situation which led to some development only to suffer break-down might merely repeat this process more frequently in a longer interval of time. On the other hand, the effect of large initial numbers is to assure at least one situation in which the whole series of schemes will win through.
12. The foregoing properties of world process are generic. They

assume that there are laws of the classical type, but they do not assume the determinate content of any particular classical law. They assume that classical laws can be combined into the circular relationships of schemes, but they do not venture to analyse the structure of any scheme whatever. They assume that there are statistical laws, but there is no assumption of the determinate content of any statistical law.

Moreover, these properties are relatively invariant. They rest on the scientist's necessary presupposition that there are classical and statistical laws to be determined. But they in no way prejudge the determination of those laws nor the manner in which they are to be combined to yield schemes of recurrence and their successive probabilities. It follows that the foregoing properties of world process cannot be upset by any amount of scientific work in the determination of classical or statistical laws.

Again, these properties are explanatory of world process. They reveal an order, a design, an intelligibility. For they account in generic fashion for numbers and time intervals, for distributions and concentrations, for blind alleys and break-downs, for enormous differentiation, for increasing systematization, for stability without necessity, for assurance without determinism, for development without chance.

Finally, the intelligibility, offered by the explanation, is immanent in world process. It exhibits the inner design of world process as an emergent probability, and from that design it concludes to the outstanding, generic features of the same process. Accordingly, since empirical method aims at such an immanent intelligibility, emergent probability is a view of world order within the limits of empirical method. As we began by inviting the reader to grasp the intelligibility immanent in the image of a cart-wheel, so now we are inviting him to perform again the same kind of act. The only difference is that, for the image of the cart-wheel, he now must substitute the main features of the universe of our experience.

## 3. CLARIFICATION BY CONTRAST

There is a clarification of ideas through contrast with their opposites. As we have argued that an acceptance of both classical and statistical laws leads to some such world view as emergent probability, so now we have to see how different methodological positions result in different world views.



### 3.1 *The Aristotelian World View*

Aristotle recognized both natural laws and statistical residues. But his natural laws lumped together in primitive confusion not only classical laws and schemes of recurrence but also an element or aspect of statistical laws. His distinction was between the necessary and the contingent. The necessary was what always happens, as in the movements of the stars. The contingent was what usually happens; thus, usually, heavy bodies fall to the earth but, sometimes, they are propped up and so do not fall.

Not only did Aristotle fail to grasp the abstract laws of nature of the classical type, but explicitly he repudiated the possibility of a theory of probability. For him all terrestrial events were contingent. No doubt, effect follows from cause; but it does so, only if some other cause does not intervene; and such intervention is a mere coincidence. It is true that any coincidence can be traced back to earlier coincidences and that from the earlier coincidences one can regress to still earlier coincidences; but one can never get out of the category of the merely coincidental, and within that category there is nothing to be grasped by any science. Hence, while Aristotle recognized statistical residues and concrete patterns of diverging series of conditions, he had no theory of probability to bring them to heel within the field of scientific knowledge.

Still, Aristotle had no intention of allowing terrestrial process to bog down in a mere morass of coincidental interferences. To exorcize such entropy, he argued from seasonal variations to the influence of celestial bodies upon terrestrial activities. Because the sun and moon, the planets and stars, operated necessarily, because they operated from successively different positions, they supplied him with a sufficient ground and cause for the periodicity and perpetuity of terrestrial change. In this fashion there arose his notion of an eternal heaven, an eternal earth, and eternal cyclic recurrence.

Emergent probability differs from the Aristotelian world view, because it rests on a different notion of science and of law. Classical laws are abstract. The alleged necessary movements of the heavens are merely schemes of recurrence that arose through the unfolding of probabilities and will survive in accord with probabilities. The regularities of terrestrial process are essentially similar, though here the schemes are more complex and the probabilities lower. Finally, eternal cyclic recurrence vanishes and in its place there comes the successive realization,

in accord with successive schedules of probabilities, of a conditioned series of ever more complex schemes of recurrence. It is not celestial necessity that assures the success of terrestrial process, but emergent probability that provides the design of all process; and that design is not an eternal, cyclic recurrence, but the realization through probability of a conditioned series of ever more developed schemes.

### 3.2 *The Galilean World View*

Galileo discovered our law of falling bodies, but he failed to recognize its abstractness. Correctly, he grasped that explanation lies beyond description, that the relations of things to our senses must be transcended, that the relations of things to one another must be grasped, and that a geometrization of nature is the key tool in performing this task. Still Galileo did not cast his methodological discoveries in the foregoing terms. Instead of speaking of the relations of things to our senses, he spoke of the merely apparent, secondary qualities of things. Instead of speaking of the relations of things to one another, he spoke of their real and objective primary qualities, and these he conceived as the mathematical dimensions of matter in motion.

Thus Galilean methodology is penetrated with philosophic assumptions about reality and objectivity and, unfortunately, those assumptions are not too happy. Their influence is evident in Descartes. Their ambiguities appear in Hobbes and Locke, Berkeley and Hume. Their final inadequacy becomes clear in Kant, where the real and objective bodies of Galilean thought prove to constitute no more than a phenomenal world.

Hitherto, on the other hand, our procedure has been to prescind severely from philosophic questions about reality and objectivity. In due course we shall have to meet them. But our present concern is the fact that Galilean laws of nature are not conceived in abstraction from sensible or, at least, imaginable elements and, consequently, that the Galilean law stands in the field, not of our abstract classical laws, but rather of our schemes of recurrence in which abstract laws and imaginable elements can combine.

From this concreteness of the conception of natural laws there follows a twofold consequence. On the one hand, there arises the hostility of incomprehension against statistical laws. On the other hand, there results a mechanistic view of the universe. For, in the abstract, classical laws possess universality and necessity. The Galilean acknowledges this



universality and necessity but cannot recognize its abstractness. For him, it is attached immediately to imaginable particles or an imaginable aether or both. For him, it is already concrete, and so it is not in need of further determinations to reach concreteness. For him, the further determinations, which would be non-systematically related to one another, simply do not exist. Accordingly, since he has no doubt of the existence of classical laws, he cannot but regard statistical laws as mere formulations of our ignorance. There is some vast aggregate of discrete or continuous but imaginable elements; they are subject to universal and necessary laws; and the business of the scientist is the hard task of determining those laws and so predicting what cannot but occur.

Moreover, within this context, the negation of statistical laws involves mechanism. A machine is a set of imaginable parts, each of which stands in determinate systematic relations to all the others. In like manner, the universe, implicit in Galilean methodology, is an aggregate of imaginable parts and each is related systematically to all the others. The sole difference is that, apart from the machine, there are other imaginable elements that can interfere with its operation, but apart from the universe of imaginable elements, what imaginable interventions can there arise? Mechanism accordingly becomes a determinism.

Until recently, this Galilean view has been dominant in scientific circles. It easily survived the rather veiled implications of Darwinism. But it seems to have suffered a crippling wound from the overt claims of Quantum Mechanics. Our argument, however, moves on a different terrain. It appeals to Darwinism and to Quantum Mechanics only as illustrations of scientific intelligence. Its proper premises lie in the dynamic structure of empirical inquiry and in the canons that govern its unfolding. In that field it has noticed that abstraction is not impoverishing but enriching, that in the sense of enriching abstraction classical laws are abstract, that a systematic unification of classical laws does not imply the possibility of imaginative synthesis, that the concentration of systematic relationships in the abstract field leaves the further determinations, needed for concrete applications, non-systematically related to one another. It follows that classical and statistical laws, so far from being opposed, are complementary. It follows that the regularities of our universe result, not from classical laws alone, but from the combination of such laws with suitable constellations of concrete circumstances. Finally, it follows that these schemes of recurrence—just as the machines

that men make—emerge and function, survive and vanish, in accord with the successive schedules of probabilities for the realization of a conditioned series of schemes.

### 3.3 *The Darwinian World View*

There are those that date the dawn of human intelligence from the publication of Darwin's *Origin of Species* in 1859. In fact, though the work does not contain any systematic statement of methodological foundations, it does present the outstanding instance of the employment of probability as a principle of explanation. For, in the first place, Darwinism proposes to explain. It offers to tell why species differ, why they are found in their observable spatio-temporal distributions, why the numbers in each species increase, or remain constant, or diminish even to the point of extinction. In the second place, the explanation presents an intelligibility immanent in the data, grounded in similarities and differences, in numbers and their rates of change, in distributions over the surface of the earth and through the epochs of geology. In the third place, this immanent intelligibility differs radically from the immanent intelligibility offered, for instance, by Newton's theory of universal gravitation or Laplace's affirmation of a single mathematical formula by which a suitably endowed intelligence might deduce any world situation from complete information on a single situation. For the follower of Laplace cannot reach any determinate conclusions, unless he is provided with fully accurate information on the basic situation. But the follower of Darwin is indifferent to the details of his basic situation, and he obtains his conclusions by appealing to the natural selection of chance variations that arise in any of a large variety of terrestrial processes from any of a large variety of initial situations.

It is not difficult to discern in Darwin's natural selection of chance variations a particular case of a more general formula. For it is not the single, isolated variation but rather a combination of variations that is significant for the evolutionary process. Again, while such combinations of variations may be attributed to chance, in the sense that the biologist is concerned, not with efficient causality, but with an immanent intelligibility, still, what is significant for evolution is the probability of emergence of such combinations of variations and not the non-systematic divergence from their probability, which is our meaning of the name, chance. Finally, as chance variation is an instance of probability of emergence, so natural selection is an instance of probability of survival.



Artificial selection is the work of the breeder, who mates the plants or animals possessing the characteristics he wishes to encourage. Natural selection is the work of nature, which gives a shorter life expectancy and so less frequent litters to the types that are less well equipped to fend for themselves. Still, nature effects this selection, not with the exact predictability of the changing phases of the moon, but only by a general tendency that admits exceptions and that increases in efficacy with the increase of numbers and the prolongation of time intervals. In a word, natural selection means survival in accord with the probabilities.

Moreover, these combinations of variations, which possess probabilities of emergence and of survival, are relevant to schemes of recurrence. For the concrete living of any plant or animal may be regarded as a set of sequences of operations. Such operations are of kinds; there are many of the same kind; and those of the same kind occur at different times. There are, then, in each set of sequences recurrent operations, and the regularity of the recurrence reveals the existence and functioning of schemes.

Within such schemes the plant or animal is only a component. The whole schematic circle of events does not occur within the living thing, but goes beyond it into the environment from which sustenance is won and into which offspring are born. No doubt, the higher the type, the greater the complexity and the greater the proportion of significant events that occur within the animal. But this greater complexity only means that the larger circle connects a series of lesser and incomplete circles. The vascular circulation occurs within the animal, but it depends upon the digestive system, which depends upon the animal's capacity to deal with its environment and, in turn, that capacity depends on the growth and nourishment secured by the vascular system.

Again, the plant or animal is a component for a range of schemes. Unlike the planets which stick to their courses in the solar system, and like the electrons which may be imagined to hop from one orbit to another, the plant or animal enters into any of a range of sets of alternative schemes. This range is limited by immanent structure and capacity. Still, though it is limited, it remains open to alternatives. For without change of structure or of basic capacity, the plant or animal continues to survive within some variations of temperature and pressure, of circumambient water or air, of sunlight and soil, of the floating population of other plants or animals on which it lives.

At this point, however, the differences between Darwinism and emergent probability begin to come to light. Emergent probability affirms a conditioned series of schemes of recurrence that are realized in accord with successive schedules of probabilities. Darwinism, on the other hand, affirms a conditioned series of species of things to be realized in accord with successive schedules of probability. The two views are parallel in their formal structures. They are related, inasmuch as species of living things emerge and function within ranges of alternative sets of schemes of recurrence. None the less, there is a profound difference. For Darwinian probabilities of emergence and survival regard, not schemes of recurrence, but underlying potential components for any schemes within a limited range, and the Darwinian series of species is a sequence of higher potentialities that exhibit their development by their capacity to function in ever greater ranges of alternative sets of schemes.

This difference prompts us to recall that the present account of emergent probability did not aim at completeness. We had not raised the question, What are things? We had not determined whether there is an answer to that question that satisfies the scientific canon of parsimony. Accordingly, we presented emergent probability in the present chapter with the qualification that later, when the notion of thing has been investigated, there might be needed a further development of the analysis.

However, it may not be amiss to add at once that inquiry into the meaning of the name, thing, will yield but a further manifestation of the dualism in uncritical thought. Just as mechanist determinism has involved an extra-scientific world view, so also has Darwinism. Just as we have replaced the natural selection of chance variations by an emergent probability of schemes of recurrence, so in Chapter VIII we shall find that a critical notion of the thing necessitates a still more significant departure from the unconscious philosophic assumptions of nineteenth-century science.

### 3.4 *Indeterminism*

By indeterminism is meant a contemporary tendency that owes its origin to the verified equations of Quantum Mechanics but goes beyond its source inasmuch as it pronounces on the nature of scientific knowledge and even on philosophic issues. While it is opposed radically to mechanical determinism, its positive features do not admit summary



description and, perhaps, our purpose will best be served by discussing successively a series of issues.

First, as Galileo distinguished between merely apparent secondary qualities and, on the other hand, the real and objective dimensions of matter in motion, so too there are indeterminists that offer a somewhat parallel disclosure of the nature of reality. The old distinction between the real and the apparent is retained, but now the real is microscopic and random, while the merely apparent is the macroscopic in which classical laws seem to be verified. However, we mention this issue only to decline an immediate discussion. Later in a philosophic context we shall attempt an explanatory account of the almost endless variety of views on reality and objectivity. For the present we shall have to be content with the canon of parsimony. The scientist may affirm what he can verify and he may not affirm what he cannot verify.

Secondly, indeterminists tend to reject the old imaginable particles and waves and to favour some type of conceptual symbolism. Here again the issue is the precise nature of reality but now, by appealing to the canon of parsimony, we can reach two conclusions. On the one hand, it would seem that the only possible verification of the imagined as imagined lies in a corresponding sensation; accordingly, if the particles are too small and the waves too subtle to be sensed as particles and waves, then the particles as imagined and the waves as imagined cannot be verified; and if they cannot be verified, they may not be affirmed by the scientist. On the other hand, it is possible to verify conceptual formulations if they possess sensible implications; for in the measure that an increasing number and variety of such implications are found to correspond to sensible experience, the verification of the conceptual formulation is approached. Thus, Special Relativity is said to be probable, not because many scientists feel that they have had a fairly good look at a four-dimensional space-time manifold, but because many scientists working on different problems have found procedures and predictions based on Special Relativity to be highly successful.

Thirdly, there occurs an argument from the haziness of data to the ultimate unverifiability of classical laws. While I do not believe it to be cogent, it is well worth attention. For it appeals to the criterion of verifiability; it rests on the solid fact of the haziness of data; and it does exclude misconceptions of the nature of classical laws.

To begin, the haziness of data is not to be denied. What of itself is determinate never is a datum and always is a concept. Of themselves,

data may be said to be determinate materially or potentially; but they become determinate formally only in the measure that they are subsumed under concepts; and this process of subsumption can be prolonged indefinitely. Thus, a greater formal determinateness of data is possible as long as scientific concepts can be revised to yield more precise objects for measurement and as long as scientific techniques can be improved to make measurements more accurate. But as long as a greater formal determinateness is possible, the determinateness that actually is attained is conjoined with an unspecified remainder of merely potential determinateness. That unspecified remainder is the haziness of data, and it will be with us as long as new concepts and more accurate measurements are possible.

However, the haziness of data alone cannot prove the unverifiability of classical laws. Logically, it is impossible for a valid conclusion to contain a term that does not appear in the premises. More concretely, it could be true that, whenever data became more determinate formally, new classical laws were discovered; and in that case the haziness of data would prove, not that classical laws were unverifiable, but that existing classical laws were always due to be revised in favour of other classical laws.

One comes closer to the issue when one argues that classical laws are conceptual formulations, that they possess all the precision and formal determinateness of concepts, that they cannot be stripped of that precision and determinateness without ceasing to be classical laws. In contrast, data are irreducibly hazy. Because measurements never can be accurate to $n$ decimal places, where $n$ is as large as one pleases, classical laws never can be more than approximative. Their essential determinacy is in radical conflict with the haziness of data; and so classical laws essentially are unverifiable.

Now this argument is valid if classical laws are interpreted concretely. For on concrete interpretation classical laws are supposed to state relations between data or between elements in strict correspondence with data. But there cannot be completely determinate relations between essentially hazy terms; and so, on concrete interpretation, classical laws must be regarded as no more than approximative.

Still, there is no need to interpret classical laws concretely. They can be statements of elements in abstract system where

(1) the abstract system is constituted by implicitly defined relations and terms,



(2) the abstract system is connected with data not directly but through the mediation of a complementary set of descriptive concepts, and

(3) the laws of the abstract system are said to be verified inasmuch as they assign limits on which, other things being equal, vast varieties of data converge.

On this showing, the completely determinate relations of classical laws are between the completely determinate terms they implicitly define. This closed structure is referred to data through a set of descriptive and so approximative concepts. Finally, the closed structure is proved relevant to data, not by exact coincidence, but by assigning the limits on which data converge.

Fourthly, the affirmation of convergence is also an admission of divergence. Is not that admission equivalent to the statement that ultimately classical laws are not verifiable?

Again, the issue is the precise nature of verification. It hardly would be claimed that any single law was not verified because it did not account for the whole of our experience. But what can hold for single laws, also can hold for the totality of classical laws. The existence of the divergence proves that classical laws are not the whole of our explanatory knowledge. But though they are not the whole, they can be a part; and the classical laws that in fact are such a part are the ones that are verified in the sense that they assign the limits on which data do converge.

Fifthly, it is claimed that Quantum Mechanics is the more general theory and that it includes, say, Newtonian mechanics as a particular case.

Here I would suggest the relevance of a distinction between logical inclusion and concrete application. I see no reason for disputing the contention that Schrödinger's time equation can plausibly be simplified into Newton's second law of motion. But it need not follow that the simplification has no analogue in the world of events. On the contrary, it would seem that such an analogue would exist if schemes of recurrence were realized perfectly; and in that case it would seem difficult to maintain that the accuracy of basic observations was not the sole limit to the accuracy of predictions. More realistically, in so far as schemes are not realized perfectly or perfect realization cannot be ascertained, at least the reason for objective divagations or subjective ignorance would be assigned.

Sixthly, it may be argued that determinism must be true or false and that we seem to be dodging the issue. But if the disjunction is admitted, one finds oneself forced into philosophic questions. At least in the present context, our contention would be that the old determinism with its philosophic implications has to give way to a new, purely methodological view that consists in a developing anticipation of a determinate object.

Such a view would remain within the limits of empirical science. It would distinguish between an antecedent component of methodological assumptions and a consequent component of probably verified laws and frequencies. Both components would be regarded as variable. The antecedent component develops; initially it consists in such vague generalities as the assertion that there is a reason for everything; subsequently, as science advances, it takes on the increasing precision of ever more accurately differentiated heuristic structures. Again, the consequent component is subject to variation, for what is regarded as verified at any time may be called into question and subjected to revision. The concrete conjunction of the two components in the minds of scientists constitutes at any time their anticipations of a determinate object; and when the components are undergoing profound change, there naturally will be some uncertainty in their anticipations.

On this view the old determinism was mistaken not only because it was involved in philosophic issues but also because it failed to envisage the possibility of development in heuristic structures. It supposed the universal validity of a type of explanation that is possible only when schematic situations are realized perfectly. It overlooked the possibility of a type of explanation in which the probabilities of the non-schematic account for the emergence of the schematic.

Indeterminism is true as a negation of the old determinism. But it cannot escape the necessity of methodological assumptions and precepts; it cannot prevent their conjunction in thought with laws and frequencies that are regarded as verified; and so it cannot succeed even in delaying the day when, from a new viewpoint, scientific anticipations once more will envisage a determinate object to be known.

However, at the present time, there is some difficulty in specifying in a universally acceptable fashion just what is the determinate object that science is to anticipate. A student of human knowledge can make suggestions that regard the antecedent component, and so I have offered a unified view that anticipates both the systematic and the non-systematic



without excluding in particular cases insight into concrete non-schematic situations. The possibility of concrete insight into the non-schematic situations of the subatomic order probably will be called into question on both practical and theoretical grounds. However, I do not propose to discuss this aspect of the issue, principally because it regards the consequent component of methodological anticipations, but also because I believe all discussions of concrete possibility to suffer from a radical ambiguity. For on any concrete issue further insight is always possible and, when it occurs, what previously seemed impossible, turns out to be quite feasible after all.

4. Let us bring this long chapter to an end. It began from the problem of apparent duality that arose from the existence of two types of insight, two heuristic structures, and two distinct methods of empirical investigation. There was no question of eliminating the duality, for the direct and the inverse types of insight both occur. There remained, then, the task of relating diverse procedures and results into a single whole. In a first section it was argued that classical and statistical investigations are complementary as cognitional activities. In a second section it was revealed how their results, whatever their precise content, can be combined into a single world view. In a third section this world view was contrasted with the Aristotelian, with that of mechanist determinism, with the Darwinian view, and with contemporary tendencies to affirm an indeterminism. In the course of the argument the problem of the thing and, with it, the problem of objectivity became increasingly apparent. But before tackling such large issues, it will be well to broaden the basis of our operations and so we turn to the notions of space and time.

CHAPTER V

# SPACE AND TIME

For a variety of reasons, attention is now directed to the notions of space and time. Not only are these notions puzzling and so interesting, but they throw considerable light on the precise nature of abstraction, they provide a concrete and familiar context for the foregoing analyses of empirical science, and they form a natural bridge over which we may advance from our examination of science to an examination of common sense.

The present chapter falls into five sections. First of all, there is set forth a problem that is peculiar to physics as distinct from other natural sciences such as chemistry and biology. Secondly, there is worked out a descriptive account of space and time. Thirdly, an attempt is made to formulate their abstract intelligibility. Fourthly, there follows a discussion of rods and clocks. Finally, the concrete intelligibility of space and time is indicated.

## 1. THE PROBLEM PECULIAR TO PHYSICS

### 1.1 *Invariant and Relative Expressions*

To formulate this problem, distinctions must be drawn

(1) between propositions and expressions, and
(2) between invariant and relative expressions.

For present purposes the distinctions between propositions and expressions will be indicated sufficiently by such illustrative statements as the following:

'It is cold' and '*Il fait froid*' are two expressions of the same proposition.

Again, '2+2=4' and '10+10=100' are respectively the decimal and binary expressions of the same proposition.

Now just as different expressions may stand for the same proposition, so the same expression under different circumstances may stand for different propositions. This fact leads to a distinction between invariant and relative expressions.

Expressions are named invariant if, when employed in any place or at any time, they stand for the same proposition.





Expressions are named relative if, when employed in different places or at different times, they stand for different propositions.

Thus, '2+2=4' stands for the same proposition no matter where or when it is uttered. It is invariant. On the other hand, 'John is here now' stands for as many different propositions as there are places in which it is uttered and times at which it is uttered. It is relative.

### 1.2 *Their Ground in Abstraction*

It is not difficult to discern the reason why some expressions are invariant and others are relative. For if an expression stands for an abstract proposition, it contains no reference to any particular place or time; if it contains no reference to particular places or times, it contains no element that might vary with variations of the place or time of the speaker. Inversely, if an expression stands for a concrete proposition, it will contain a reference to a particular place or time and so it will include an element that can vary with variations of the speaker's position and time.

The point may be illustrated by contrasting the use of the copula, 'is', in the two expressions, 'John is here', and 'Pure water is $H_2O$'. In the first expression, which stands for a concrete proposition, the copula is relative to the time of utterance; the grammatical present tense of the verb, *to be*, has its proper force; and saying that John is here has no implication that John was or was not here, or that John will or will not be here. On the other hand, to say that pure water is $H_2O$ is to utter an abstract law of nature; grammatically, the copula occurs in the present tense, but it is not intended to confine the force of the expression to the present time. For if really it is true that pure water is $H_2O$, then necessarily pure water was $H_2O$ even before oxygen was discovered and pure water will remain $H_2O$ even after an atom-bomb has eliminated anyone interested in chemistry. In brief, the copula, 'is', in abstract expressions occurs not in the ordinary present tense but rather in an invariant tense that abstracts from particular times.

### 1.3 *Abstraction in Physics*

Now if the invariance or relativity of expressions follows from the abstractness or concreteness of the propositions for which they stand, then, since all mathematical principles and all natural laws of the classical type are abstract, it follows that their appropriate expression must be invariant.

In fact, such invariance of expression is secured automatically in mathematics, in chemistry, and in biology. There never arose any tendency to write out the multiplication table or to state the binomial theorem differently in Germany and France, in the nineteenth or twentieth centuries. In like manner it would be impossible to find relative expressions for the hundreds of thousands of formulae for chemical compounds. Such statements simply contain no reference to space or time, and so cannot vary with variations of the speaker's position or epoch.

However, the science of physics does not enjoy the same immunity. It investigates local movements, and it cannot state their laws without some reference to places and times. Since laws contain a reference to places and times, they include an element that can vary with variations of the speaker's position and time. Accordingly, there arises a problem peculiar to physics. Just as ordinary language develops an invariant copula to express general truths, so too the physicist has to find spatio-temporal invariants, if he is to employ the appropriate invariant expressions in stating laws of local motion.

## 2. THE DESCRIPTION OF SPACE AND TIME

Before tackling the problem peculiar to physics, it will be well to review the materials or data that are involved. Such a review is a task for description and, as we have seen, descriptions are cast in terms of experiential conjugates. Accordingly, we shall begin from elementary experiences, work out the resultant notions of space and time, and show how they necessarily involve the use of frames of reference and of transformations.

### 2.1 *Extensions and Durations*

There exist certain elementary and familiar experiences of looking, moving about, grasping, etc.

The experiences themselves have a duration. They occur, not all at once, but over time. Moreover, correlative to the duration of looking, there is the duration of what is looked at. Correlative to the duration of the moving, there is the duration of what is moved through or over. Correlative to the duration of the grasping, there is the duration of what is grasped. Descriptively, then, duration is either an immanent aspect or quality of an experience or a correlative aspect or quality of what is experienced.



While duration is commonly attributed both to the experience and to the experienced, extension is attributed only to the latter. The colours I see, the surfaces I grasp, the volumes through which I move, all have extension. But it would seem paradoxical to speak of the extension of the experience of seeing, of the experience of grasping, of the experience of moving. Descriptively, then, extensions are correlative to certain elementary and familiar experiences but they are in the experienced and not in the experiencing.

### 2.2 *Descriptive Definitions*

Let us now define Space as the ordered totality of concrete extensions, and Time as the ordered totality of concrete durations. Further, let us give notice that henceforth, when Space and Time are written with capital letters, the words will be employed in accord with the foregoing definitions.

For besides the totalities of concrete extensions and concrete durations, there also are merely imaginary totalities. What a man experiences, he also can imagine. As he experiences extension, he also imagines extension. As he experiences duration, he also imagines duration. Our concern is not with imaginary extensions or imaginary durations but with the concrete extensions and durations correlative to experience.

Immediately, however, there arises an obvious difficulty. For neither the totality of concrete extensions nor the totality of concrete durations falls within the experience of the human race, let alone the human individual. For this reason the definition refers, not to any totalities, but to ordered totalities. It is true enough that only a fragment of concrete extension and of concrete duration falls within human experience. Still, one can take that fragment as origin. Beyond the extension that is experienced, there is further extension; and since it is continuous with the extension of experience, it is not merely imagined. Similarly, beyond the duration of experience, there is further duration, and since it is continuous with the duration of experience, it is not merely imagined.

There follows a simple criterion for distinguishing between the notion of concrete Space or Time, and, on the other hand, merely imaginary space or time. Within concrete Space there is some extension that is correlative to experience, all other extension in Space is related to that concrete extension; and in virtue of that relation all other extension in Space is concrete. Similarly, a notion of concrete Time is

constructed about a nucleus of experienced duration. On the other hand, merely imaginary space or time contains no part that is correlative to actual experience.

From the criterion, there follows a corollary. Imaginary space or time may or may not be structured about an origin. But notions of concrete Space or Time must be structured about an origin. For only fragments of concrete Space or Time enter into human experience, and so it is only by a relational structure to given extensions or durations that totalities of extensions or durations can be concrete. In other words, frames of reference are essential to the notions of Space and Time.

### 2.3 *Frames of Reference*

Frames of reference are structures of relations employed to order totalities of extensions and/or durations. They fall into three main classes: the personal, the public, and the special.

First, everyone has his personal reference frame. It moves when he moves, turns when he turns, and keeps its 'now' synchronized with his psychological present. The existence of this personal reference frame is witnessed by the correlation between the place and time of the speaker and, on the other hand, the meaning of such words as *here*, *there*, *near*, *far*, *right*, *left*, *above*, *below*, *in front*, *behind*, *now*, *then*, *soon*, *recently*, *long ago*, etc.

Secondly, there are public reference frames. Thus, men become familiar with the plans of buildings, the network of streets in which they move, the maps of their cities, countries, continents. Similarly, they are familiar with the alternation of night and day, with the succession of weeks and months, with the use of clocks and calendars. Now such relational schemes knit together extensions and durations. But they are not personal reference frames that shift about with an individual's movements. On the contrary, they are public, common to many individuals, and employed to translate the *here* and *now* of the personal reference frame into generally intelligible locations and dates. Finally, the difference between personal and public reference frames comes out clearly in the occurrence of such questions as, Where am I? What time is it? What is the date? Everyone is always aware that he is here and now. But further knowledge is required to correlate one's *here* with a place on a map and one's *now* with the reading of a clock or a calendar.

Thirdly, there are special reference frames. A basic position, direction, and instant are selected. Co-ordinate axes are drawn. Divisions on the axes are specified, and so any point at any instant can be denoted univocally as an $(x, y, z, t)$.



Special reference frames may be mathematical or physical. They are mathematical if they order an imaginary space and time. They are physical if they order concrete Space and Time. The distinction is brought to light by selecting any $(x, y, z, t)$ and asking where and when it is. For if the frame is physical, the answer will indicate some precise point in Space and some precise instant in Time. But if the frame is mathematical, the answer will be that any point-instant whatever will do.

### 2.4 *Transformations*

There can be as many distinct reference frames of any kind, as there are possible origins and orientations.

From this multiplicity there follows the problem of transposing from statements relative to one reference frame to statements relative to another.

Solutions may be particular, and then they are obtained by inspection and insight. Thus, when two men face each other, one may observe that the region of Space to the right of one man is to the left of the other, and so one concludes that under such circumstances what for one is 'right' for the other is 'left'. In like manner, maps of different countries may be correlated by turning to the map of the continent that includes both countries, and clocks in different positions may be synchronized by appealing to the earth's spin.

Special reference frames admit a more general solution. Let the point $(x, y, z)$ in the frame $K$, be identical with the point specified as $(x', y', z')$ in the frame, $K'$. From geometrical considerations it will be possible to find three equations relating $x$, $y$, and $z$, respectively to $x'$, $y'$, and $z'$ and, further, to show that these equations hold for any point $(x, y, z)$. In this fashion there are obtained transformation equations and by the simple process of substitution any statement in terms of $x$, $y$, $z$, can be transformed into a statement in terms of $x'$, $y'$, $z'$.

For example, the wave-front of a light signal emitted from the origin of a frame, $K$, might be the sphere

$$x^2 + y^2 + z^2 = c^2t^2.$$

The equations for transforming from the frame, $K$, to a frame, $K'$, might be

$$x = x' - vt', \; y = y'; \; z = z'; \; t = t'.$$

On substituting, one would obtain the equation of the wave-front in the frame, $K'$, namely:

$$(x'-vt')^2+y'^2+z'^2=c^2t'^2.$$

### 2.5 *Generalized Geometry*

In the foregoing consideration of transformations, the procedure in the special case was based upon geometrical considerations. It is worth noting that the inverse procedure is possible, that is, that from a consideration of transformations one can work out the general theory of geometries.

Consider any function of $n$ variables, say,

$$F(x_1, x_2, \ldots)=0 \qquad (1)$$

and any $n$ arbitrary transformation equations, say,

$$\begin{aligned} x_1&=x_1(x'_1, x'_2, \ldots)\\ x_2&=x_2(x'_1, x'_2, \ldots)\end{aligned} \qquad (2)$$

which on substitution yield the new function, say,

$$G(x'_1, x'_2, \ldots)=0. \qquad (3)$$

Let these mathematical expressions have a geometrical interpretation, so that the initial variables in $x_i$ refer to positions along the axes of a co-ordinate system, $K$, and the subsequent variables in $x'_i$ refer to positions along the axes of another co-ordinate system, $K'$, and the transformation equations represent a shift from the reference frame, $K$, to the frame, $K'$.

Now the mathematical expressions have the same meaning, stand for the same propositions, and require the same geometrical interpretation, if they have the same symbolic form. For the meaning of a mathematical expression resides, not in the material symbols employed, but in the form of their combination to indicate operations of adding, multiplying, and so forth.

Accordingly, when the symbolic form of a mathematical expression is unchanged by a transformation, the meaning of the expression is unchanged. But a transformation is a shift from one spatio-temporal standpoint to another and, when expressions do not change their meaning under such shifts, then, as we have seen above, the expressions are invariant and the ground of that invariance is that the expressions stand for abstract and generally valid propositions.

Now the principles and laws of a geometry are abstract and generally valid propositions. It follows that the mathematical expression of the principles and laws of a geometry will be invariant under the permissible transformations of that geometry.



Such is the general principle, and it admits at least two applications. In the first application, one specifies successive sets of transformation equations, determines the mathematical expressions invariant under those transformations, and concludes that the successive sets of invariants represent the principles and laws of successive geometries. In this fashion one may differentiate Euclidean, affine, projective and topological geometries.*

A second, slightly different application of the general principle occurs in the theory of Riemannian manifolds. The one basic law governing all such manifolds is given by the equation for the infinitesimal interval, namely,

$$ds^2=\Sigma g_{ij}dx_i dx_j \quad [i, j=1, 2 \ldots n]$$

where $dx_1$, $dx_2$, . . . are differentials of the co-ordinates, where the coefficients, $g_{ij}$, are functions of the co-ordinates, and where in general there are $n^2$ products under the summation. Since this equation defines the infinitesimal interval, it must be invariant under all permissible transformations. However, instead of working out successive sets of transformations, one considers any transformations to be permissible and effects the differentiation of different manifolds by imposing restrictions upon the coefficients. This is done by appealing to the tensor calculus. For tensors are defined by their transformation properties and it can be shown that, in the present case, if the coefficients, $g_{ij}$, are any instance of a covariant tensor of the second degree, then the expression for the infinitesimal interval will be invariant under arbitrary transformations. It follows that there are as many instances of the Riemannian manifold and so as many distinct geometries, as there are instances of covariant tensors of the second degree employed to specify the coefficients, $g_{ij}$. Thus, in the familiar Euclidean instance, $g_{ij}$ is unity when $i$ equals $j$; it is zero when $i$ does not equal $j$; and there are three dimensions. In Minkowski space, the $g_{ij}$ is unity or zero as before, but there are four dimensions, and $x_4$ equals $ict$. In the General Theory of Relativity, the coefficients are symmetrical, so that $g_{ij}$ equals $g_{ji}$; and in the Generalized Theory of Gravitation, the coefficients are anti-symmetrical.

* See, for instance, the summary outline offered by V. Lenzen in his *Nature of Physical Theory*, New York 1931, pp. 59 ff.

### 2.6 A Logical Note

It is to be observed that transformation equations, operations of transforming, the definition of tensors by their transformation properties, and the whole foregoing account of the differentiation of geometrical manifolds belong to higher-order statements.

For distinct reference frames assign different specifications to the same points and instants and they assign the same specifications (numbers) to different points and instants. Accordingly, they must belong to different universes of logical discourse, else endless ambiguities would result. Now the relations between different universes of discourse can be stated only in a further, higher-order universe of discourse; in other words, the relations between different universes of discourse regard, not the things specified in those universes, but the specifications employed to denote the things. Thus, a transformation equation does not relate points or instants, but it does relate different ways of specifying the same points and instants. Similarly, such a property as invariance is a property, not of a geometrical entity, but of an expression regarding geometrical or other entities.

## 3. THE ABSTRACT INTELLIGIBILITY OF SPACE AND TIME

The argument began from a problem peculiar to physics. Because that science deals with objects in their spatial and temporal relations, the expression of its principles and laws does not automatically attain the invariance proper to such abstract propositions. However, as was shown in Chapter II, this difficulty can be turned to profit, inasmuch as the physicist can posit a postulate of invariance and then employ that postulate as a heuristic norm in determining which expressions can represent physical principles and laws.

The second strand of the argument consisted in an outline of the descriptive notions of Space and Time. It began from experience of concrete extensions and durations and it showed that we can form notions of all concrete extensions and of all concrete durations if, and only if, these totalities are ordered by frames of reference. Essentially, then, the descriptive notion of Space is of Space-for-us and the descriptive notion of Time is of Time-for-us. Again, one might say that these notions necessarily contain, on the one hand, an empirical or material element and, on the other hand, an intelligible or formal element. The empirical or material element consists of concrete extensions and of



concrete durations. The intelligible or formal element orders these materials into singular totalities. Moreover, without this intervention of ordering intelligence, the notion of Space cannot be both concrete and all-embracing, and similarly the notion of Time cannot regard the totality of concrete durations.

Still, these descriptive notions of Space and Time cannot contain the intelligibility that is explanatory of Space and Time. It is true that they contain an intelligible or formal component. But that component is the order of a reference frame, and reference frames are an infinity. They can be the intelligibility of Space-for-us and of Time-for-us, that is, they can be the manners in which we intelligently order extensions and durations in accord with the convenience of the moment. But they cannot be the immanent intelligibility that is explanatory of Space nor the immanent intelligibility that is explanatory of Time, for reference frames are infinite, but correct explanations are unique.

However, this gives rise to a further problem. On the one hand, if we retain reference frames, we are dealing with infinities of formally different notions of Space and Time. On the other hand, if we drop reference frames, then our inquiry is confined either to merely imaginary space and time or else to the relatively few extensions and durations that fall within our experience. It is this dilemma that reveals the significance of transformations and invariance under transformations. For, while such considerations belong to a higher-order universe of discourse which directly regards not objects but expressions referring to objects, still they can serve to point the way to grasping the intelligibilities immanent in Space and in Time. Inasmuch as we say what we think, the properties of our expressions reflect the properties of our thoughts. Inasmuch as we think intelligently, the properties of our thoughts reflect the properties of our insights. In this fashion, the invariance of expression has already been traced to the abstractness of what is thought or meant and, at an earlier stage of the inquiry, the abstractness of classical laws was grounded on the enriching contribution of insight. Accordingly, we shall not be venturing into a new line of thought, if we argue that the set of insights, by which we grasp the intelligibility immanent in Space and Time, will be the set that is formulated in spatial and temporal principles and laws invariant under transformations of reference frames.

Clearly enough, this conclusion gives no more than a generic answer to our question. It amounts to saying that the immanent intelligibility

of Space and of Time will be formulated in one of the geometries that fall under the generalized notion of geometry. There remains the task of assigning the specific geometry that governs concrete extensions and concrete durations. Still, one has only to mention this task to be reminded that there is a problem peculiar to the empirical science of physics, that this problem arises in physics inasmuch as it is involved in spatial and temporal relations, and that the general form of its solution is to postulate the invariance of physical principles and laws.

### 3.1 *The Theorem*

It is time to turn from talk about what we propose to do and settle down to the work of doing it.

The abstract formulation of the intelligibility immanent in Space and in Time will be one of the possible sets of definitions, postulates, and inferences that systematically unify the relations of extensions and of durations. All such possible sets of definitions, postulates, and inferences are geometries. Therefore, the abstract formulation of the intelligibility immanent in Space and in Time will be a geometry.

The expression of the principles and laws of any geometry will be invariant. For principles and laws are independent of particular places and times, and so their proper expression cannot vary with variations of spatio-temporal standpoints.

Moreover, a geometry cannot refer to Space or to Time except through a reference frame. Accordingly, the invariance proper to the expression of geometrical principles and laws is an invariance under transformations of reference frames.

There follows at once the generic solution. The abstract formulation of the intelligibility of Space and Time consists in a set of invariants under transformations of reference frames. However, there is a range of such sets of invariants, and so there remains the task of determining the specific solution.

We note, accordingly, that the relevant intelligibility is immanent in concrete extensions and in concrete durations. It is an intelligibility that belongs not to the imagined but to the experienced. Now the empirical canon of complete explanation has already assigned to natural science the duty of doing for experienced extensions and durations exactly what is done for experienced colours, experienced sounds, experienced heat, experienced electromagnetic phenomena. Further, physics is the natural science on which this duty falls, as appears from its peculiar



problem of invariance. Again, if the physicist solves his peculiar problem and arrives at an invariant expression of his principles and laws under transformations of reference frames, he cannot avoid reaching the specific solution which we are seeking. For the specific solution we are seeking is the set of invariants under transformations that is verifiable in experienced extensions and durations.

The abstract formulation, then, of the intelligibility immanent in Space and in Time is, generically, a set of invariants under transformations of reference frames and, specifically, the set verified by physicists in establishing the invariant formulation of their abstract principles and laws.

A corollary may be added. The intelligibility immanent in Space and in Time is identical with the intelligibility reached by physicists investigating objects as involved in spatial and temporal relations. Hence, to eliminate the concrete objects of physics would be to eliminate the intelligibility of Space and of Time. Again, inasmuch as physical objects are involved differently in spatial and temporal relations, there result different intelligibilities of Space and of Time. This conclusion may be illustrated by the possibility of different types of tensors being employed to secure the covariance of different sets of physical principles and laws.

### 3.2 *Euclidean Geometry*

While the foregoing argument of itself says nothing for or against the verifiability of Euclidean geometry, still it supposes that Euclidean geometry is not the one and only true geometry, and it admits the possibility of other geometries being verifiable.

The supposition is, of course, far more fundamental than the admission. It is difficult not to find the inspiration of rationalism, which deduces everything else from alleged self-evident principles, in the notion that Euclid formulated the one and only true geometry. After all, the supreme rationalist wrote on his title page, *Ethica ordine geometrico demonstrata*. Still, these high matters lie beyond the range of present considerations though, in due course, we hope to meet this issue with a distinction between analytic propositions which are not far from tautologies, and analytic principles, whose terms and relations are verifiable in the existent.

At any rate, present concern has to be confined to meeting claims that Euclidean geometry obviously is verified in concrete extensions and that ordinary notions of simultaneity obviously are verified in concrete durations.

Clearly, there is a sense in which these claims are true. It has been seen that one cannot form a notion of Space without invoking a frame of reference. It is plain that men form notions of Space and, no less, that the frames of reference they construct satisfy Euclidean requirements. Similarly, one cannot form a notion of Time without introducing a frame of reference, and the frame ordinarily introduced is necessarily in complete accord with ordinary notions on simultaneity. Not for a moment would I dispute the contention that Euclidean geometry and the common view of simultaneity are both verifiable and verified in the descriptive notions men form of Space and Time.

However, after granting all that is obvious, we must now add that it is quite beside the point. The analysis of descriptive notions of Space and Time has its significance, but that significance is anthropological. It reveals how men commonly proceed from the extensions and durations of experience to the totalities named Space and Time. On the other hand, when we admit that Euclidean geometry might not be verifiable, we are speaking of a verification, not in human notions, but in concrete extensions and durations. We are not asking how men find it convenient to conceive Space and Time; we are asking how scientists may correctly explain Space and Time. Were the scientists in question the psychologists, one might appeal to what is obvious in the mentality of Western man. But the scientists in question happen to be physicists, and the data of consciousness, however clear, are not among the data proper to physics.

So much then, for the sweeping claim that our conclusion must be wrong because its error is obvious. It remains that objections may be less sweeping, and these must now be met.

### 3.3 *Absolute Space*

The absolute space and the absolute time of Newtonian thought possess the twofold merit of exhibiting an 'obvious' view and of inviting criticism that goes to the root of the matter.

Suppose a penny to fall to the floor of a moving train, and ask for an account of the trajectory of the fall. Unfortunately, there are many accounts. Relatively to the floor, the trajectory is a vertical straight line. Relatively to the earth, it is a parabola. Relatively to the axes fixed in the sun, it is a more complicated curve that takes into account the spin and orbit of the earth's movements. Relatively to the receding nebulae, it contains still further components. But there is only one



penny in question, and there is only one fall. Which, really, is the trajectory?

Newton would answer by distinguishing between true and apparent motion. Both are relative. But, while apparent motion is relative to other bodies, such as the train, the earth, the sun, the nebulae, true motion is relative to an eternal set of immutable places named absolute space. If one thinks of apparent motion, one can say that the penny moves relatively to the train, the train relatively to the earth, the earth relatively to the sun, and the sun relatively to the nebulae. But if one thinks of true motion, one can say that, perhaps, the penny, the train, the earth, the sun, and the nebulae have a common velocity relatively to a set of eternal and immutable places.

Moreover, if Newton named his absolute space mathematical, he also considered it real. He admitted the difficulty of determining when there was a true motion. But he was far from acknowledging such a conclusion as impossible. On the contrary, he performed his famous bucket experiment to show that true motion relative to absolute space could be detected. A bucket of water was suspended from a twisted rope. The bucket spun and, for a while, the surface of the water remained flat. The surface then hollowed out into a paraboloid. Eventually, the bucket ceased to spin, but the surface remained hollow. Finally, the surface became flat again. Now the hollowing of the surface of the water was due to the rotation of the water and, as this hollowing occurred both while the bucket was spinning and while the bucket was not spinning, it could not be merely an apparent motion relative to the bucket. Therefore, it was true motion relative to absolute space.

Let us now turn to criticism.

First of all, the bucket experiment does not establish the existence of an absolute space. From the experiment one might conclude that really and truly the water was rotating; for in the hollowing of the surface one might verify a centrifugal acceleration; and if there is a verified centrifugal acceleration, there is a verified motion. However, true motion in the sense of verified motion is one thing; and true motion in the sense of motion relative to absolute space is quite another. The bucket experiment does not establish true motion in this second sense. Indeed, the sole link between the experiment and absolute space lies in an equivocal use of the term, true.

Secondly, the Newtonian distinction between true and apparent motion involves the use of an extra-scientific category. There are the

data of experience. There are inquiries, insights, and formulations. There are verifications of formulations. But just as Galileo impugned given colours, sounds, heat, and the like as merely apparent, so Newton impugned as apparent the observable changes of relative position of observable bodies. Just as Galileo affirmed as real and objective the primary qualities that are mathematical dimensions of matter in motion, so Newton, after eliminating experienced motions as apparent, acknowledged as true the motions relative to a non-experienced absolute space. What is this truth of true motion? Though Newton confused it with the truth of experiment and verification, it has to be something else; otherwise, there would be no confusion. What, then, is it?

A fuller account will be attempted when we treat the notion of objectivity. For the present, it will suffice to recall that the Galilean assertion of the reality and objectivity of primary qualities was not in accord with the canon of parsimony but, as we have seen, extra-scientific (See Chapter III, § 5). In simpler terms, Galileo's real and objective was the residue left in the popular category of the 'really out there', after colours, sounds, heat, etc., had been eliminated. By parallel reasoning, Newton's absolute space was the 'really out there' but emptied not only of Galileo's secondary qualities but also of his own apparent motions. From this position to Kant's, it is an easy step. For Kant, as for his scientific predecessors, all sensible presentations were phenomenal. But, while Newton secured a metaphysical status for his absolute space by naming it the *divine sensorium*,* Kant gave this empty 'really out there' a critical status by making it an *a priori* form of human sensibility.

Thirdly, Galileo, Newton, and Kant were looking for some sort of absolute, but they were looking in the wrong places. They sought the real as opposed to the apparent, only to end up with everything apparent, the notion of the real included. Let us follow a different tack. Then every content of experience will be equally valid, for all are equally given, and all equally are to be explained. Next, explanations result from enriching abstraction, and so they are abstract, and their proper expression must be invariant. Thirdly, not every explanation is equally correct; some can be verified, and some cannot. There follows at once the conclusion that the real, objective, true consists of what is

* See E. A. Burtt, *The Metaphysical Foundations of Modern Science*, London and New York 1925, pp. 257 ff.



known by formulating and verifying invariant principles and laws. Our account of Space is simply a particular case of that conclusion.

Fourthly, let us attempt to meet the problem of the trajectory of the penny. As we have seen, possible frames of reference are infinite; but in any determinate frame of reference, there is only one correct trajectory for the penny. Next, while some possible frames of reference are more convenient than others, still all are equally valid, and so there are many correct trajectories for the penny. Further, this involves no contradiction; just as what is to my right can be to your left, so the one fall of the one penny can be a straight line in one frame of reference and a parabola in another frame of reference; there would be a contradiction only if the same fall were both a straight line and a parabola in the same frame of reference.

Finally, this position is not unsatisfactory. As long as we are speaking of particular things at particular times in particular places, we cannot avoid employing relative expressions; for it is through our senses that we know the particular; and our senses are in particular places at particular times. On the other hand, invariant expression, which is independent of the spatio-temporal standpoint of particular thinkers, is a property of abstract propositions; it can be demanded only of the principles and laws of a science; and the trajectory of the fall of a particular penny is not a principle or a law in any science.

### 3.4 *Simultaneity*

The common view of simultaneity possesses, perhaps, a larger and more resolute following than Newton's absolute space. If two events are at the same time for any observer, then, we shall be told, they must be at the same time for every observer.

The first line of defence will be, no doubt, the principle of contradiction. The same events cannot be both at the same time and not at the same time. Therefore, to say that the same events are at the same time for one observer and not at the same time for another, is simply to violate the principle of contradiction.

Still, this first line can be turned. What is 'now' for me writing is not 'now' for you reading. If the same event can be both now (for me) and not now (for you), it may be true that 'at the same time' belongs to the same class of relative terms as does 'now'; and if it does, then there is no more a contradiction in saying that events, simultaneous for one observer, are not simultaneous for another, than there is in saying that

events of the present for one observer will be events of the past for another.

The issue is not the principle of contradiction. The issue is simply whether or not 'at the same time' is to be listed along with such relative terms as 'now' and 'soon', 'here' and 'there', 'right' and 'left'.

The simplest approach to the issue is to analyse elementary apprehensions of simultaneity. Already we have remarked that we experience duration both in the sense that the experiencing is over time and in the sense that the experienced endures through time. Now we have to add that these two aspects of the experience of duration stand in a certain order. Thus, when I watch a man crossing a street, I look out and inspect the distance that he traverses, but I cannot look out and inspect in the same manner the time he takes to cross. Nor is this surprising. The whole distance traversed is there to be inspected all at once, but the duration of the traversing is there to be inspected, not all at once, but only in successive bits. Moreover, what is true of the traversing is also true of the inspecting; it too is, not all at once, but over time. If one supposed the possibility of a timeless inspecting, one might infer the inspection of a four-dimensional continuum in which both distances and durations were presented in exactly the same fashion. But when inspecting takes time, then the time of the inspecting runs concurrently with the time of the inspected.

Such remarks on the apprehension of durations seem relevant to an account of the apprehension of simultaneous durations. Instead of watching one man cross a street, I might watch two men crossing a street at the same time. Since it would be perfectly obvious that they were crossing at the same time, it should be equally obvious that there is some time that is one and the same. What time, then, obviously is the same? It must be the time of the watching. For, in the first place, the watching has a duration, for it is not all at once. In the second place, the duration of the watching runs concurrently with the duration of what is watched. In the third place, when two movements are the object of one and the same watching, there are, in all, three durations, namely, one in each movement and one in the watching; but it is the duration of the watching that is apprehended as running concurrently both with the duration of one movement and with the duration of the other; and so it is the duration of the watching that is the one and same time at which both the movements are occurring.

This analysis is confirmed by a consideration of apprehensions of



'apparent' simultaneity. If you stand beside a man swinging a hammer, then the sight and the sound of the blow are at the same time. If you stand off at a distance of a few hundred feet, the sight of the blow is prior to the sound. In the first case, the sight and sound are at the same time. In the second case, the sight and the sound are not at the same time. Still, the blow is always the simultaneous source of both light-waves and sound-waves. The reason why there are different, 'apparent' simultaneities must be that the 'appearance' of simultaneity has its ground in the duration immanent in the flow of consciousness.

Such seem to be the facts and, like the facts of relative motion, they give rise to a problem. Is one to follow Galileo and Newton and insist that, beyond the multiplicity of merely apparent simultaneities, there is a real, objective, and true simultaneity that is unique? If so, one can omit further mention of the observer, and one will end up with an absolute time that flows equably everywhere at once. It will not be the time of clocks, which run fast or slow. It will not be the time of the spinning earth, for under the action of the tides and the receding moon, that spin is decelerating. It will be an exact, constant velocity that at every point in the universe perpetually separates the present from the past and the future in precisely the same manner.

Still, this absolute time will not be what we have defined as Time. For Time, as we have defined it, is an ordered totality of concrete durations. It includes the concrete durations both of our experiencing and of what we experience. Through an ordering structure or reference frame it reaches out to embrace in a single totality all the other concrete durations which, though not experienced, are related to the concrete durations that are experienced. In contrast with this Time, absolute time simply lies outside experience. It meets the requirements of a mathematical ideal and, strangely enough, unlike other mathematical ideals, it is said to be 'really out there'. Rather, it once was thought to be really out there. For the Newtonian rejection of experienced durations as apparent time in favour of a non-experienced absolute time promptly was followed by Kant's transformation of absolute time into an *a priori* form of human sensibility.

Nor is this the only complaint against the Newtonian procedure. As absolute space, so absolute time is a result of looking for the absolute where the absolute does not exist. If it were true that events, simultaneous for one observer, must be simultaneous for every other observer, then it would be true that expressions of simultaneity are

invariant. But there is no reason to expect invariant expressions of simultaneity, for invariance results from abstractness, and no statement regarding the particular times of particular events is abstract. From the very structure of our cognitional apparatus, particulars are known through our senses, and our senses operate under spatio-temporal conditions. They cannot escape relativity and so, if an absolute is wanted, it must be sought on the level of intelligence which by abstraction from particulars provides a ground for invariant expressions.

### 3.5 *Motion and Time*

We have been speaking of the elementary durations and simultaneities of the personal reference frame. But, besides personal reference frames, there are public and special reference frames, and they call for a few remarks.

Aristotle defined time as the number and measure of local motion derived from successively traversed distances. Such is the time of the spinning earth and of clocks. 'Two o'clock' is a number and 'two hours' is a measure. Both are reached from the local motion of the hands over the face of a dial.

However, there are many local motions, and every one successively traverses a series of distances. It follows that, though all do not yield numbers and measures indicating time, still all could do so. Objectively, then, and fundamentally there are many times.

This implication of the Aristotelian position was noted by Aquinas. However it seemed to him not an important truth, but rather an objection to be answered. Time must be one, and so he appealed to the *primum mobile*, the outermost sphere, and it had only one local motion. Moreover, as it grounded all other local motions both in the sky and on the earth, the time of its movement must be the ground of all other times.*

One will be inclined, I think, to agree that as long as Aristotle's *primum mobile* was supposed to exist, our universe was supplied with a single, standard time. On the other hand, once Copernicus eliminated the Ptolemaic system, that standard time no longer was possible and, in its place, there arose the problem of synchronization, of making many movements yield a single time for public and special reference frames.

* See S. Thomas Aquinas, *In IV lib. Phys. Arist., lect.* 17, ed. Leon. Rome 1884, vol 2, p. 202, § 3, 4.



Suppose, then, an aggregate of clocks scattered about the universe. Let their relative positions be constant, and let them be known in terms of some reference frame, *K*. Let light signals be sent from the origin of co-ordinates to the clocks and reflected from the clocks back to the origin. Then, a synchronization of clocks might be effected by laying down the rule,

$$2t = t' + t''$$

where $t$ is the reading of the distant clock when the light signal is received and reflected, and where $t'$ and $t''$ are the readings of the clock at the origin when the light signal is emitted and when it returns.

However, synchronization by this rule would be successful, only if the outward and the return journeys of the light signal took the same length of time. To satisfy this requirement, one might distinguish between basic and derived synchronizations and demand that the basic synchronization take place with clocks that are at rest with respect to the aether and in a reference frame that similarly is at rest. Then, synchronization in moving frames would be the synchronization of their clocks with the clocks of the basic frame, and there would follow for all point-instants an observable time that conformed to the properties of Newton's absolute time.

There is, however, one difficulty to this solution. One can in principle suppose any number of reference frames exhibiting as many varieties of relative motion as one pleases. One can supply each frame with clocks that, relatively to the frame, are at rest. But a difficulty arises when one attempts to select the frame that absolutely is at rest and, if one cannot determine the basic synchronization, much less can one reach the derived synchronizations.

Still there is an alternative. Instead of seeking the absolute in the field of particular reference frames, one can seek it in the field of abstract propositions and invariant expressions. Accordingly, one may postulate that the mathematical expression of physical principles and laws be invariant under inertial transformations, and one may note that from the postulate it follows that in all reference frames moving with a relative uniform motion the velocity of light will be the same.*

3.6 Before closing this section, it will be well to set forth briefly the principles that have guided us in determining the abstract intelligibility

* For the consequent derivation of the Einstein-Lorentz transformation and of Minkowski space, the reader may be referred to Lindsay and Margenau, pp. 333 ff.

of Space and Time and, no less, to indicate the grounds that lead to different views.

Our position follows from our account of abstraction. Because the principle or law is abstract, its expression cannot vary with variations of spatio-temporal standpoint. On the other hand, because we know particulars through spatio-temporally conditioned senses, we know them from some point and instant within Space and Time. It follows that concrete places and times are apprehended only as relative to an observer, that their totalities can be embraced only through the device of reference frames, that reference frames will be many, and that transformations of reference frames can involve changes in the relativity of places and times to observers. Accordingly, it would be a mistake to look for the fixed or absolute on the level of particular places and times; the only absolute relevant to Space and Time resides in the abstract propositions whose expression remains invariant under permissible transformations of reference frame.

On the other hand, opposed positions take their stand on the premise that something fixed or absolute is to be acknowledged on the level of sense. In the Aristotelian world view, this was supplied by the outermost celestial sphere which bounded effective Space and, for Aquinas at least, provided the universe with a standard time. Newton's absolute space and absolute time were in the first instance imaginary mathematical constructions; but they were objectified through a confusion of the truth of verification and the truth, prior to intelligence and thought, that resides in a 'really out there'; finally, they were given a metaphysical status by being connected with the omnipresence and the eternity of God. Kant simplified this position by making Newton's empty space and time into *a priori* forms of sensibility.

## 4. RODS AND CLOCKS

On Galilean and Newtonian suppositions, measurements of distance and of duration are invariant, so that if a measurement is correct in any frame of reference, the same measurement must be correct in all frames of reference that are permissible.

On the Special Theory of Relativity the invariant is the four-dimensional interval, $ds$, where

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2.$$

Hence, if the value of $ds$ is correct in any reference frame, the same value must be correct in all permissible frames. On the other hand, the values of the spatial components, $dx$, $dy$, $dz$, and the value of the temporal component, $dt$, can be correct in one reference frame without therefore being correct in other permissible frames. As is clear from the above equation, the spatial and temporal components can assume any values compatible with the constancy of the interval, $ds$.



Clearly enough, this theory necessitates some revision of earlier notions on measurable magnitudes, standard units, measuring, and measurement. For on the earlier view a measurement of a distance or duration is some single number valid in all reference frames. On the new view a measurement of a distance or a duration seems to be a series of numbers in correspondence with a series of reference frames.

Such a revision is not easy. Ordinarily people form their notions of measurements at a time when they take Newtonian presuppositions for granted. Later, when they are confronted with relativity, they are apt to be content to make obvious alterations without thinking things through to a fully coherent position. There results a piecemeal and inadequate revision of basic concepts and thus manifests itself in a parade of alleged Einsteinian paradoxes.

Our proposal is to attempt a thorough revision. First, we shall examine the elementary paradox that the measuring rods of one reference frame are both shorter and longer than those of another, and that the clocks of one frame run both slower and faster than those of another.* Secondly, we shall work out a generic notion of measurement that is independent of differences between Galileo and Einstein. Thirdly, we shall show how the same generic notion admits differentiation into the two different specific views.

### 4.1 *The Elementary Paradox*

Consider the pair of point-instants, $P$ and $Q$, which in a frame of reference, $K$, have the co-ordinates, $(x_1, t_1)$ and $(x_2, t_2)$, and in a frame, $K'$, moving with a relative constant velocity, $u$, have the co-ordinates $(x'_1, t'_1)$ and $(x'_2, t'_2)$. Then by the Lorentz-Einstein transformation, writing

$$H = 1/(1 - u^2/c^2)^{\frac{1}{2}}$$

one easily obtains the equations

$$x'_2 - x'_1 = (x_2 - x_1)H - (t_2 - t_1)uH \qquad (1)$$

$$t'_2 - t'_1 = (t_2 - t_1)H - (x_2 - x_1)uH/c^2. \qquad (2)$$

* For an exposition, see Lindsay and Margenau, pp. 339 ff.

It is to be noted that if either of the equations (1) and (2) can be obtained then both can be obtained. Moreover, by transforming in the opposite direction from $K'$ to $K$, there are to be obtained two other equations similar to (1) and (2).

Now these equations admit both a spatial and a temporal application, and to each application three interpretations can be given. The spatial application is to suppose that $P$ and $Q$ are the simultaneous end positions of a standard rod of unit length in $K$ so that

$$x_2 - x_1 = 1 \tag{3}$$
$$t_2 - t_1 = 0 \tag{4}$$

whence by equations (1) and (2)

$$x'_2 - x'_1 = H \tag{5}$$
$$t'_2 - t'_1 = -uH/c^2. \tag{6}$$

The temporal application is to suppose that $P$ and $Q$ are readings at successive seconds on a stationary standard clock in $K$ so that

$$x_2 - x_1 = 0 \tag{7}$$
$$t_2 - t_1 = 1 \tag{8}$$

whence by equations (1) and (2)

$$x'_2 - x'_1 = -uH \tag{9}$$
$$t'_2 - t'_1 = H. \tag{10}$$

Accordingly, inasmuch as standard units of distance and of time are expected to transform invariantly, a problem of interpretation arises and three answers may be given.

A first interpretation seems inspired by the Fitzgerald contraction. Since $H$ is greater than unity, it is concluded from equations (3) and (5) that the standard rod in $K'$ is shorter than the standard rod in $K$. Similarly, it is concluded from equations (8) and (10) that the unit of time in $K'$ is shorter than the unit of time in $K$. Moreover, the opposite conclusions are reached from the equations obtained by transforming from $K'$ to $K$. But quite apart from its paradox, this interpretation has the defect of saying very little about equations (4) and (6), (7) and (9).

A second interpretation begins by noting that in Special Relativity clocks are synchronized in each frame of reference by assuming, not that simultaneity is identical, but that the velocity of light is the same constant in all frames of reference. Accordingly, on this interpretation equations (5) and (6) are taken together, and at once it is apparent that a distance between simultaneous positions in $K$ has been transformed into a distance between positions that are not simultaneous in $K'$. But even Cinderella's foot would seem large if one measured the distance between the tip of her toe at one instant and the back of her heel at another; and such is the view in $K'$ of the standard unit of length in $K$. Similarly, equations (9) and (10) are taken together to reveal that, what for $K$ is a time interval on the same stationary clock, for $K'$ is a difference in time between clocks in different positions. It follows that the difference in time given by equation (10) results not only from the difference in time given by equation (8) but also from the fact, underlying the transformation equations, that in every frame of reference clocks in different positions are synchronized by assuming the velocity of light to be the same constant in all frames. Indeed, while one may find this method of synchronization to be strange, while one may even find it strange that there is any problem of synchronization, still, granted that initial oddity, there is no further oddity brought to light by equations (3) to (10) or by the similar equations obtained when one transforms from $K'$ to $K$.



A third interpretation is in terms of Minkowski space. It asserts that, within the context of Special Relativity, it is a blunder to suppose that a difference of position is a merely spatial entity or that a difference of time is a merely temporal entity. Hence, a standard rod is spatio-temporal: it is not merely a distance between two positions; it is a distance between a position, $x_1$, at a time, $t_1$, and a position, $x_2$, at a time, $t_2$. Similarly, a standard clock is spatio-temporal: it does not assign merely temporal differences; it assigns a difference between a time, $t_1$, at a position, $x_1$, and a time, $t_2$, at a position, $x_2$. Moreover, a unit on any standard rod determines one and the same invariant spatio-temporal interval for all frames of reference, namely, unity; and a unit on any standard clock determines one and the same invariant spatio-temporal interval for all frames of reference, namely, $ic$.* However, while standard rods and clocks determine the same spatio-temporal intervals for all frames of reference, still these invariant intervals divide differently into spatial and temporal components in different frames of

* This invariant interval, $s$, may be obtained from the equations

$$s^2 = (x_2 - x_1)^2 - c^2(t_2 - t_1)^2 = (x'_2 - x'_1)^2 - c^2(t'_2 - t'_1)^2.$$

Thus, one will find that substitutions from equations (3) and (4) will yield the same result, unity, as substitutions from equations (5) and (6); similarly, substitutions from equations (7) and (8) will yield the same result, $ic$, as substitutions from equations (9) and (10).

reference. Hence one may distinguish between normal and abnormal frames by introducing the definitions:

> A reference frame is normal to measurements if differences of position have a temporal component that is zero and differences of time have a spatial component that is zero.
>
> A reference frame is abnormal to measurements if differences of position have a temporal component that is not zero and differences of time have a spatial component that is not zero.

Operationally this means that reference frames, rods, clocks, and measurable objects should be relatively at rest if one's measuring is not to be complicated by the ambiguities of the elementary paradox.

Finally, it may be noted that, while the first interpretation differs from the other two, the second and third are compatible and complementary. For the second explains the differences that arise on transforming units of distance and time by remarking that, when the relative velocity is not zero, the transformation equations cover over a peculiar technique in synchronization, while the third interpretation systematizes the whole matter by adverting to spatio-temporal invariants and by noting that these invariants divide differently into spatial and temporal components in different reference frames. It remains, however, that something be said on the general notion of measurement presupposed by the second and third interpretations.

### 4.2 *The Generic Notion of Measurement*

Empirical inquiry has been conceived as a process from description to explanation. We begin from things as related to our senses. We end with things as related to one another. Initial classifications are based upon sensible similarities. But as correlations, laws, theories, systems are developed, initial classifications undergo a revision. Sensible similarity has ceased to be significant, and definitions consist of technical terms that have been invented as a consequence of scientific advance. In this fashion biological classifications have felt the imprint of the theory of evolution. Chemical compounds are defined by appealing to chemical elements. Chemical elements are defined by their relations to one another in a periodic table that has room for elements that, as yet, have not been discovered or synthesized. The basic notions of physics are a mass that is distinct from weight, a temperature that differs from the intensity of the feeling of heat, and the electromagnetic vector fields.



Now the principal technique in effecting the transition from description to explanation is measurement. We move away from colours as seen, from sounds as heard, from heat and pressure as felt. In their place, we determine the numbers named measurements. In virtue of this substitution, we are able to turn from the relations of sensible terms, which are correlative to our senses, to the relations of numbers, which are correlative to one another. Such is the fundamental significance and function of measurement.

Further, in constructing these numerical relations of things to one another, there is introduced an almost necessary simplification of arrangement. If it would be theoretically possible, it would not be practicable to relate things to one another by stating separately the relations of each to all the others. The procedure that is both simpler and more systematic is to select one type of thing or magnitude, to relate all others directly to it, and to leave to deductive inference the relations of the others among themselves. Thus, instead of noting that Tom is $\frac{1}{10}$ taller than Dick, Dick $\frac{1}{20}$ shorter than Harry, and Harry $\frac{8}{100}$ shorter than Tom, one selects some arbitrary magnitude as standard unit and measures Tom, Dick and Harry, not in terms of one another, but in terms of feet or centimetres.

A standard unit, then, is a physical magnitude among other similar physical magnitudes. Its position of privilege is due to the systematic simplicity of implying the relations of each of these magnitudes to all the others by stating only the relations of all to some one.

In selecting and determining standard units, there is a conventional, arbitrary element and, as well, there is a far larger theoretical element. It is a matter of convention that the standard foot is the length between notches on a bar at a certain temperature in a given place. It is arbitrary that the foot happens to have the length it has, neither more nor less. On the other hand, the remaining aspects of the standard unit have their basis in presumed or acquired theoretical knowledge. What is length? Does length vary with temperature? Does length vary with change of place or of time? Does length vary with changes of frames of reference? These are relevant questions. If their answers rest on the results of empirical science, they are subject to revision when those results are revised. If their answers can be obtained only by appealing to the field of basic presuppositions and presumptions, they will be methodological and subject to the revisions of methodology.

The fundamental point to be grasped here is a point that already has been made. The absolute resides not on the level of sensible presentations but in the field of abstract propositions and invariant expressions. The constancy in time of the length of a standard metal bar cannot be ascertained by comparing its length yesterday with its length today; the field of observables is limited to the present place and time; today's length of the bar can be observed, if today you are in the right place; but yesterday's length has passed out of the field of observables and tomorrow's has not yet been ushered in. It remains that the constancy in time of the length of the bar is a conclusion based on general knowledge. One ascertains, as best one can, all the manners in which metal bars can change in length; one takes precautions to prevent the occurrence of any such changes in the standard; and one concludes that, as far as one knows, no such change has taken place. In other words, the constancy of the standard is a conclusion based upon the invariance of laws, and a revision of the laws will lead to a new determination of standard requirements.

This possible revision of standards sets a logical puzzle. How, one may ask, can one reach new laws except through measurements based on old standards? How can the new laws be correct if the old standards are wrong? How can incorrect laws lead to the correction of old standards? Behind such questions there lies a mistaken presupposition. Science does not advance by deducing new conclusions from old premises. Deduction is an operation that occurs only in the field of concepts and propositions. But the advance of science, as we have seen, is a circuit, from data to inquiry, from inquiry to insight, from insight to the formulation of premises and the deduction of their implications, from such formulation to material operations, which yield fresh data and, in the limit, generate the new set of insights named a higher viewpoint. A basic revision, then, is a leap. At a stroke, it is a grasp of the insufficiency both of the old laws and of the old standards. At a stroke, it generates both the new laws and the new standards. Finally, by the same verification, it establishes that both the new laws and the new standards satisfy the data.

What holds for standards, also holds for their use. It is necessary to define as accurately as possible the precise type of magnitude that is to be measured. It is necessary to define the precise procedure that leads from the measurable magnitude and the standard unit to the determination of the number named a measurement. At each stage in the development of a science, these definitions will be formed in the light of acquired or presumed knowledge. But at every subsequent stage, there is the possibility of further acquisitions and of new presumptions and so of a revision of the definitions. Such a revision involves, not the deduction of new conclusions from old premises, but a leap to fresh premises.



Such then, is the generic notion of measurement. Clearly, it contains within itself the possibility of successive differentiations that result from revisions that occur in the abstract field of definitions, principles, and laws. We have now to turn our attention to the revision involved in the notions of spatial and temporal measurements by the Special Theory of Relativity.

### 4.3 *Differentiations of the Generic Notion of Measurement*

Let us begin by distinguishing

(1) size,
(2) length, and
(3) measurement.

By size will be meant magnitude apart from any geometrical conceptions. It is an elementary, experiential conjugate, and it is to be characterized in terms of simple experiences.

Thus, spatial size may be indicated sufficiently by saying that it varies in two manners. It varies in an external fashion, inasmuch as the nearer it is, the bigger it looks. Also it varies in an internal fashion, inasmuch as it expands or contracts.

Temporal size similarly varies in two manners. There is the external variation, named psychological time, which rushes by when we are interested and lags when we are bored. There are also internal differences between the sizes of durations; twenty years is a long time, even if one is not in prison; and a second is a short time, even if one is.

By length will be meant size as fitted into a geometrical construction.

Spatial length, at a first approximation, seems simply to be size in a single direction or dimension. Still, one does have to use some such expression as direction or dimension. This fact recalls, not only the analysis of size into length, breadth, and depth, but also the requirement that length has to be taken along a straight line or geodetic. Further, the ends of a straight line or geodetic are points, but the ends of a size are hardly just points; it follows that the size of the material object must have been submitted to some detailed geometrical analysis,

so that boundaries of the size stand in some unique correspondence with points on a straight line. Finally, material objects may be varying internally in size, and they may be moving locally; an expanding or contracting object has a series of lengths at a series of instants; a moving object successively lies between two series of bounding positions; its length is not the distance between present and past bounding positions; and so it follows that the length of an object depends, not only on a geometry of space, but also upon determinations of the instant and of simultaneity.

The length of a duration can be determined only by adding mechanical to geometrical analysis. There has to be discovered some constant velocity or some regular periodicity. The spatial size traversed by the velocity has to be conceived in terms of length and divided into equal parts. Finally, while the length of a single duration may be determined by counting traversed parts or recurring periods, still there are many durations; they have to be related to one another in some fashion; and so there must be worked out some general determination of simultaneity or synchronization.

It has been noted that sizes differ in two manners; internally, in virtue of expansions and contractions, prolongations and curtailments; externally, in virtue of the relative position of our senses and the quality of our subjective states. The obvious advantage of the notion of length is that it eliminates merely external differences of size. Still, one must not jump to the conclusion that, therefore, length will prove invariant. As has been seen, determinations of length depend upon determinations of simultaneity, and it may be that simultaneity is not invariant. Again, determinations of length depend upon the supposition of some specific geometry, and it may happen that the specific geometry, verified in Space and Time, does not regard length as invariant.

There remains measurement. On Newtonian suppositions, a measurement is a number that stands to unity as the length of the measured magnitude stands to the length of a standard unit. Thus, to say that a room is twenty feet long is to say that the length of the room stands to the length of a foot-rule as the number, twenty, stands to unity. Again, to say that a process lasts five seconds is to say that the length of the process stands to the length of a standard second as the number, five, stands to unity. Finally, lengths are invariant under permissible transformations, and so measurements valid in one reference frame are valid in all permissible frames.



Now the transition to the suppositions of Special Relativity may be effected very simply by noting an oversight in the foregoing account of measurement. Two rods, $AP$ and $BQ$, are equal in length if and only if $A$ coincides with $B$ at the same time as $P$ coincides with $Q$. In particular, if $A$ coincides with $B$ at one moment and $P$ coincides with $Q$ at another moment, relative motion could occur during the interval and so equality could not be asserted. Similarly, two clocks, $R$ and $S$, are synchronous if and only if readings taken at the same time agree. In particular, synchronization cannot be asserted on the ground that the readings from $R$ at one series of moments agree with the readings from S at another series of moments.

Moreover, not only is an exact determination of the meaning of simultaneity an essential condition in measuring spatial and temporal differences but also, as has been seen, it cannot be presumed that that meaning is identical for all spatio-temporal standpoints. Indeed, since simultaneity is a relation between particular events occurring at particular times in particular places, it may be expected to be analogous to such notions as 'now' and 'here'.

Further, to escape the relativity of simultaneity, appeal must be made to some absolute. But the absolute in measurement, as the absolute in space and time, resides in the realm of principles and laws. For principles and laws, because they abstract from particular places and particular times, cannot vary with variations in place and time.

Hence, the basic supposition of measurement in Special Relativity will coincide with its basic postulate that the mathematical expression of physical principles and laws is invariant under inertial transformations. It follows that the appropriate geometry into which sizes must be fitted to yield lengths will be Minkowski space. Further, it follows that the correct notion of simultaneity will be the notion implicit

(1) theoretically in the Lorentz-Einstein transformation, and
(2) operationally in the fact that in all reference frames clocks are synchronized by light signals and the velocity of light is always the same constant.

Hence, in Special Relativity the measurement of any spatial or temporal difference determines a spatio-temporal interval

(1) that is invariant for all reference frames, but
(2) that resolves into different spatial and temporal components in different relatively moving frames.

Further, a distinction may be drawn between normal and abnormal reference frames. For if a measured magnitude is purely spatial, in a normal frame it will have a temporal component that is zero, but in an abnormal frame it will have a temporal component that is not zero. Similarly, if a measured magnitude is purely temporal, in a normal frame it will have a spatial component that is zero, but in an abnormal frame it will have a spatial component that is not zero. It follows that in actual measuring only normal frames should be used if one is to avoid the complexity of discovering the temporal component in a spatial difference and the spatial component in a temporal difference.

It may be remarked that on the present analysis there seems to vanish the apparently arbitrary division of the universe into rods and clocks on the one hand and, on the other, everything else.* For the fundamental point is the relativity of simultaneity, and that relativity enters into the very notion of a determinate measurement. Hence, while measurements are relations between rods and clocks on the one hand and, on the other, all other spatial and temporal magnitudes, still there is no peculiarity in rods that is lacking in other spatial magnitudes and there is no peculiarity in clocks that is lacking in other temporal magnitudes.

Finally, it is perhaps unnecessary to note that our account of measurement makes no attempt to treat either the notion of measurement implicit in General Relativity or the problems that arise when the activity of measuring introduces a coincidental or non-systematic element into the objects under investigation. No doubt, these issues could not be omitted in a general treatment of the subject, but our purpose has been to reinforce the point that absolutes do not lie in the field of sensible particulars and to disassociate our account of the abstract intelligibility of Space and Time from the paradoxes that too readily have been supposed to be inherent in the Special Theory of Relativity.

### 5. THE CONCRETE INTELLIGIBILITY OF SPACE AND TIME

Space and Time have been defined as ordered totalities of concrete extensions and of concrete durations.

They are distinct from imaginary space and imaginary time, which are totalities of merely imagined extensions and of merely imagined durations. Moreover, the existence of this distinction reveals that

* See the autobiography in *Albert Einstein, Philosopher-Scientist*, edited by P. A. Schilpp, The Library of Living Philosophers, New York. 1949 and 1951. P. 59.



notions of Space and Time begin from experienced extensions and experienced durations and employ reference frames to reach out and embrace the totality of other concrete extensions and concrete durations.

Since reference frames are an endless multiplicity, their intelligible order cannot be more than descriptive. If one would understand, not men's notions of Space and Time, but the intelligibility immanent in Space and Time, then one must advance from reference frames to the geometrical principles and laws whose expression is invariant under transformations. Moreover, the geometry to be reached will coincide with the geometry determined by physicists in securing invariant expression for physical principles and laws.

However, such a geometry is abstract. It is abstract, not indeed in the sense that it is not verified (for what is wanted is a geometry verified by physicists), but in the sense that it consists in a set of abstract propositions and invariant expressions and that, while applicable to concrete extensions and durations, still it is applied differently from different spatio-temporal viewpoints. Thus, as long as men remain on the level of invariant expressions, they are not considering any concrete extension and duration; inversely, as soon as men consider concrete extensions and durations, each views them differently. The endless multiplicity of different spatio-temporal standpoints and of different frames of reference, so far from being transcended, reappears with every return from the abstract to the concrete.

There is a parallel point to be made. The abstract intelligibility of Space and Time is coincident with the solution of a problem in physics. It is the intelligibility, not so much of Space and Time, as of physical objects in their spatio-temporal relations. May one not expect an intelligibility proper to Space and proper to Time?

Such, then, is the question envisaged by this section on the concrete intelligibility of Space and Time. What is wanted is an intelligibility grasped in the totality of concrete extensions and durations and, indeed, identical for all spatio-temporal viewpoints.

The answer is easily reached. One has only to shift from the classical type of inquiry, which has been under consideration, to the complementary statistical type. It has been argued that a theory of emergent probability exhibits generically the intelligibility immanent in world process. Emergent probability is the successive realization of the possibilities of concrete situations in accord with their probabilities. The

concrete intelligibility of Space is that it grounds the possibility of those simultaneous multiplicities named situations. The concrete intelligibility of Time is that it grounds the possibility of successive realizations in accord with probabilities. In other words, concrete extensions and concrete durations are the field or matter or potency in which emergent probability is the immanent form or intelligibility.

CHAPTER VI

# COMMON SENSE AND ITS SUBJECT

The illustrative basis of our study must now be broadened. In the previous five chapters, precision was our primary objective, and so our examples were taken from the fields of mathematics and physics. Still the occurrence of insight is not restricted to the minds of mathematicians, when doing mathematics, and to the minds of physicists, when engaged in that department of science. On the contrary, one meets intelligence in every walk of life. There are intelligent farmers and craftsmen, intelligent employers and workers, intelligent technicians and mechanics, intelligent doctors and lawyers, intelligent politicians and diplomats. There is intelligence in industry and commerce, in finance and taxation, in journalism and public relations. There is intelligence in the home and in friendship, in conversation and in sport, in the arts and in entertainment. In every case, the man or woman of intelligence is marked by a greater readiness in catching on, in getting the point, in seeing the issue, in grasping implications, in acquiring know-how. In their speech and action the same characteristics can be discerned, as were set forth in describing the act that released Archimedes' 'Eureka!' For insight is ever the same, and even its most modest achievements are rendered conspicuous by the contrasting, if reassuring, occurrence of examples of obtuseness and stupidity.

## 1. COMMON SENSE AS INTELLECTUAL

The light and drive of intelligent inquiry unfolds methodically in mathematics and empirical science. In the human child it is a secret wonder that, once the mystery of language has been unravelled, rushes forth in a cascade of questions. Far too soon, the questions get out of hand, and weary adults are driven to ever more frequent use of the blanket, 'My dear, you cannot understand that yet.' The child would understand everything at once. It does not suspect that there is a strategy in the accumulation of insights, that the answers to many questions depend on answers to still other questions, that, often enough, advertence to these other questions arises only from the insight that to meet interesting questions one has to begin from quite uninteresting

ones. There is, then, common to all men, the very spirit of inquiry that constitutes the scientific attitude. But in its native state it is untutored. Our intellectual careers begin to bud in the incessant 'What?' and 'Why?' of childhood. They flower only if we are willing, or constrained, to learn how to learn. They bring forth fruit only after the discovery that, if we really would master the answers, we somehow have to find them out ourselves.

Just as there is spontaneous inquiry, so too there is a spontaneous accumulation of related insights. For questions are not an aggregate of isolated monads. In so far as any question is followed by an insight, one has only to act, or to talk, or perhaps merely to think on the basis of that insight, for its incompleteness to come to light and thereby generate a further question. In so far as the further question is in turn met by the gratifying response of a further insight, once more the same process will reveal another aspect of incompleteness to give rise to still further questions and still further insights. Such is the spontaneous process of learning. It is an accumulation of insights in which each successive act complements the accuracy and covers over the deficiency of those that went before. Just as the mathematician advances from images through insights and formulations to symbols that stimulate further insights, just as the scientist advances from data through insights and formulations to experiments that stimulate further insights, so too the spontaneous and self-correcting process of learning is a circuit in which insights reveal their shortcomings by putting forth deeds or words or thoughts and, through that revelation, prompt the further questions that lead to complementary insights.

Such learning is not without teaching. For teaching is the communication of insight. It throws out the clues, the pointed hints, that lead to insight. It cajoles attention to drive away the distracting images that stand in insight's way. It puts the further questions that reveal the need of further insights to modify and complement the acquired store. It has grasped the strategy of developing intelligence, and so begins from the simple to advance to the more complex. Deliberately and explicitly, all this is done by professional teachers that know their job. But the point we would make is that it also is done, though unconsciously and implicitly, by parents with their offspring and by equals among themselves. Talking is a basic human art. By it each communicates to others what he knows and, at the same time, provokes the contradictions that direct his attention to what he has overlooked. Again, far more impressive



than talking is doing. Deeds excite our admiration and stir us to emulation. We watch to see how things are done. We experiment to see if we can do them ourselves. We watch again to discover the oversights that led to our failures. In this fashion, the discoveries and inventions of individuals pass into the possession of many, to be checked against their experience, to undergo the scrutiny of their further questions, to be modified by their improvements. By the same token, the spontaneous collaboration of individuals is also the communal development of intelligence in the family, the tribe, the nation, the race. Not only are men born with a native drive to inquire and understand; they are born into a community that possesses a common fund of tested answers, and from that fund each may draw his variable share, measured by his capacity, his interests, and his energy. Not only does the self-correcting process of learning unfold within the private consciousness of the individual; for by speech and, still more, by example, there is effected a sustained communication that at once disseminates and tests and improves every advance to make the achievement of each successive generation the starting-point of the next.

From a spontaneous inquiry, the spontaneous accumulation of related insights, and the spontaneous collaboration of communication, we have worked towards the notion of common sense as an intellectual development. Naturally enough, there will arise the question of the precise inventory of this public store. How does it define its terms? What are its postulates? What are the conclusions it infers from the premises? But if the question is obvious enough, the answer is more difficult. For the answer rests on one of those queer insights that merely grasps the false supposition of the question. Definitions, postulates, and inferences are the formulation of general knowledge. They regard, not the particular but the universal, not the concrete but the abstract. Common sense, unlike the sciences, is a specialization of intelligence in the particular and the concrete. It is common without being general, for it consists in a set of insights that remains incomplete, until there is added at least one further insight into the situation in hand; and, once that situation has passed, the added insight is no longer relevant, so that common sense at once reverts to its normal state of incompleteness. Thus, common sense may seem to argue from analogy, but its analogies defy logical formulation. The analogy that the logician can examine is merely an instance of the heuristic premise that similars are similarly understood. It can yield a valid argument, only if the two concrete

situations exhibit no significant dissimilarity. But common sense, because it does not have to be articulate, can operate directly from its accumulated insights. In correspondence with the similarities of the situation, it can appeal to an incomplete set of insights. In correspondence with the significant difference of situations, it can add the different insights relevant to each. Again, common sense may seem to generalize. But a generalization proposed by common sense has quite a different meaning from a generalization proposed by science. The scientific generalization aims to offer a premise from which correct deductions can be drawn. But the generalizations issued by common sense are not meant to be premises for deductions. Rather they would communicate pointers that ordinarily it is well to bear in mind. Proverbs are older far than principles and, like rules of grammar, they do not lose their validity because of their numerous exceptions. For they aim to express, not the scientist's rounded set of insights that either holds in every instance or in none at all, but the incomplete set of insights which is called upon in every concrete instance but becomes proximately relevant only after a good look around has resulted in the needed additional insights. Look before you leap!

Not only does common sense differ from logic and from science in the meaning it attaches to analogies and generalizations. In all its utterances it operates from a distinctive viewpoint and pursues an ideal of its own. The heuristic assumptions of science anticipate the determination of natures that always act in the same fashion under similar circumstances and, as well, the determination of ideal norms of probability from which events diverge only in a non-systematic manner. Though the scientist is aware that he will reach these determinations only through a series of approximations, still he also knows that even approximate determinations must have the logical properties of abstract truth. Terms, then, must be defined unambiguously and they must always be employed exactly in that unambiguous meaning. Postulates must be stated; their presuppositions must be examined; their implications must be explored. Automatically there results a technical language and a formal mode of speech. Not only is one compelled to say what one means and to mean what one says, but the correspondence that obtains between saying and meaning has the exact simplicity of such primitive utterances as, This is a cat.

Common sense, on the other hand, never aspires to universally valid knowledge and it never attempts exhaustive communication. Its concern is the concrete and particular. Its function is to master each situation as it arises. Its procedure is to reach an incomplete set of insights that is to be completed only by adding on each occasion the further insights that scrutiny of the occasion reveals. It would be an error for common sense to attempt to formulate its incomplete set of insights in definitions and postulates and to work out their presuppositions and implications. For the incomplete set is not the understanding either of any concrete situation or of any general truth. Equally, it would be an error for common sense to attempt a systematic formulation of its completed set of insights in some particular case; for every systematic formulation envisages the universal; and every concrete situation is particular.



It follows that common sense has no use for a technical language and no tendency towards a formal mode of speech. It agrees that one must say what one means and mean what one says. But its correspondence between saying and meaning is at once subtle and fluid. As the proverb has it, a wink is as good as a nod. For common sense not merely says what it means; it says it to some one; it begins by exploring the other fellow's intelligence; it advances by determining what further insights have to be communicated to him; it undertakes the communication, not as an exercise in formal logic, but as a work of art; and it has at its disposal not merely all the resources of language but also the support of modulated tone and changing volume, the eloquence of facial expression, the emphasis of gestures, the effectiveness of pauses, the suggestiveness of questions, the significance of omissions. It follows that the only interpreter of common-sense utterances is common sense. For the relation between saying and meaning is the relation between sensible presentations and intellectual grasp, and if that relation can be as simple and exact as in the statement, This is a cat, it can also take on all the delicacy and subtlety, all the rapidity and effectiveness, with which one incarnate intelligence can communicate its grasp to another by grasping what the other has yet to grasp and what act or sound or sign would make him grasp it. Such a procedure, clearly, is logical, if by 'logical' you mean 'intelligent and reasonable'. With equal clearness, such a procedure is not logical, if by 'logical' you mean conformity to a set of general rules valid in every instance of a defined range; for no set of general rules can keep pace with the resourcefulness of intelligence in its adaptations to the possibilities and exigencies of concrete tasks of self-communication.

Just as the elliptical utterances of common sense have a deeper ground

than many logicians and practically all controversialists have managed to reach, so too the plane of reality envisaged by common-sense meaning is quite distinct from the plane that the sciences explore. It has been said that the advance of science is from description to explanation, from things as related to our senses through measurements to things as related to one another. It is clear that common sense is not concerned with the relations of things to one another, and that it does not employ the technical terms that scientists invent to express those relations. Still, this obvious difference provides no premise for the inference that the object of scientific description is the same as the object of common-sense communication. It is true enough that both types of utterance deal with the things as related to our senses. But also it is true that they do so from different viewpoints and with different ends. Scientific description is the work of a trained scientific observer. It satisfies the logician's demand for complete articulateness and exhaustive statement. It reveals the imprint of the scientist's anticipation of attainment of the pure conjugates that express the relations of things to one another. For, though scientific description deals with things as related to our senses, it does so with an ulterior purpose and under the guidance of a method that strains towards its realization.

Common sense, on the other hand, has no theoretical inclinations. It remains completely in the familiar world of things for us. The further questions, by which it accumulates insights, are bounded by the interests and concerns of human living, by the successful performance of daily tasks, by the discovery of immediate solutions that will work. Indeed, the supreme canon of common sense is the restriction of further questions to the realm of the concrete and particular, the immediate and practical. To advance in common sense is to restrain the omnivorous drive of inquiring intelligence and to brush aside as irrelevant, if not silly, any question whose answer would not make an immediately palpable difference. Just as the scientist rises in stern protest against the introduction into his field of metaphysical questions that do not satisfy his canon of selection, so the man of common sense (and nothing else) is ever on his guard against all theory, ever blandly asking the proponent of ideas what difference they would make and, if the answer is less vivid and less rapid than an advertisement, then solely concerned with thinking up an excuse for getting rid of the fellow. After all, men of common sense are busy. They have the world's work to do.



Still, how can the world's work be done either intelligently or efficiently, if it is done by men of common sense that never bother their heads a minute about scientific method? That question can be answered, I think, if we begin from another. Why is it that scientists need scientific method? Why must such intelligent men be encumbered with the paraphernalia of laboratories and the dull books of specialized libraries? Why should they be trained in observation and logic? Why should they be tied down by abstruse technical terms and abstract reasoning? Clearly it is because their inquiry moves off from the familiar to the unfamiliar, from the obvious to the recondite. They have to attend to things as related to us in the manner that leads to things as related to one another. When they reach the universal relations of things to one another, they are straining beyond the native range of insight into sensible presentations and they need the crutches of method to fix their gaze on things as neither sensibly given nor concrete nor particular.

Common sense, on the other hand, has no such aspirations. It clings to the immediate and practical, the concrete and particular. It remains within the familiar world of things for us. Rockets and space platforms are superfluous, if you intend to remain on this earth. So also is scientific method superfluous in the performance of the tasks of common sense. Like the sciences, it is an accumulation of related insights into the data of experience. Like the sciences, it is the fruit of a vast collaboration. Like the sciences, it has been tested by its practical results. Still there is a profound difference. For the sciences have theoretical aspirations, and common sense has none. The sciences would speak precisely and with universal validity, but common sense would speak only to persons and only about the concrete and particular. The sciences need methods to reach their abstract and universal objects; but scientists need common sense to apply methods properly in executing the concrete tasks of particular investigations, just as logicians need common sense if they are to grasp what is meant in each concrete act of human utterance. It has been argued that there exists a complementarity between classical and statistical investigations; perhaps it now is evident that the whole of science, with logic thrown in, is a development of intelligence that is complementary to the development named common sense. Rational choice is not between science and common sense: it is a choice of both, of science to master the universal, and of common sense to deal with the particular.

There remains to be mentioned the differentiation of common sense. Far more than the sciences, common sense is divided into specialized departments. For every difference of geography, for every difference of occupation, for every difference of social arrangements, there is an appropriate variation of common sense. At a given place, in a given job, among a given group of people, a man can be at intelligent ease in every situation in which he is called upon to speak or act. He always knows just what is up, just the right thing to say, just what needs to be done, just how to go about it. His experience has taken him through the cycle of eventualities that occur in his milieu. His intelligence has ever been alert. He has made his mistakes and from them he has learnt not to make them twice. He has developed the acumen that notices movements away from the familiar routine, the poise that sizes them up before embarking on a course of action, the resourcefulness that hits upon the response that meets the new issue. He is an embodiment of the ideal of common sense, yet his achievement is relevant only to its environment. Put him among others in another place or at another job and, until they become familiar, until he has accumulated a fresh set of insights, he cannot avoid hesitancy and awkwardness. Once more he must learn his way about, catch on to the tricks of a new trade, discern in little signs the changing moods of those with whom he deals. Such, then, is the specialization of common sense. At once, it adapts individuals in every walk of life to the work they have chosen or the lot that has befallen them and, no less, it generates all those minute differences of viewpoint and mentality that separate men and women, old and young, town and country until, in the limit, one reaches the cumulative differences and mutual incomprehension of different strata of society, different nations, different civilizations, and different epochs of human history.

We have been endeavouring to conceive the intellectual component in common sense. Our effort began from spontaneous questions, spontaneous accumulations of insights, spontaneous collaboration in testing and improving them. Next, there was formulated the central notion of an habitual but incomplete set of insights that was completed with appropriate variations in each concrete set of circumstances that called for speech or action. It was shown that such an intellectual development not only aimed at mastering the concrete and particular but also achieved its aim in a concrete and particular manner that contrasted with the general rules of logic and the general methods of science yet



provided a necessary complement both for the concrete use of general techniques and the concrete application of general conclusions. Finally, attention was drawn to the differentiations of common sense which multiply, not by theoretical differences as do the departments of sciences, but by the empirical differences of place and time, circumstance and environment.

## 2. THE SUBJECTIVE FIELD OF COMMON SENSE

If there is a parallel between a scientific and a common-sense accumulation of insights, there also exists a difference. Where the scientist seeks the relations of things to one another, common sense is concerned with the relations of things to us. Where the scientist's correlations serve to define the things that he relates to one another, common sense not merely relates objects to a subject but also constitutes relations of the subject to objects. Where the scientist is primarily engaged in knowing, common sense cannot develop without changing the subjective term in the object-to-subject relations that it knows.

There is, then, a subtle ambiguity in the apparently evident statement that common sense relates things to us. For who are we? Do we not change? Is not the acquisition of common sense itself a change in us? Clearly, an account of common sense cannot be adequate without an investigation of its subjective field. To this end we propose in the present section to introduce the notion of patterns of experience, to distinguish biological, aesthetic, intellectual, and dramatic patterns, to contrast the patterns of consciousness with the unconscious patterns of neural process and, finally, to indicate the connection between a flight from insight and, on the other hand, repression, inhibition, slips of the tongue, dreams, screening memories, abnormality, and psychotherapy.

### 2.1 *Patterns of Experience*

The notion of the pattern of experience may best be approached by remarking how abstract it is to speak of a sensation. No doubt, we are all familiar with acts of seeing, hearing, touching, tasting, smelling. Still, such acts never occur in isolation both from one another and from all other events. On the contrary, they have a bodily basis; they are functionally related to bodily movements; and they occur in some dynamic context that somehow unifies a manifold of sensed contents and of acts of sensing.

Thus, without eyes, there is no seeing; and when I would see with

my eyes, I open them, turn my head, approach, focus my gaze. Without ears, there is no hearing; and to escape noise, I must move beyond its range or else build myself sound-proof walls. Without a palate, there is no tasting; and when I would taste, there are involved movements of the body and arms, of hands and fingers, of lips and tongue and jaws. Sensation has a bodily basis and functionally it is linked to bodily movements.

Nor is this all. Both the sensations and the bodily movements are subject to an organizing control. Besides the systematic links between senses and sense organs, there is, immanent in experience, a factor variously named conation, interest, attention, purpose. We speak of consciousness as a stream, but the stream involves not only the temporal succession of different contents but also direction, striving, effort. Moreover, this direction of the stream is variable. Thales was so intent upon the stars that he did not see the well into which he tumbled. The milkmaid was so indifferent to the stars that she could not overlook the well. Still, Thales could have seen the well, for he was not blind and, perhaps, the milkmaid could have been interested in the stars, for she was human.

There are, then, different dynamic patterns of experience, nor is it difficult for us to say just what we mean by such a pattern. As conceived, it is the formulation of an insight; but all insight arises from sensitive or imaginative presentations; and in the present case the relevant presentations are simply the various elements in the experience that is organized by the pattern.

### 2.2 *The Biological Pattern of Experience*

A plant draws its sustenance from its environment by remaining in a single place and by performing a slowly varying set of routines in interaction with a slowly varying set of things. In contrast, the effective environment of a carnivorous animal is a floating population of other animals that move over a range of places and are more or less well equipped to deceive or elude their pursuers. Both plant and animal are alive, for in both aggregates of events insight discerns an intelligible unity that commonly is formulated in terms of biological drive or purpose. But plants adapt slowly, animals rapidly, to changing situations; and if we endeavour to understand the sudden twists and turns both of fleeing quarry and pursuing beast of prey, we ascribe to them a flow of experience not unlike our own. Outer senses are the heralds of biological opportunities and dangers. Memory is the file of supplementary



information. Imagination is the projection of courses of action. Conation and emotion are the pent-up pressure of elemental purposiveness. Finally, the complex sequence of delicately co-ordinated bodily movements is at once the consequence of striving and a cause of the continuous shift of sensible presentations.

In such an illustration insight grasps the biological pattern of experience. By such a pattern is not meant the visible or imaginative focus of attention offered by the characteristic shape and appearance of an animal. Nor, again, is the pattern reached by grasping that spatially and temporally distinct data all belong to a single living thing, for plants no less than animals are alive and, as yet, we have not satisfied ourselves upon the validity of the notion of the thing. Rather, the pattern is a set of intelligible relations that link together sequences of sensations, memories, images, conations, emotions, and bodily movements; and to name the pattern biological is simply to affirm that the sequences converge upon terminal activities of intussusception or reproduction or, when negative in scope, self-preservation. Accordingly, the notion of the pattern takes us beyond behaviourism, inasmuch as attention is not confined to external data; it takes us beyond a narrow positivism, inasmuch as the canon of relevance leads us to acknowledge that there is a content to insight; but it observes the canon of parsimony by adding no more than a set of intelligible relations to elements of experience.

A more informative characterization of the biological pattern of experience is to be obtained by comparing animals and plants. For conscious living is only a part of the animal's total living. As in the plant, so in the animal there go forward immanent vital processes without the benefit of any conscious control. The formation and nutrition of organic structures and of their skeletal supports, the distribution and neural control of muscles, the physics of the vascular system, the chemistry of digestion, the metabolism of the cell, all are sequences of events that fit into intelligible patterns of biological significance. Yet it is only when their functioning is disturbed that they enter into consciousness. Indeed, not only is a large part of animal living non-conscious, but the conscious part itself is intermittent. Animals sleep. It is as though the full-time business of living called forth consciousness as a part-time employee, occasionally to meet problems of malfunctioning, but regularly to deal rapidly, effectively, and economically with the external situations in which sustenance is to be won and into which offspring are to be born.

Thus extroversion is a basic characteristic of the biological pattern of

experience. The bodily basis of the senses in sense organs, the functional correlation of sensations with the positions and movements of the organs, the imaginative, conative, emotive consequences of sensible presentations, and the resulting local movements of the body, all indicate that elementary experience is concerned, not with the immanent aspects of living, but with its external conditions and opportunities. Within the full pattern of living, there is a partial, intermittent, extroverted pattern of conscious living.

It is this extroversion of function that underpins the confrontational element of consciousness itself. Conation, emotion, and bodily movement are a response to stimulus; but the stimulus is ever against the response; it is a presentation through sense and memory and imagination of what is responded to, of what is to be dealt with. The stimulating elements are the elementary object; the responding elements are the elementary subject. When the object fails to stimulate, the subject is indifferent; and when non-conscious vital process has no need of outer objects, the subject dozes and falls asleep.

### 2.3 *The Aesthetic Pattern of Experience*

There exists in man an exuberance above and beyond the biological account-books of purposeful pleasure and pain. Conscious living is itself a joy that reveals its spontaneous authenticity in the untiring play of children, in the strenuous games of youth, in the exhilaration of sun-lit morning air, in the sweep of a broad perspective, in the swing of a melody. Such delight is not, perhaps, exclusively human, for kittens play and snakes are charmed. But neither is it merely biological. One can well suspect that health and exercise are not the dominant motive in the world of sport; and it seems a little narrow to claim that good meals and fair women are the only instances of the aesthetic. Rather, one is led to acknowledge that experience can occur for the sake of experiencing, that it can slip beyond the confines of serious-minded biological purpose, and that this very liberation is a spontaneous, self-justifying joy.

Moreover, just as the mathematician grasps intelligible forms in schematic images, just as the scientist seeks intelligible systems that cover the data of his field, so too the artist exercises his intelligence in discovering ever novel forms that unify and relate the contents and acts of aesthetic experience.* Still, sense does not escape one master merely to

* Insight in musical composition is described by S. K. Langer, *Feeling and Form* (New York 1953), pp. 121 ff.



fall into the clutches of another. Art is a twofold freedom. As it liberates experience from the drag of biological purposiveness, so it liberates intelligence from the wearying constraints of mathematical proofs, scientific verifications, and common-sense factualness. For the validation of the artistic idea is the artistic deed. The artist establishes his insights, not by proof or verification, but by skilfully embodying them in colours and shapes, in sounds and movements, in the unfolding situations and actions of fiction. To the spontaneous joy of conscious living, there is added the spontaneous joy of free intellectual creation.

The aesthetic and artistic are symbolic. Free experience and free creation are prone to justify themselves by an ulterior purpose or significance. Art, then, becomes symbolic, but what is symbolized is obscure. It is an expression of the human subject outside the limits of adequate intellectual formulation or appraisal. It seeks to mean, to convey, to impart something that is to be reached, not through science or philosophy, but through a participation and, in some fashion, a re-enactment of the artist's inspiration and intention. Pre-scientific and pre-philosophic, it may strain for truth and value without defining them. Post-biological, it may reflect the psychological depths yet, by that very fact, it will go beyond them.

Indeed, the very obscurity of art is in a sense its most generic meaning. Prior to the neatly formulated questions of systematizing intelligence, there is the deep-set wonder in which all questions have their source and ground. As an expression of the subject, art would show forth that wonder in its elemental sweep. Again, as a twofold liberation of sense and of intelligence, art would exhibit the reality of the primary object for that wonder. For the animals, safely sheathed in biological routines, are not questions to themselves. But man's artistry testifies to his freedom. As he can do, so he can be what he pleases. What is he to be? Why? Art may offer attractive or repellent answers to those questions but, in its subtler forms, it is content to communicate any of the moods in which such questions arise, to convey any of the tones in which they may be answered or ignored.

### 2.4 *The Intellectual Pattern of Experience*

The aesthetic liberation and the free artistic control of the flow of sensations and images, of emotions and bodily movements, not merely break the bonds of biological drive but also generate in experience a

existence of the variable. Men will claim that they work because they must live; but it is plain that they work so hard because they must make their living dignified. To lack that dignity is to suffer embarrassment, shame, degradation; it is to invite amusement, laughter, ridicule. Inversely, to grant free rein to man's impulse for artistically manifested dignity is to set so-called hard-headed industrialists and financiers to the task of stimulating artistic imagination with advertisements and of meeting its demands with the raw materials of the earth and with the technology of an age of science.

Such artistry is dramatic. It is in the presence of others, and the others too are also actors in the primordial drama that the theatre only imitates. If aesthetic values, realized in one's own living, yield one the satisfaction of good performance, still it is well to have the objectivity of that satisfaction confirmed by the admiration of others; it is better to be united with others by winning their approval; it is best to be bound to them by deserving and obtaining their respect and even their affection. For man is a social animal. He is born in one family only to found another of his own. His artistry and his knowledge accumulate over the centuries because he imitates and learns from others. The execution of his practical schemes requires the collaboration of others. Still, the network of man's social relationships has not the fixity of organization of the hive or the ant-hill; nor, again, is it primarily the product of pure intelligence devising blue-prints for human behaviour. Its ground is aesthetic liberation and artistic creativity, where the artistry is limited by biological exigence, inspired by example and emulation, confirmed by admiration and approval, sustained by respect and affection.

The characters in this drama of living are moulded by the drama itself. As other insights emerge and accumulate, so too do the insights that govern the imaginative projects of dramatic living. As other insights are corrected through the trial and error that give rise to further questions and yield still further complementary insights, so too does each individual discover and develop the possible roles he might play and, under the pressure of artistic and affective criteria, work out his own selection and adaptation. Out of the plasticity and exuberance of childhood through the discipline and the play of education there gradually is formed the character of the man. It is a process in which rational consciousness with its reflection and criticism, its deliberation and choice, exerts a decisive influence. Still there is no deliberation or choice about becoming stamped with some character; there is no deliberation



about the fact that our past behaviour determines our present habitual attitudes; nor is there any appreciable effect from our present good resolutions upon our future spontaneity. Before there can be reflection or criticism, evaluation or deliberation, our imaginations and intelligence must collaborate in representing the projected course of action that is to be submitted to reflection and criticism, to evaluation and decision. Already in the prior collaboration of imagination and intelligence, the dramatic pattern is operative, outlining how we might behave before others and charging the outline with an artistic transformation of a more elementary aggressivity and affectivity. Ordinary living is not ordinary drama. It is not learning a role and developing in oneself the feelings appropriate to its performance. It is not the prior task of assembling materials and through insight imposing upon them an artistic pattern. For in ordinary living there are not first the materials and then the pattern, nor first the role and then the feelings. On the contrary, the materials that emerge in consciousness are already patterned, and the pattern is already charged emotionally and conatively.

### 2.6 *Elements in the Dramatic Subject*

The first condition of drama is the possibility of acting it out, of the subordination of neural process to psychic determinations. Now in the animals this subordination can reach a high degree of complexity to ensure large differentiations of response to nuanced differences of stimuli. None the less, this complexity, so far from being an optional acquisition, seems rather to be a natural endowment and to leave the animal with a relatively small capacity for learning new ways and for mastering other than native skills. In contrast, man's bodily movements are, as it were, initially detached from the conative, sensitive, and emotive elements that direct and release them; and the initial plasticity and indeterminacy ground the later variety. Were the pianist's arms, hands, and fingers locked from birth in natural routines of biological stimulus and response, they never could learn to respond quickly and accurately to the sight of a musical score. To take another illustration, the production of sound is a complicated set of correlated oscillations and movements; but the wailing and gurgling of infants develop through the prattle of children into articulate speech, and this vocal activity can be complemented with the visual and manual activities of reading and writing; the whole structure rests upon conventional signs, yet the endlessly complex correlations that are involved between the psychic and the

neural have become automatic and spontaneous in a language that one knows.

Inverse to the control of the psychic over the neural, are the demands of neural patterns and processes for psychic representation and conscious integration. Just as an appropriate, schematic image specifies and leads to a corresponding insight, so patterns of change in the optic nerve and the cerebrum specify and lead to corresponding acts of seeing. What is true of sight, is also true of the other outer senses and, though the matter is far from fully explored, one may presume that memory and imagination, conation and emotion, pleasure and pain, all have their counterparts in corresponding neural processes and originate from their specific demands.

It would be a mistake, however, to suppose that such demands are unconditional. Perceiving is a function not only of position relative to an object, of the intensity of the light, of the healthiness of eyes, but also of interest, anticipation, and activity. Besides the demands of neural processes, there also is the pattern of experience in which their demands are met; and as the elements that enter consciousness are already within a pattern, there must be exercised some preconscious selection and arrangement. Already we have noticed, in treating the intellectual pattern of experience, how the detached spirit of inquiry cuts off the interference of emotion and conation, how it penetrates observation with the abstruse classifications of science, how it puts the unconscious to work to have it bring forth the suggestions, the clues, the perspectives that emerge at unexpected moments to release insight and call forth a delighted 'Eureka!' In similar fashion, the dramatic pattern of experience penetrates below the surface of consciousness to exercise its own domination and control and to effect, prior to conscious discrimination, its own selections and arrangements. Nor is this aspect of the dramatic pattern either surprising or novel; there cannot be selection and arrangement without rejection and exclusion; and the function that excludes elements from emerging in consciousness is now familiar as Freud's censor.

Since, then, the demands of neural patterns and processes are subject to control and selection, they are better named demand functions. They call for some psychic representation and some conscious integration, but their specific requirements can be met in a variety of different manners. In the biological pattern of experience, where both unconscious vital process and conscious striving pursue the same end, there is,



indeed, little room for diversification of psychic contents. But aesthetic liberation, artistic creativity, and the constant shifting of the dramatic setting open up vast potentialities. All the world's a stage and not only does each in his time play many parts but also the many parts vary with changes of locality, period, and social milieu. Still, there are limits to this versatility and flexibility. The demand functions of neural patterns and processes constitute the exigence of the organism for its conscious complement; and to violate that exigence is to invite the anguish of abnormality.

### 2.7 *Dramatic Bias*

Just as insight can be desired, so too it can be unwanted. Besides the love of light, there can be a love of darkness. If prepossessions and prejudices notoriously vitiate theoretical investigations, much more easily can elementary passions bias understanding in practical and personal matters. To exclude an insight is also to exclude the further questions that would arise from it and the complementary insights that would carry it towards a rounded and balanced viewpoint. To lack that fuller view results in behaviour that generates misunderstanding both in ourselves and in others. To suffer such incomprehension favours a withdrawal from the outer drama of human living into the inner drama of phantasy. This introversion, which overcomes the extroversion native to the biological pattern of experience, generates a differentiation of the *persona* that appears before others and the more intimate *ego* that in the day-dream is at once the main actor and the sole spectator. Finally, the incomprehension, isolation, and duality rob the development of one's common sense of some part, greater or less, of the corrections and the assurance that result from learning accurately the tested insights of others and from submitting one's own insights to the criticism based on others' experience and development.

2.7.1 *Scotosis.* Let us name such an aberration of understanding a scotosis, and let us call the resultant blind spot a scotoma. Fundamentally, the scotosis is an unconscious process. It arises, not in conscious acts, but in the censorship that governs the emergence of psychic contents. None the less, the whole process is not hidden from us, for the merely spontaneous exclusion of unwanted insights is not equal to the total range of eventualities. Contrary insights do emerge. But they may be accepted as correct, only to suffer the eclipse that the bias brings about

by excluding the relevant further questions. Again, they may be rejected as incorrect, as mere bright ideas without a solid foundation in fact; and this rejection tends to be connected with rationalization of the scotosis and with an effort to accumulate evidence in its favour. Again, consideration of the contrary insight may not reach the level of reflective and critical consciousness; it may occur only to be brushed aside in an emotional reaction of distaste, pride, dread, horror, revulsion. Again, there are the inverse phenomena. Insights that expand the scotosis can appear to lack plausibility; they will be subjected to scrutiny; and as the subject shifts to and from his sounder viewpoint, they will oscillate wildly between an appearance of nonsense and an appearance of truth. Thus, in a variety of manners, the scotosis can remain fundamentally unconscious yet suffer the attacks and crises that generate in the mind a mist of obscurity and bewilderment, of suspicion and reassurance, of doubt and rationalization, of insecurity and disquiet.

2.7.2 *Repression.* Nor is it only the mind that is troubled. The scotosis is an aberration, not only of the understanding, but also of the censorship. Just as wanting an insight penetrates below the surface to bring forth schematic images that give rise to the insight, so not wanting an insight has the opposite effect of repressing from consciousness a scheme that would suggest the insight. Now this aberration of the censorship is inverse to it. Primarily, the censorship is constructive; it selects and arranges materials that emerge in consciousness in a perspective that gives rise to an insight; this positive activity has by implication a negative aspect, for other materials are left behind and other perspectives are not brought to light; still, this negative aspect of positive activity does not introduce any arrangement or perspective into the unconscious demand functions of neural patterns and processes. In contrast, the aberration of the censorship is primarily repressive; its positive activity is to prevent the emergence into consciousness of perspectives that would give rise to unwanted insights; it introduces, so to speak, the exclusion of arrangements into the field of the unconscious; it dictates the manner in which neural demand functions are not to be met; and the negative aspect of its positive activity is the admission to consciousness of any materials in any other arrangement or perspective. Finally, both the censorship and its aberration differ from conscious advertence to a possible mode of behaviour and conscious refusal to behave in that fashion. For the censorship and its aberration are operative prior to conscious



advertence and they regard directly not how we are to behave but what we are to understand. A refusal to behave in a given manner is not a refusal to understand; so far from preventing conscious advertence, the refusal intensifies it and makes its recurrence more likely; and, finally, while it is true that conscious refusal is connected with a cessation of the conscious advertence, still this connection rests, not on an obnubilation of intelligence, but on a shift of effort, interest, preoccupation. Accordingly, we are led to restrict the name, repression, to the exercise of the aberrant censorship that is engaged in preventing insight.

2.7.3 *Inhibition.* The effect of the repression is an inhibition imposed upon neural demand functions. However, if we distinguish between demands for images and demands for affects, it becomes clear that the inhibition will not block both in the same fashion. For insights arise, not from the experience of affects, but rather from imaginative presentations. Hence, to prevent insights, repression will have to inhibit demands for images. On the other hand, it need inhibit demands for affects only if they are coupled with the undesired images. Accordingly, the repression will not inhibit a demand for affects, if that demand becomes detached from its apprehensive component, slips along some association path, and attaches itself to some other apprehensive component. Inversely, when there emerges into consciousness an affect coupled with an incongruous object, then one can investigate association paths, argue from the incongruous to the initial object of the affect, and conclude that this combination of initial object and affect had been inhibited by a repression. Nor is this conclusion to be rejected as preposterous because the discovered combination of image and affect is utterly alien to conscious behaviour. For the combination was inhibited, precisely because it was alien. Insights are unwanted, not because they confirm our current viewpoints and behaviour, but because they lead to their correction and revision. Inasmuch as the scotosis grounds the conscious, affective attitudes of the *persona* performing before others, it also involves the repression of opposite combinations of neural demand functions; and these demands will emerge into consciousness with the affect detached from its initial object and attached to some associated and more or less incongruous object. Again, inasmuch as the scotosis grounds the conscious, affective attitudes of the *ego* performing in his own private theatre, it also involves the repression of opposite combinations of neural demand functions; and in like

manner these demands make their way into consciousness with the affect detached from its initial object and attached to some other more or less incongruous object. In a systematization of Jung's terminology, the conscious *ego* is matched with an inverse non-conscious shadow, and the conscious *persona* is matched with an inverse non-conscious *anima*. Thus, the *persona* of the dispassionate intellectual is coupled with a sentimental *anima*, and an *ego* with a message for mankind is linked to a diffident shadow.

2.7.4 *Performance.* Apprehension and affect are for operations but, as one would expect, the complex consequences of the scotosis tend to defeat the efforts of the dramatic actor to offer a smooth performance. To speak fluently or to play a musical instrument, one has to be able to confine attention to higher-level controls and to leave the infinite details of the execution to acquired habit. But the division of conscious living between the two patterns of the *ego* and *persona* can hamper attention to the higher-level controls and allow the sentiments of the *ego* or shadow to slip into the performance of the *persona*. Thus, a friend of mine, who had been out of town, asked me how my work was getting on. I answered with a dreaded didactic monologue on the connection between insight and depth psychology. His laudatory comment ended with the remark, 'Certainly, while I have been away, you have not been wasting my time.'

Besides the waking performance of the dramatic actor, there is also the strange succession of fragmentary scenes that emerge in sleep. Then experience is not dominated by a pattern. Not only are there lacking the critical reflection and deliberate choosing that make waking consciousness reasonable, but also the preconscious activity of the censor, selecting and arranging neural demands, is carried out in a half-hearted and perfunctory manner. This relaxation of the censorship, however, not only accounts for the defective pattern of experience in dream-land but also explains the preponderant influence of the other determinant of conscious contents, namely, the neural demand functions. Claims ignored during the day become effective in sleep. The objects and affects of the *persona* and of the *ego* make an overt appearance and with them mingle the covert affects of the shadow and the *anima* attached to their incongruous objects.

The basic meaning of the dream is its function. In the animal, consciousness functions as a higher technique for the effective prosecution



of biological ends. In man, not only does it fulfil this purpose but also provides the centre for the operations of the self-constituting dramatic actor. Sleep is the negation of consciousness. It is the opportunity needed by unconscious vital process to offset without interference the wear and tear suffered by nerves during the busy day. Within this function of sleep lies the function of the dream. Not only have nerves their physical and chemical basis but also they contain dynamic patterns that can be restored to an easy equilibrium only through the offices of psychic representations and interplay. Besides restoring the organism, sleep has to knit up the ravelled sleave of care, and it does so by adding dreams in which are met ignored claims of neural demand functions.

Functionally, then, the dream is a psychic flexibility that matches and complements the flexibility of neural demands. If consciousness is to yield to the preoccupations of the intellectual or of the dramatic actor, it cannot be simply a function of neural patterns and processes. Inversely, if neural demands, ignored by consciousness, are to be met without violating the liberation of the artistic, intellectual, or dramatic pattern of experience, then they find their opportunity in the dream.

There is a further aspect to this twofold flexibility. The liberation of consciousness is founded on a control of apprehensions; as has been seen, the censorship selects and arranges materials for insight or, in its aberration, excludes the arrangements that would yield insight. Inversely, the imperious neural demands are not for apprehensive psychic contents but for the conations and emotions that are far more closely linked with activity; thus, while we imagine much as we please, our feelings are quite another matter. Accordingly, since the dream is the psychic safety-valve for ignored neural demands, and since the imperious neural demands are affective rather than apprehensive, the dream will appear as a wish fulfilment. This statement, of course, must not be taken in the sense that the unconscious has wishes which are fulfilled in dreams, for wishing is a conscious activity. Nor again does it mean that the wishes fulfilled in dreams are those of the conscious subject, for inverse to the *ego* is the shadow and inverse to the *persona* is the *anima*. The accurate statement is that dreams are determined by neural demands for conscious affects, and the affects in question may be characteristic not only of the *ego* or the *persona* but also of the shadow or the *anima*. However, as has been seen, if the affects emergent in the dream are characteristic of the shadow or the *anima*, they emerge disassociated from their initial objects and attached to some incongruous object;

and in this fact there now may easily be discerned a functional significance. The affects of the shadow and *anima* are alien to the conscious performer; were they to emerge into consciousness with their proper objects, not only would they interfere with his sleep but also they would violate his aesthetic liberation. The disguise of the dream is essential to its function of securing a balance between neural demands and psychic events while preserving the integrity of the conscious stream of experience.

Hence, to penetrate to the latent content of the dream is to bring to light a secret that, so to speak, has purposely been hidden. To equip an animal with intelligence constitutes not only the possibility of culture and of science but also the possibility of every abomination that has occurred in the course of human history. To affirm the latter human potentiality in abstract terms is somewhat unpleasant. To proceed syllogistically from the universal to the particular is distasteful. To assert that potentialities inherent in human nature exist in one's acquaintances, one's relatives, one's parents, oneself, is logical enough yet outrageous. Yet far more vivid than the utterance of such truths is their apprehension through insights into images that are affectively charged. In his waking hours man may preclude the occurrence of such insights. Even if his unconscious patterns and processes have been so stimulated as to demand them, the demand can be met in a dream in which the disassociation of the affect from its proper object respects the direction of the stream of consciousness.

A similar functional significance may be found in the formation of screening memories. Of our childhood we are apt to remember only a few vivid scenes and, when these are submitted to scrutiny and investigation, they are likely to prove mere fictions. Freud has divined such false memories to be screens. Behind them are actions which later understanding would view in a fashion unsuspected by the child that performed them. If the memory of such actions is not to enter consciousness, it has to be repressed; if it is repressed, it undergoes the disassociation and recombination that result from inhibition. In this fashion there is formed the false and screening memory that enables the dramatic actor to play his present role with all the more conviction because he does not believe his past to differ too strikingly from his present.

2.7.5 *A Common Problem.* Our study of the dramatic bias has worked from a refusal to understand through the series of its consequences.



There result in the mind a scotosis, a weakening of the development of common sense, a differentiation of the *persona* and the *ego*, an alternation of suspicion and reassurance, of doubt and rationalization. There follow an aberration of the censorship, the inhibition of unwanted imaginative schemes, the disassociation of affects from their initial objects and their attachment to incongruous yet related materials, the release of affective neural demands in dreams, and the functionally similar formation of screening memories.

However, if the account has made no explicit mention of sex, this must not be taken to imply that the depth psychologists have been on the wrong track. On the contrary, the peculiarities of sexual development make it the ordinary source of materials for the scotosis. Because hunger and sex are vital, they constitute the areas in which experience can be contracted from its dramatic to its biological pattern. But hunger is present from birth and its manifestations do not greatly change. Sexual development, on the contrary, is prolonged and, indeed, both organic and psychological. From birth to puberty there occur successive specializations of the neural demand functions; and their term is not some free combination of movements, like playing the piano, but a naturally determined sequence of apprehensions, affects, and movements that admit only superficial modifications from the inventive dramatist. Interdependent with this change, there is a psychological transformation in which the affective and submissive attitudes of the child within the family give place to the man self-reliantly orientating himself in the universe and determining to found a family of his own.

During the course of this long and intricate process, there is room not only for waywardness motivated by strange pleasure but also for accidents, incomprehension, blunders, secretiveness. If adverse situations and mistakes occur at random, they can be offset by the excretory function of the dream, by the pressures and attractions of a healthy environment, by suitable and opportune instruction, by some form of inner acceptance of the drive to understanding and truth with its aesthetic and moral implications. If thy eye be single, thy whole body will be lightsome. On the other hand, one adverse situation can follow another; the error and waywardness of each previous occasion can make still more probable the mishandling of the next. A scotosis becomes established. As an aberration of the understanding, it stands in the way of the proper development of affective attitudes. As an aberration of the censorship, it loads the neural demand functions with inhibi-

tions. Affective demands are shifted to incongruous dream objects. The incongruous objects may chance to function as do normal stimuli for affects, and waywardness may solidify the connection. The shadow and the *anima* can become organized as demands for integrated attitudes of love or hatred. Eventually, a point is reached where the immanently determined direction of the misled stream of consciousness is no longer capable of providing psychic representation and conscious integration for the distorted neural demand functions. Then neural demands assert themselves in waking consciousness through the inadequacies, compulsions, pains, and anxieties of the psychoneuroses. Dramatic living has forfeited its autonomy and only through delusions can it pretend to its old mastery.

Still, before this point is reached, there can occur the intermediate phenomena studied by Freud in his *Totem and Taboo*. It was remarked above that the dream provides release from the random repressions that are more or less inevitable and that the development of scotosis results from the cumulative effect of successive adverse situations. Now, when adverse situations become the rule for most members of a society, then the society can survive only by providing a regular public equivalent for the dream. Such prophylactic group therapy will exist whenever unconscious needs are met in a disguised manner. Dr. Stekel's description of the theatre as mass therapy echoes Aristotle's statement that tragedy effects a catharsis of fear and pity. Nor is the invention of such therapy in a primitive culture any more difficult than the invention of the cultural organization itself. For the constraints of the organization give rise to corresponding dreams; the relief afforded by the dreams can be noticed; this advertence may be given dramatic expression; the dramatic expression would meet in a disguised form the unconscious needs of the community; and if the dramatic expression is not included in the cultural organization, then the culture will not survive to be investigated by anthropologists.

This basic mechanism admits a series of applications that range from knowledge issuing forth in prophylactic purpose through successive stages of intellectual obnubilation to close approximations to abnormal phenomena. Man's capacity for art and science, psychology and philosophy, religion and morality, operates in the primitive and in the uneducated without awareness of the differences between these departments and without any sharp distinction between them and underlying impulses or needs. In the complex phenomena of totemism, in the rites



of the Mother Goddess, in the myths of the Sky Gods, there appear reflections not only of the social organization of hunters, agriculturalists, and parasitic nomads but also of human sexuality; nor did the Mosaic proscription of images prevent the backsliding lamented by the prophets, nor the mystical flight from sense of the Buddhists eliminate the earlier Brahminism, nor the rational criticism of the Greeks forestall popular hatred of the Christians.

Again, there is a nice distinction between the sensitive mechanism that enforces a taboo and the rational judgment that imposes a moral obligation. Freud was aware that his path would have been easier if he had glozed over the more shocking elements in his discoveries; yet to take the easier course would have involved not only a violation of his intellectual convictions but also a conquest of his moral feelings. Still, such a coincidence of conscience and moral feeling can be procured not only by an adaptation of feeling to moral judgment but also by the determination of judgment in accord with the feelings instilled through parental and social influence. Once feeling takes the lead, critical reflection can prevent an arbitrary extension of the moral code. But in the primitive and in the child, not only is critical reflection undeveloped and unequipped but also there is little capacity to distinguish between the outer constraint of commands imposed through affection and fear and the inner implications through which reasonable judgment entails reasonable living. Then moral feelings are free to develop according to the psychological laws that link affects to successively associated objects. The taboo not only operates but also tends to expand in much the same fashion as the compulsion neurosis.

2.7.6 *A Piece of Evidence.* It is not an easy matter to connect our outline of dramatic bias with the evidence provided by specialists in the field of psychotherapy. Not only are they not directly concerned with human intelligence but also the scope of our work leaves no room for an account of the existence, on the level of the sensitive psyche, of an initiating factor that operates in a parallel fashion to the flight from understanding. It is true that later, in the first section of Chapter XVII, an earlier exploration of genetic method will have made it possible to offer some indications on this score. But we must strike the iron while it is hot and so we propose to prescind from all questions of causal origins and to view our account of dramatic bias simply as a functional correlation. For if we cannot expect the reader to believe that a flight

from insight is the infantile beginning of psychic trouble, we cannot but claim that there is some connection between it and, on the other hand, repression and inhibition, the slips of waking consciousness and the function of dreams, the aberrations of religion and morality and, as a limit, the psychoneuroses. Moreover, our claim would be more than Freud seems to have implied when, in his *History of the Psychoanalytic Movement*, he prefaced his indictment of the secessionists, Adler and Jung, with the remark that he had always asserted that repressions and the sustaining resistance *might* involve a suspension of understanding. Accordingly, we raise the question whether any specialists in the field of psychiatry offer any evidence for a correlation that links both psychic trouble with a flight from insight and psychic recovery with an intellectual illumination.

To this precise question an affirmative answer seems to be offered by Dr. Wilhelm Stekel's *Technique of Analytic Psychotherapy*.* The work, which is thoroughly practical in conception and purpose, consistently considers analytic treatment as a retrospective education. Once the differential diagnosis has excluded both somatic disorder and the imminence of psychosis, the working hypothesis becomes the assumption that the analysand is the subject of a scotoma. A favourable prognosis requires that the patient's critical reflection and deliberate choice be allied with the analyst; but along with this rational attitude there exists a flight from knowledge that is to be cured by knowledge. During the analysis this flight continues to manifest itself in two manners named the resistance and the transference. Just as in the rest of his living the patient's understanding spontaneously finds measures of self-defence and thereby nourishes the scotoma, so in the intimate drama of the analysis the patient is engaged both in devising means to prevent the coming revelation and, at the same time, repressing the insights that would explain to him his own conduct. Such is the resistance; it is plausible, ingenious, resourceful; it adapts itself to each new situation; yet so far from being deliberate, it is at least fundamentally non-conscious. There is also the transference. The development of the scotoma has involved the repression of feelings of love or hatred for persons in the patient's milieu; this repression and the consequent inhibition mean that the patient is the subject of neural demands for affects that, however, are detached from their initial objects; the transference is the emergence into consciousness of these affects directed upon the person of the analyst.

* The Bodley Head, London, 1939.



Just as the disorder is linked with a refusal to understand, so its cure is an insight, a 'lightning flash of illumination'. Just as the refusal excluded not some single insight but an expanding series, so the cure consists in the occurrence of at least the principal insights that were blocked. It is the re-formation of the patient's mentality. Moreover, these insights must occur, not in the detached and disinterested intellectual pattern of experience, but in the dramatic pattern in which images are tinged with affects. Otherwise the insights will occur but they will not undo the inhibitions that account for the patient's affective disorders; there will result a development of theoretical intelligence without a change in sensitive spontaneity. Finally, the patient is not to be thought capable of curing himself; for the cure consists precisely in the insights which arise from the schematic images that spontaneously the patient represses; and even if by an extraordinary effort of intellectual detachment the patient succeeded in grasping in part what he was refusing to understand, this grasp would occur in the intellectual pattern of experience and so would prove ineffectual; indeed, the effort would be likely to produce an obsession with analytic notions, and there would be some danger that such merely theoretical insight would tend to inoculate the patient against the benefit of a true analytical experience with its dramatic overtones.

The analyst, then, is needed. To perform the differential diagnosis, he must know medicine. Otherwise he will risk not merely ministering to the mind when the body is ill but also attempting to treat psychotics and so acquiring the reputation of driving people insane. Further, the analyst must himself be free from scotoma; a bias in his understanding of himself will also be a bias in his understanding of others; and this is all the more dangerous if he attempts to follow Dr. Stekel's active therapy. This active therapy rests upon knowledge that is parallel in structure to common sense. As has been seen, common sense consists in a basic accumulation of insights to which must be added further insights derived from the situation in hand. Similarly, the analyst's knowledge has two parts. There is the basic accumulation derived from an academic formation and from personal experience. It consists in an understanding of the psychoneuroses or parapathies in their origins, their development, their results; it is a grasp of a vast manifold of possibilities; it involves an ability to proceed from a patient's biography and behaviour, his dreams and associations, to a grasp of his precise flight from knowledge. However, that precise flight was the hidden

component of an individual history; it possesses not merely typical features but also its own particular twists and turns; and it continues to be operative in the analytical situation. The analyst has to outwit the resistance. He has to discern the transference, be able to make capital of it, and know when to end it. He has to be able to wait for favourable opportunities, ready to take the initiative when the occasion calls for it, capable of giving up when he is defeated, and ingenious in keeping things going when he sees he can win. In this complicated and dangerous chess-game, he is to be gaining insight into the patient's basic trouble, winning his confidence by the explanation and removal of superficial symptoms, and preparing the way for the discovery of the profound secret. Finally, he has to be able to end the analysis, stiffen the analysand to self-reliance, contribute what he can to the happy ending in which both need of the analyst and disturbing memories of the analysis pass away.

It is time to revert to our question. Does there exist empirical evidence for the assertion that the suspension of understanding is not merely a possible consequence but also a regular factor in psychogenic disorder? Unfortunately, there are divisions among specialists in the field and so, instead of giving a single answer, I must give two.

To those not disinclined to agree with Dr. Stekel, one may say that there exists empirical evidence for a psychotherapeutic notion in the measure that the notion is operative in actual treatment, that it is operative in the treatment of all types of disorder rather than in a partial selection of types, that it survives prolonged and varied experience, that the survival contrasts with a readiness to drop unverified notions, that failures cannot be traced to the notion in question. Now Dr. Stekel has attained an international position both as an analyst and as a writer of technical works; he is able to describe his *Technique of Analytic Psychotherapy* as the fruit of thirty years' experience; in that book the analyst's working hypothesis is that the patient is suffering from a scotoma and the analyst's goal is to lead the patient towards a 'lightning flash of illumination'; this view dominates the whole treatment and is relevant to the whole class of parapathies or psychoneuroses; finally, there is a good deal of evidence for Dr. Stekel's independence of mind and his readiness to abide by results.

However, there perhaps are those to whom Dr. Stekel's favour for an opinion provides presumptive evidence that the opinion is erroneous or at least rash. To them I would point out that the present issue is not



the validity of the whole of Dr. Stekel's theory and practice but solely the existence of empirical evidence for a single correlation. I am not asking for the adoption of Dr. Stekel's active method; I am not even urging that analytic treatment is desirable; my concern is restricted to a theoretical issue, and my question is whether or not evidence exists. It seems to me that a negative answer is impossible. Even if one prescinds entirely from Dr. Stekel and his pupils, still there occur other analytic treatments in which the cure operates through knowledge;* but the knowledge in question is of a particular kind; it is not sensitive knowledge apart from organization through insights, for hypnosis is not a satisfactory method; it is not knowledge on the level of critical reflection and judgment, for delusions are not the principal characteristic of psychoneurosis; it is the intermediate factor that we have been investigating under the name of insight, and on the present theoretical level it makes no difference whether the patient be led to the insight by an active method or left to discover it for himself by a passive method.

2.7.7 *A Note on Method.* There is a final point to be made, and it regards the significance for depth psychology of recent developments in scientific methodology. At the turn of the century mechanist determinism was still the world view dominant in scientific circles. Freud's discovery and development of the notion of psychogenic disorder came at the ambiguous moment when the old outlook was about to dissolve and, as one might expect, the ambiguity of the moment forced am-

* Otto Fenichel [*Problems of Psychoanalytic Technique.* Translated by D. Brunswick. Albany, N.Y., 1941] asks how it is that interpretation works (p. 52) and discusses the process in which the patient appropriates an interpretation (pp. 76 ff.). Gregory Zilboorg [The Emotional Problem and the Therapeutic Role of Insight. *The Psychoanalytic Quarterly*, XXI (1952), 1–24] experiences difficulty in defining insight, grants that Freud's basic hope was an enlargement of our capacity to understand, insists that therapeutically significant insight arises only as a consequence to psychic liberation. *A Study of Interpersonal Relations* [edited by P. Mullahy, New York 1949] included three papers on insight by E. D. Hutchinson. From these papers Clara Thompson [*Psychoanalysis: Evolution and Development.* With the collaboration of P. Mullahy. London 1952] derives a definition of insight and applies it to a therapeutic process that ends with an illuminating moment in which previous thinking falls into perspective and sensitive spontaneity undergoes an effortless change (pp. 238–40). In *The Interpersonal Theory of Psychiatry*, H. S. Sullivan speaks a number of times of a marvellous 'selective inattention' to what is significant (see the Index) and depicts a patient suddenly seeing the point of a dream (pp. 338 f.). In brief, there is an essential difference between the task of the teacher and the task of the psychotherapist, but teaching individuals something about themselves is a significant component in psychotherapy; and in the light of the present analysis of human knowledge successful teaching is a communication of insights.

biguity upon the interpretation of his work. Were mechanist determinism correct, then neither normality nor disorder could be psychogenic; Laplace's demon could calculate both from the world distribution of atoms in any basic situation; Freud could be said to introduce a new name and a new technique inasmuch as he dealt with collocations of atoms through their psychic appearances; but Freud could not be credited with the discovery of an autonomous science. On the other hand, if mechanist determinism is incorrect, the category of the psychogenic promptly assumes a profound significance. Let us attempt to clarify this point.

As we have seen, empirical science is the determination of correlations verified in observables.* Mechanism is the additional determination to invent what is neither a correlation nor verified nor observable. What is so invented, is pronounced real and objective; and in comparison with this fictitious treasure, the observable becomes the merely apparent. Thus, in nineteenth-century physical theory, the aether is real and objective, and its properties resembling, say, a sponge-like vortex are what make electromagnetic equations true. Nor is this all. Because verified correlations are attributed to atoms or aether, as imagined, they are not abstract but concrete; and once classical correlations are considered to be concrete, determinism follows and the possibility of statistical laws, except as a confession of ignorance, rigorously is excluded.

Now Freud's own investigations threw some doubt on the scientific character of mechanist objectivation. He was aware of the importance of extroversion in the object-finding that pertains to the psychic side of sexual development. He could appeal to projection to account for the transformation of the unconscious ill-will of primitives to deceased relatives into the explicit ill-will of the departed spirits to the bereaved mourners. But he had no intention of going back over the path traversed by Galileo and Descartes, Hobbes and Berkeley, Hume and Kant. Nor did the methodology of then contemporary science provide him with a canon of parsimony that restricted scientific affirmation to verified correlations and to observables. On the contrary, on many occasions Freud represents the outlook of his time and tends to regard observable psychic events as appearance and unobservable entities as reality. What precisely is the libido? Is it what is known either by observing nerves or by observing psychic events or by correlating these

* Here 'observable' denotes, not the physical variable of Quantum Mechanics, but simply what can be observed.



observables or by verifying these correlations? Or is it a construct that stands to Freud's verified correlations in much the same manner as the sponge-vortex aether once stood to electromagnetic equations? To resolve the ambiguity, if it can be resolved, would call for an investigation by a trained expert in the history of science.*

Again, Freud was professedly determinist. But in so far as determinism is operative in Freud's work, it amounts to the postulate that there is a reason for everything, even for numbers that one appears to select at random. But if one admits that some reasons are non-systematic, that postulate becomes compatible with statistical laws; and if Laplace has failed to exclude probability from physics, there is little likelihood of its being excluded from psychology.

Still, whatever may have been Freud's involvement in mechanist determinism, it remains that his scientific discovery was psychogenic disease. It is not science but a philosophy that pronounces conscious events to be appearance and some underlying ultimate to be reality. It is not science but a philosophy that confers upon atoms the exclusive role of ruling the course of conscious events. On the other hand, it is Freud's discovery that reveals the psychogenic to be more than an illusory name; and it is a sacrifice of mechanist determinism that opens the way to the recognition of the psychogenic as a genuine category.

For, in the first place, an acknowledgement of the non-systematic leads to an affirmation of successive levels of scientific inquiry. If the non-systematic exists on the level of physics, then on that level there are coincidental manifolds that can be systematized by a higher chemical level without violating any physical law. If the non-systematic exists on the level of chemistry, then on that level there are coincidental manifolds that can be systematized by a higher biological level without violating any chemical law. If the non-systematic exists on the level of biology, then on that level there are coincidental manifolds that can be systematized by a higher psychic level without violating any biological law. If the non-systematic exists on the level of the psyche, then on that level there are coincidental manifolds that can be systematized by a higher level of insight and reflection, deliberation and choice, without

* There is a long and nuanced chapter on 'Freud's Theory of Mind' in the first volume of Dr Ernest Jones' *The Life and Work of Sigmund Freud* (New York 1953). Contrast the influence of contemporary physics in the relational concepts elaborated in *Culture and Personality* (Proceedings of an Inter-disciplinary Conference held under the auspices of the Viking Fund. 14 East 71st Street, New York City, 7 and 8 November 1947. Published by the Viking Fund, 1949. Reprinted 1950. Pp. 9, 11, 13 ff., 48 ff., 175 ff.).

violating any law of the psyche. In brief, an acknowledgement of the non-systematic makes it possible to conceive

(1) psychic health as a harmonious unfolding of a process that moves at once on distinct yet related levels,

(2) psychic aberration as an orientation of the stream of consciousness in conflict with its function of systematizing underlying manifolds, and

(3) analytic treatment as an effort to reorientate an aberrant stream of consciousness and to effect a release from unconscious obstructions with a psychic origin.

Again, an acknowledgement that the real is the verified makes it possible to affirm the reality no less of the higher system than of the underlying manifold. The chemical is as real as the physical, the biological as real as the chemical; the psychic as real as the biological; and insight as real as the psychic. At once the psychogenic ceases to be merely a name, for the psychic becomes a real source of organization that controls underlying manifolds in a manner beyond the reach of their laws.

By the same stroke there is laid the spectre that at least popularly is associated with Freud's discoveries. For the latent content of the dream, so far from revealing the 'real' man, now merely exhibits potentialities that are rejected not only by waking but also by dreaming consciousness. Though the potentialities are for parricide and cannibalism, incest and suicide, still they are only potentialities and commonly they are rejected. What has been found shocking or revolting is not the affirmation of the possibility of what, after all, does occur. It is the affirmation that under the disguise of a phenomenal consciousness there lurks a monster that is the reality of each of us and the effective master of our lives.

Finally, it is to be observed that, if this note on method draws upon previous discussion, it also is perhaps excessively proleptic. Only in Chapter VIII shall we be in a position either to say what is meant by the thing, the man, the person, or to extend to things and persons the notion of emergent probability. Only in Chapter XIV shall we be able to come to grips with the philosophic problems of reality and objectivity. Only in Chapter XV shall we be able to attempt a systematic account of genetic method. But if our present suggestions cannot avoid a conspicuous lack of precision and detail, perhaps they possess the pedagogical value of opening a perspective and promising a fuller though later scrutiny.

CHAPTER VII

# COMMON SENSE AS OBJECT

The apparently modest and secure undertaking of common sense is to understand things in their relations to us. Unfortunately, we change; even the acquisition of common sense is a change in us; and so in the preceding section we attempted an investigation of the biological, aesthetic, artistic, intellectual, dramatic subject to which common sense relates things. But if the development of common sense is a change in its subject, still more obviously does it involve a change in its object. Common sense is practical. It seeks knowledge, not for the sake of the pleasure of contemplation, but to use knowledge in making and doing. Moreover, this making and doing involve a transformation of man and his environment, so that the common sense of a primitive culture is not the common sense of an urban civilization, nor the common sense of one civilization the common sense of another. However elaborate the experiments of the pure scientist, his goal is always to come closer to natural objects and natural relationships. But the practicality of common sense engenders and maintains enormous structures of technology, economics, politics, and culture, that not only separate man from nature but also add a series of new levels or dimensions in the network of human relationships. No less than the subjective, the objective field of common sense must be explored, for the development of common sense involves a change not only in us, to whom things are related, but also in the things, which are related to us.

## 1. PRACTICAL COMMON SENSE

In the drama of human living, human intelligence is not only artistic but practical. At first, there appears little to differentiate man from the beasts, for in primitive fruit-gathering cultures, hunger is linked to eating by a simple sequence of bodily movements. But primitive hunters take time out from hunting to make spears, and primitive fishers take time out from fishing to make nets. Neither spears nor nets in themselves are objects of desire. Still, with notable ingenuity and effort, they are fashioned because, for practical intelligence, desires are recurrent, labour is recurrent, and the comparatively brief time spent making

spears or nets is amply compensated by the greater ease with which more game or fish is taken on an indefinite series of occasions.

Moreover, such an intervention of intelligence is itself recurrent. As products of human ingenuity, spears and nets illustrate not only the idea of the old mechanical arts but also the more recondite idea of modern technology. As pieces of material equipment, the same objects are initial instances of the idea of capital formation. Now the history of man's material progress lies essentially in the expansion of these ideas. As inventions accumulate, they set problems calling for more inventions. The new inventions complement the old to suggest further improvements, to reveal fresh possibilities and, eventually, to call forth in turn the succession of mechanical and technological higher viewpoints that mark epochs in man's material progress. Moreover, this advance of practical intelligence is registered not merely in memory and, later, in books, but more obviously in concrete products, in tools and buildings, in the ever increasing manifold of appurtenances of labourers, craftsmen, merchants, and carriers. Thus, in correspondence with each stage in the development of practical intelligence, there is a measure and structure of capital formation, that is, of things produced and arranged not because they themselves are desired but because they expedite and accelerate the process of supplying the goods and services that are wanted by consumers. Again, in correspondence with each advance of practical intelligence, there is a technological obsolescence of capital equipment. The old shops still have their shelves and counters; the old machines may suffer no material or mechanical defect. But the new models produce better goods more efficiently; and trade now walks on different streets.

The concrete realization of the succession of new practical ideas does not take place without human co-operation. It demands a division of labour and, at the same time, it defines the lines along which labour is divisible. It invites men to specialize in the skilful use of particular tools and the expeditious performance of particular tasks. It calls forth some economic system, some procedure that sets the balance between the production of consumer goods and new capital formation, some method that settles what quantities of what goods and services are to be supplied, some device for assigning tasks to individuals and for distributing among them the common product.

As technology evokes the economy, so the economy evokes the polity. Most men get ideas, but the ideas reside in different minds, and



the different minds do not quite agree. Of itself, communication only reveals the disparity. What is wanted is persuasion, and the most effective persuader becomes a leader, a chief, a politician, a statesman. For the problem of effective agreement is recurrent. Each step in the process of technological and economic development is an occasion on which minds differ, new insights have to be communicated, enthusiasm has to be roused, and a common decision must be reached. Beyond the common sense of the labourer, the technician, the entrepreneur, there is the political specialization of common sense. Its task is to provide the catalyst that brings men of common sense together. It is an incomplete accumulation of insights to be complemented and modified by the further insights that arise from the situation in hand. It involves some understanding of industry and of commerce but its special field is dealing with men. It has to discern when to push for full performance and when to compromise, when delay is wisdom and when it spells disaster, when widespread consent must be awaited and when action must be taken in spite of opposition. It has to be able to command attention and to win confidence, to set forth concretely the essentials of a case, to make its own decisions and secure the agreement of others, to initiate and carry through some section of that seriation of social responses meeting social challenges that Arnold Toynbee in his *Study of History* has so lavishly and brilliantly illustrated.

## 2. THE DYNAMIC STRUCTURE

As in the fields of physics, chemistry, and biology, so in the field of human events and relationships there are classical and statistical laws that combine concretely in cumulating sets of schemes of recurrence. For the advent of man does not abrogate the rule of emergent probability. Human actions are recurrent; their recurrence is regular; and the regularity is the functioning of a scheme, of a patterned set of relations that yields conclusions of the type, If an $X$ occurs, then an $X$ will recur. Children are born only to grow, mature, and beget children of their own. Inventions outlive their inventors and the memory of their origins. Capital is capital because its utility lies not in itself but in the acceleration it imparts to the stream of useful things. The political machinery of agreement and decision is the permanent yet self-adapting source of an indefinite series of agreements and decisions. Clearly, schemes of recurrence exist and function. No less clearly, their functioning is not inevitable. A population can decline, dwindle,

vanish. A vast technological expansion, robbed of its technicians, would become a monument more intricate but no more useful than the pyramids. An economy can falter, though resources and capital equipment abound, though skill cries for its opportunity and desire for skill's product, though labour asks for work and industry is eager to employ it; then one can prime the pumps and make *X* occur; but because the schemes are not functioning properly, *X* fails to recur. As the economy, so too the polity can fall apart. In a revolution violence goes unchecked; laws lose their meaning; governments issue unheeded decrees; until from sheer weariness with disorder men are ready to accept any authority that can assert itself effectively. Yet a revolution is merely a passing stroke of paralysis in the state. There are deeper ills that show themselves in the long-sustained decline of nations and, in the limit, in the disintegration and decay of whole civilizations. Schemes that once flourished lose their efficacy and cease to function; in an ever more rapid succession, as crises multiply and remedies have less effect, new schemes are introduced; feverish effort is followed by listlessness; the situation becomes regarded as hopeless; in a twilight of straitened but gracious living men await the catalytic trifle that will reveal to a surprised world the end of a once brilliant day.

Still, if human affairs fall under the dominion of emergent probability, they do so in their own way. A planetary system results from the conjunction of the abstract laws of mechanics with a suitable concrete set of mass-velocities. In parallel fashion, there are human schemes that emerge and function automatically, once there occurs an appropriate conjunction of abstract laws and concrete circumstances. But, as human intelligence develops, there is a significant change of roles. Less and less importance attaches to the probabilities of appropriate constellations of circumstances. More and more importance attaches to the probabilities of the occurrence of insight, communication, agreement, decision. Man does not have to wait for his environment to make him. His dramatic living needs only the clues and the opportunities to originate and maintain its own setting. The advance of technology, the formation of capital, the development of the economy, the evolution of the state are not only intelligible but also intelligent. Because they are intelligible, they can be understood as are the workings of emergent probability in the fields of physics, chemistry, and biology. But because they also are increasingly intelligent, increasingly the fruit of insight and decision, the analogy of merely natural process



becomes less and less relevant. What possesses a high probability in one country, or period, or civilization, may possess no probability in another; and the ground of the difference may lie only slightly in outward and palpable material factors and almost entirely in the set of insights that are accessible, persuasive, and potentially operative in the community. Just as in the individual the stream of consciousness normally selects its own course out of the range of neurally determined alternatives, so too in the group commonly accessible insights, disseminated by communication and persuasion, modify and adjust mentalities to determine the course of history out of the alternatives offered by emergent probability.

Such is the high significance of practical common sense, and it will not be amiss, I believe, to pause and make certain that we are not misconceiving it. For the practical common sense of a group, like all common sense, is an incomplete set of insights that is ever to be completed differently in each concrete situation. Its adaptation is too continuous and rapid for it ever to stand fixed in some set of definitions, postulates, and deductions; even were it outfitted, like David in Saul's armour, with such a logical panoply, it could be validated neither in any abstract realm of relations of things to one another nor in all members of any class of concrete situations. As its adaptation is continuous, so its growth is as secret as the germination, the division, the differentiation of cells in seed and shoot and plant. Only ideal republics spring in full stature from the mind of man; the civil communities that exist and function know only a story of their origins, only an outline of their development, only an estimate of their present complexion. For the practical common sense, operative in a community, does not exist entire in the mind of any one man. It is parcelled out among many, to provide each with an understanding of his role and task, to make every cobbler an expert at his last, and no one an expert in another's field. So it is that to understand the working of even a static social structure, one must inquire from many men in many walks of life and, as best one can, discover the functional unity that organically binds together the endlessly varied pieces of an enormous jig-saw puzzle.

### 3. INTERSUBJECTIVITY AND SOCIAL ORDER

Though I just spoke of a functional unity to be discovered, really there is a duality to be grasped. As intelligent, man sponsors the order imposed by common sense. But man is not a pure intelligence. Initially

and spontaneously, he identifies the good with the object of desire, and this desire is not to be confused either with animal impulse or with egoistic scheming. Man is an artist. His practicality is part of his dramatic pursuit of dignified living. His aim is not for raw and isolated satisfactions. If he never dreams of disregarding the little matter of food and drink, still what he wants is a sustained succession of varied and artistically transformed acquisitions and attainments. If he never forgets his personal interest, still his person is no Leibnizian monad; for he was born of his parents' love; he grew and developed in the gravitational field of their affection; he asserted his own independence only to fall in love and provide himself with his own hostages to fortune. As the members of the hive or herd belong together and function together, so too men are social animals and the primordial basis of their community is not the discovery of an idea but a spontaneous intersubjectivity.

Thus, primitive community is intersubjective. Its schemes of recurrence are simple prolongations of prehuman attainment, too obvious to be discussed or criticized, too closely linked with more elementary processes to be distinguished sharply from them. The bond of mother and child, man and wife, father and son, reaches into a past of ancestors to give meaning and cohesion to the clan or tribe or nation. A sense of belonging together provides the dynamic premise for common enterprise, for mutual aid and succour, for the sympathy that augments joys and divides sorrows. Even after civilization is attained, intersubjective community survives in the family with its circle of relatives and its accretion of friends, in customs and folk-ways, in basic arts and crafts and skills, in language and song and dance, and most concretely of all in the inner psychology and radiating influence of women. Nor is the abiding significance and efficacy of the intersubjective overlooked, when motley states name themselves nations, when constitutions are attributed to founding fathers, when image and symbol, anthem and assembly, emotion and sentiment are invoked to impart an elemental vigour and pitch to the vast and cold, technological, economic, and political structures of human invention and convention. Finally, as intersubjective community precedes civilization and underpins it, so also it remains when civilization suffers disintegration and decay. The collapse of Imperial Rome was the resurgence of family and clan, feudal dynasty and nation.

Though civil community has its obscure origins in human intersubjectivity, though it develops imperceptibly, though it decks itself out with more primitive attractions, still it is a new creation. The time comes when men begin to ask about the difference between φύσις and νόμος, between nature and convention. There arises the need of the apologue to explain to the different classes of society that together they form a functional unity and that no group should complain of its lot any more than a man's feet, which do all the walking, complain of his mouth, which does all the eating. The question may be evaded and the apologue may convince, but the fact is that human society has shifted away from its initial basis of intersubjectivity and has attempted a more grandiose undertaking. The discoveries of practical intelligence, which once were an incidental addition to the spontaneous fabric of human living, now penetrate and overwhelm its every aspect. For just as technology and capital formation interpose their schemes of recurrence between man and the rhythms of nature, so economics and politics are vast structures of interdependence invented by practical intelligence for the mastery not of nature but of man.



This transformation forces on man a new notion of the good. In primitive society it is possible to identify the good simply with the object of desire; but in civil community there has to be acknowledged a further component, which we propose to name the good of order. It consists in an intelligible pattern of relationships that condition the fulfilment of each man's desires by his contributions to the fulfilment of the desires of others and, similarly, protect each from the object of his fears in the measure he contributes to warding off the objects feared by others. This good of order is not some entity dwelling apart from human actions and attainments. Nor is it any unrealized ideal that ought to be but is not. But though it is not abstract but concrete, not ideal but real, still it cannot be identified either with desires or with their objects or with their satisfactions. For these are palpable and particular, but the good of order is intelligible and all-embracing. A single order ramifies through the whole community to constitute the link between conditioning actions and conditioned results and to close the circuit of interlocked schemes of recurrence. Again, economic break-down and political decay are not the absence of this or that object of desire or the presence of this or that object of fear; they are the break-down and decay of the good of order, the failure of schemes of recurrence to function. Man's practical intelligence devises arrangements for human living; and in the measure that such arrangements are understood and

accepted, there necessarily results the intelligible pattern of relationships that we have named the good of order.

In a simple yet inexorable fashion, this order, originated by human invention and convention, ceases to be an optional adjunct and becomes an indispensable constituent of human living. For the long-run effects of technological advance and new capital formation consist in some combination of increased population, reduced work, and improved living standards. In the course of a century the differences in all three respects may be so great that any return to an earlier state of affairs is regarded as preposterous and is to be brought about only by violence or disaster. But concomitant with the technological and the material development, there also takes place a complementary series of economic and political innovations. Each of these is motivated, to a greater or less extent, by the underlying technical and material changes; each, sooner or later, undergoes the adaptations demanded by subsequent changes; and so at any given moment all together present a united front that can be broken only by the destructive turmoil of a revolution or a conquest. Moreover, ideas have no geographical frontiers, and profits accrue to traders not only from domestic but also from foreign markets. Material and social progress refuses to be confined to a single country; like an incoming tide, first it reaches the promontories, then it penetrates the bays, and finally it pours up the estuaries. In an intricate pattern of lags and variations, new ideas spread over most of the earth to bind together in an astounding interdependence the fortunes of individuals living disparate lives in widely separated lands.

### 4. THE TENSION OF COMMUNITY

Intersubjective spontaneity and intelligently devised social order have their ground in a duality immanent in man himself. As intelligent, man is the originator and sponsor of the social systems within which, as an individual, he desires and labours, enjoys and suffers. As intelligent, man is a legislator but, as an individual, he is subject to his own laws. By his insights he grasps standard solutions to recurrent problems, but by his experience he provides the instances that are to be subsumed under the standard solutions. From the viewpoint of intelligence, the satisfactions allotted to individuals are to be measured by the ingenuity and diligence of each in contributing to the satisfactions of all; from the same high viewpoint the desires of each are to be regarded quite coolly as the motive power that keeps the social system functioning. But besides the detached and disinterested stand of intelligence, there is the more spontaneous viewpoint of the individual subjected to needs and wants, pleasures and pains, labour and leisure, enjoyment and privation. To each man his own desires, precisely because they are his own, possess an insistence that the desires of others can never have for him. To each man his own labours, because they are his own, have a dimension of reality that is lacking in his apprehension of the labours of others. To each man his own joys and sorrows have an expansive or contracting immediacy that others can know only through their own experience of joy and sorrow. Yet the ineluctable privacy of each one's experience provides no premise for a monadic theory of man. For the bonds of intersubjectivity make the experience of each resonate to the experience of others; and, besides this elementary communion, there are operative in all a drive to understand and an insistence on behaving intelligently that generate and implement common ways, common manners, common undertakings, common commitments.



For this reason, it would seem a mistake to conceive the sociological as simply a matter of external constraint. It is true enough that society constrains the individual in a thousand ways. It is true enough that the individual has but a slight understanding of the genesis and growth of the civilization into which he was born. It is true enough that many of the things he must do are imposed upon him in a merely external fashion. Yet within the walls of his individuality, there is more than a Trojan horse. He has no choice about wanting to understand; he is committed not by any decision of his own but by nature to intelligent behaviour; and as these determinants are responsible for the emergence of social orders in the past, so they account for their development, their maintenance, their reformation. Spontaneously every collapse is followed by a reconstruction, every disaster by a new beginning, every revolution by a new era. Commonly, men want a different social order but, left to themselves, they never consent to a complete anarchy.

There is, then, a radical tension of community. Intersubjective spontaneity and intelligently devised social order possess different properties and different tendencies. Yet to both by his very nature man is committed. Intelligence cannot but devise general solutions and general rules. The individual is intelligent and so he cannot enjoy peace of mind unless he subsumes his own feelings and actions under the general rules that he regards as intelligent. Yet feeling and spontaneous action

have their home in the intersubjective group and it is only with an effort and then only in favoured times that the intersubjective groups fit harmoniously within the larger pattern of social order.

Thus it is that in the history of human societies there are halcyon periods of easy peace and tranquillity that alternate with times of crisis and trouble. In the periods of relaxed tension, the good of order has come to terms with the intersubjective groups. It commands their esteem by its palpable benefits; it has explained its intricate demands in some approximate yet sufficient fashion; it has adapted to its own requirements the play of imagination, the resonance of sentiment, the strength of habit, the ease of familiarity, the impetus of enthusiasm, the power of agreement and consent. Then a man's interest is in happy coincidence with his work; his country is also his homeland; its ways are the obviously right ways; its glory and peril are his own.

As the serenity of the good old days rests on an integration of common sense and human feeling, so the troubled times of crisis demand the discovery and communication of new insights and a consequent adaptation of spontaneous attitudes. Unfortunately, common sense does not include an inventory of its own contents. It does not reside, whole and entire, in a single mind. It cannot point to any recorded set of experiments for its justification. It cannot assert itself in any of the inflexible generalizations that characterize logic, mathematics, and science. Common sense knows, but it does not know what it knows nor how it knows nor how to correct and complement its own inadequacies. Only the blind and destructive blows, inevitable in even a partial break-down of social order, can impress on practical common sense that there are limits to its competence and that, if it would master the new situation, it must first consent to learn. Still, what is to be learnt? The problem may baffle what experts are available. A theoretical solution need not lead automatically to its popular presentation. Even when that is achieved, the reorientation of spontaneous attitudes will remain to be effected. The time of crisis can be prolonged, and in the midst of the suffering it entails and of the aimless questioning it engenders, the intersubjective groups within a society tend to fall apart in bickering, insinuations, recriminations, while unhappy individuals begin to long for the idyllic simplicity of primitive living in which large accumulations of insights would be superfluous and human fellow-feeling would have a more dominant role.



## 5. THE DIALECTIC OF COMMUNITY

The name, dialectic, has been employed in a variety of meanings. In Plato, it denoted the art of philosophic dialogue and was contrasted with eristic. In Aristotle, it referred to an effort to discover clues to the truth by reviewing and scrutinizing the opinions of others. For the Schoolmen, it became the application of logical rules to public disputation. Hegel employed the word to refer to his triadic process from the concept of being to the Absolute Idea. Marx inverted Hegel and so conceived as dialectical a non-mechanical, materialist process. Summarily, then, dialectic denotes a combination of the concrete, the dynamic, and the contradictory; but this combination may be found in a dialogue, in the history of philosophic opinions, or in historical process generally.

For the sake of greater precision, let us say that a dialectic is a concrete unfolding of linked but opposed principles of change. Thus, there will be a dialectic, if

(1) there is an aggregate of events of a determinate character,
(2) the events may be traced to either or both of two principles,
(3) the principles are opposed yet bound together, and
(4) they are modified by the changes that successively result from them.

For example, the dramatic bias, described above,* was dialectical. The contents and affects emerging into consciousness provide the requisite aggregate of events of a determinate kind; these events originate from two principles, namely, neural demand functions and the exercise of the constructive or repressive censorship; the two principles are linked as patterned and patterning; they are opposed inasmuch as the censorship not only constructs but also represses and, again, inasmuch as a misguided censorship results in neglected neural demands forcing their way into consciousness; finally, change is cumulative, for the orientation of the censorship at any time and the neural demands to be met both depend on the past history of the stream of consciousness.

Now as there is a dialectic of the dramatic subject, so also there is a larger dialectic of community. Social events can be traced to the two principles of human intersubjectivity and practical common sense. The two principles are linked, for the spontaneous, intersubjective individual strives to understand and wants to behave intelligently; and inversely, intelligence would have nothing to put in order were there not

* Chapter VI, § 2.7

the desires and fears, labours and satisfactions of individuals. Again, these linked principles are opposed, for it is their opposition that accounts for the tension of community. Finally, these linked and opposed principles are modified by the changes that result from them; the development of common sense consists in the further questions and insights that arise from the situations produced by previous operations of practical common sense; and the alternations of social tranquillity and social crisis mark successive stages in the adaptation of human spontaneity and sensibility to the demands of developing intelligence.

In two manners this dialectic of community differs from the dialectic of the dramatic subject. First, there is a difference in extent, for the dialectic of community regards the history of human relationships, while the inner dialectic of the subject regards the biography of an individual. Secondly, there is a difference in the level of activity, for the dialectic of community is concerned with the interplay of more or less conscious intelligence and more or less conscious spontaneity in an aggregate of individuals, while the dialectic of the subject is concerned with the entry of neural demands into consciousness. Accordingly, one might say that a single dialectic of community is related to a manifold of individual sets of neural demand functions through a manifold of individual dialectics. In this relationship, the dialectic of community holds the dominant position, for it gives rise to the situations that stimulate neural demands and it moulds the orientation of intelligence that preconsciously exercises the censorship. Still, as is clear, one must not suppose this dominance to be absolute, for both covertly and overtly, neural demands conspire with an obnubilation of intelligence, and what happens in isolated individuals tends to bring them together and so to provide a focal point from which aberrant social attitudes originate.

This raises the basic question of a bias in common sense. Four distinct aspects call for attention. There is the already mentioned bias arising from the psychological depths, and commonly it is marked by its sexual overtones. There also are the individual bias of egoism, the group bias with its class conflicts, and a general bias that tends to set common sense against science and philosophy. On these three something must now be said.

### 6. INDIVIDUAL BIAS

There is a rather notable obscurity in the meaning of the terms, egoism and altruism. When a carnivorous animal stalks and kills its prey, it is not properly egoistic; for it is simply following its instincts and, in general, for animals to follow their instincts is for them to secure the biological ends of individual and specific survival. By parity of reasoning, when a female animal fosters its young, it too is following its instincts; though it contributes to a general biological end, still it does so rather by the scheming of nature than by altruism in its proper sense. Finally, if animal spontaneity is neither egoistic nor altruistic, it seems to follow that the same must be said of human spontaneity; men are led by their intersubjectivity both to satisfy their own appetites and to help others in the attainment of their satisfaction, but neither type of activity is necessarily either egoistic or altruistic.



There is a further aspect to the matter. In his *Ethics*, Aristotle asked whether a good friend loved himself. His answer was that while true friendship excluded self-love in the popular sense, none the less it demanded self-love in a higher sense; for a man loves himself, if he wants for himself the finest things in the world, namely, virtue and wisdom; and without virtue and wisdom a man can be a true friend neither to himself nor to anyone else. Accordingly, as Aristotle's answer suggests, when one turns from the realm of spontaneity to that of intelligence and reasonableness, one does not find that egoism and altruism provide ultimate categories. For intelligence and reasonableness with their implications automatically assume the ultimate position; and from their detached viewpoint there is set up a social order in which, as in the animal kingdom, both taking care of oneself and contributing to the well-being of others have their legitimate place and necessary function.

None the less, it remains that there is a sense in which egoism is always wrong and altruism its proper corrective. For man does not live exclusively either on the level of intersubjectivity or on the level of detached intelligence. On the contrary, his living is a dialectical resultant springing from those opposed but linked principles, and in the tension of that union of opposites, the root of egoism is readily to be discerned. For intelligence is a principle of universalization and of ultimate synthesis; it understands similars in the same manner; and it gives rise to further questions on each issue until all relevant data are understood. On the other hand, spontaneity is concerned with the present, the immediate, the palpable; intersubjectivity radiates from the self as from a centre, and its efficacy diminishes rapidly with distance in place or time. Egoism is neither mere spontaneity nor pure intelligence but an interference of spontaneity with the development of intelligence.

With remarkable acumen one solves one's own problems. With startling modesty one does not venture to raise the relevant further questions, Can one's solution be generalized? Is it compatible with the social order that exists? Is it compatible with any social order that proximately or even remotely is possible?

The precise nature of egoistic interference with intellectual process calls for attention. It is not to be thought that the egoist is devoid of the disinterestedness and detachment of intelligent inquiry. More than many others, he has developed a capacity to face issues squarely and to think them through. The cool schemer, the shrewd calculator, the hard-headed self-seeker are very far from indulging in mere wishful thinking. Without the detachment of intelligence, they cannot invent and implement stratagems that work. Without the disinterestedness of intelligence, they cannot raise and meet every further question that is relevant within their restricted terms of reference. Nor can one say that egoism consists in making intelligence the instrument of more elementary desires and fears. For as long as the egoist is engaged upon his problems, the immanent norms of intelligent inquiry overrule any interference from desire or fear; and while the egoist refuses to put the still further questions that would lead to a profound modification of his solution, still that refusal does not make intelligence an instrument but merely brushes it aside.

Egoism, then, is an incomplete development of intelligence. It rises above a merely inherited mentality. It has the boldness to strike out and think for itself. But it fails to pivot from the initial and preliminary motivation, provided by desires and fears, to the self-abnegation involved in allowing complete free play to intelligent inquiry. Its inquiry is reinforced by spontaneous desires and fears; by the same stroke it is restrained from a consideration of any broader field.

Necessarily, such an incompleteness of development is an exclusion of correct understanding. Just as in the sciences, intelligence begins from hypotheses that prove insufficient and advances to further hypotheses that successively prove more and more satisfactory, so too in practical living it is through the cumulative process of further questions and further insights that an adequate understanding is reached. As in the sciences, so also in practical living, individuality pertains to the empirical residue, so that there is not one course of action that is intelligent when I am concerned and quite a different course when anyone else is involved. What is sauce for the goose, is sauce for the gander. But



egoistic emancipation rests on a rejection of merely proverbial wisdom yet fails to attain the development of personal intelligence that would re-establish the old sayings.

Thus, the golden rule is to do to others as you would have them do to you. One may object that common sense is never complete until the concrete situation is reached, and that no two concrete situations are identical. Still, it does not follow that the golden rule is that there is no golden rule. For the old rule did not advocate identical behaviour in significantly different situations; on the contrary, it contended that the mere interchange of individual roles would not by itself constitute a significant difference in concrete situations.

Nor is the egoist totally unaware of his self-deception. Even in the bias and scotosis of the dramatic subject, which operates preconsciously, there is a measure of self-suspicion and disquiet. In the egoist there are additional grounds for an uneasy conscience, for it is not by sheer inadvertence but also by a conscious self-orientation that he devotes his energies to sizing up the social order, ferreting out its weak points and its loop-holes, and discovering devices that give access to its rewards while evading its demands for proportionate contributions. As has been insisted already, egoism is not spontaneous, self-regarding appetite. Though it may result automatically from an incomplete development of intelligence, it does not automatically remain in that position. There have to be overcome both the drive of intelligence to raise the relevant further questions that upset egoistic solutions and, as well, the spontaneous demands of intersubjectivity which, if they lack the breadth of a purely intellectual viewpoint with its golden rule, at least are commonly broader in their regard for others than is intelligent selfishness. Hence it is that, however much the egoist may appreciate the efforts of philosophers to assure him that intelligence is instrumental, he will be aware that, in his cool calculations, intelligence is boss and that, in his refusal to consider further questions, intelligence is not made into a servant but merely ruled out of court. Again, however much he may reassure himself by praising the pragmatists, still he suffers from the realization that the pragmatic success of his scheming falls short of a justification; for prior to the criteria of truth invented by philosophers, there is the dynamic criterion of the further question immanent in intelligence itself. The egoist's uneasy conscience is his awareness of his sin against the light. Operative within him, there is the Eros of the mind, the desire and drive to understand; he knows its value, for he

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gives it free rein where his own interests are concerned; yet he also repudiates its mastery, for he will not grant serious consideration to its further relevant questions.

### 7. GROUP BIAS

As individual bias, so also group bias rests on an interference with the development of practical common sense. But while individual bias has to overcome normal intersubjective feeling, group bias finds itself supported by such feeling. Again, while individual bias leads to attitudes that conflict with ordinary common sense, group bias operates in the very genesis of common-sense views.

Basically, social groups are defined implicitly by the pattern of relations of a social order, and they are constituted by the realization of those dynamic relations. In its technological aspect the social order generates the distinctions between scientists and engineers, technicians and workers, skilled and unskilled labour. In its economic aspect, it differentiates the formation of capital from the production of consumer goods and services, distinguishes income groups by offering proportionate rewards to contributions, and organizes contributors in hierarchies of employees, foremen, supervisors, superintendents, managers, and directors. In its political aspect, it distinguishes legislative, judicial, diplomatic, and executive functions with their myriad ramifications, and it works out some system in which the various offices are to be filled and the tasks performed.

However, in the dialectic of community there is the operation not only of practical common sense but also of human intersubjectivity. If human intelligence takes the lead in developments, still its products do not function smoothly until there is effected a suitable adaptation of sensitive spontaneity. In a school, a regiment, a factory, a trade, a profession, a prison, there develops an ethos that at once subtly and flexibly provides concrete premises and norms for practical decisions. For in human affairs the decisive factor is what one can expect of the other fellow. Such expectations rest on recognized codes of behaviour; they appeal to past performance, acquired habit, reputation; they attain a maximum of precision and reliability among those frequently brought together, engaged in similar work, guided by similar motives, sharing the same prosperity or adversity. Among strangers we are at a loss what to say or do. The social order not only gathers men together in functional groups but also consolidates its gains and expedites its operations



by turning to its own ends the vast resources of human imagination and emotion, sentiment and confidence, familiarity and loyalty.

However, this formation of social groups, specifically adapted to the smooth attainment of social ends, merely tends to replace one inertial force with another. Human sensitivity is not human intelligence and, if sensitivity can be adapted to implement easily and readily one set of intelligent dictates, it has to undergo a fresh adaptation before it will cease resisting a second set of more intelligent dictates. Now social progress is a succession of changes. Each new idea gradually modifies the social situation to call forth further new ideas and bring about still further modifications. Moreover, the new ideas are practical; they are applicable to concrete situations; they occur to those engaged in the situations to which they are to be applied. However, while the practical common sense of a community may be a single whole, its parts reside separately in the minds of members of social groups, and its development occurs as each group intelligently responds to the succession of situations with which it immediately deals. Were all the responses made by pure intelligences, continuous progress might be inevitable. In fact, the responses are made by intelligences that are coupled with the ethos and the interests of groups and, while intelligence heads for change, group spontaneity does not regard all changes in the same cold light of the general good of society. Just as the individual egoist puts further questions up to a point, but desists before reaching conclusions incompatible with his egoism, so also the group is prone to have a blind spot for the insights that reveal its well-being to be excessive or its usefulness at an end.

Thus group bias leads to a bias in the generative principle of a developing social order. At a first approximation, one thinks of the course of social change as a succession of insights, courses of action, changed situations, and fresh insights. At each turn of the wheel, one has to distinguish between fresh insights that are mere bright ideas of no practical moment and, on the other hand, the fresh insights that squarely meet the demands of the concrete situation.

Group bias, however, calls for a further distinction. Truly practical insights have to be divided into operative and inoperative; both satisfy the criteria of practical intelligence; but the operative insights alone go into effect for they alone either meet with no group resistance or else find favour with groups powerful enough to overcome what resistance there is.

The bias of development involves a distortion. The advantage of one group commonly is disadvantageous to another, and so some part of the energies of all groups is diverted to the supererogatory activity of devising and implementing offensive and defensive mechanisms. Groups differ in their possession of native talent, opportunities, initiative, and resources; those in favoured circumstances find success the key to still further success; those unable to make operative the new ideas that are to their advantage fall behind in the process of social development. Society becomes stratified; its flower is far in advance of average attainment; its roots appear to be the survival of the rude achievement of a forgotten age. Classes become distinguished, not merely by social function, but also by social success; and the new differentiation finds expression not only in conceptual labels but also in deep feelings of frustration, resentment, bitterness, and hatred.

Moreover, the course of development has been twisted. The social order that has been realized does not correspond to any coherently developed set of practical ideas. It represents the fraction of practical ideas that were made operative by their conjunction with power, the mutilated remnants of once excellent schemes that issued from the mill of compromise, the otiose structures that equip groups for their offensive and defensive activities. Again, ideas are general, but the stratification of society has blocked their realization in their proper generality. Ideas possess retinues of complementary ideas that add further adjustments and improvements; but these needed complements were submitted to the sifting of group interests and to the alterations of compromise.

Still, this process of aberration creates the principles for its own reversal. When a concrete situation first yields a new idea and demands its realization, it is unlikely that the idea will occur to anyone outside the group specialized in dealing with situations of that type. But when some ideas of a coherent set have been realized, or when they are realized in a partial manner, or when their realization does not attain its proper generality, or when it is not complemented with a needed retinue of improvements and adjustments, then there is no need to call upon experts and specialists to discover whether anything has gone wrong nor even to hit upon a roughly accurate account of what can be done. The sins of group bias may be secret and almost unconscious. But what originally was a neglected possibility, in time becomes a grotesquely distorted reality. Few may grasp the initial possibilities;



but the ultimate concrete distortions are exposed to the inspection of the multitude. Nor has the bias of social development revealed the ideas that were neglected without also supplying the power that will realize them. For the bias generates unsuccessful as well as successful classes; and the sentiments of the unsuccessful can be crystallized into militant force by the crusading of a reformer or a revolutionary.

The ensuing conflict admits a variety of forms. The dominant groups may be reactionary or progressive or any mixture of the two. In so far as they are reactionary, they are out to block any correction of the effects of group bias and they employ for this purpose whatever power they possess in whatever manner they deem appropriate and effective. On the other hand, in so far as they are progressive, they make it their aim both to correct existing distortions and to find the means that will prevent their future recurrence. Now to a great extent the attitude of the dominant groups determines the attitude of the depressed groups. Reactionaries are opposed by revolutionaries. Progressives are met by liberals. In the former case the situation heads towards violence. In the latter case there is a general agreement about ends with disagreement about the pace of change and the mode and measure of its execution.

### 8. General Bias

To err is human, and common sense is very human. Besides the bias of the dramatic subject, of the individual egoist, of the member of a given class or nation, there is a further bias to which all men are prone. For men are rational animals, but full development of their animality is both more common and more rapid than a full development of their intelligence and reasonableness. A traditional view credits children of seven years of age with the attainment of an elementary reasonableness. The law regards as a minor anyone under twenty-one years of age. Experts in the field of public entertainment address themselves to a mental age of about twelve years. Still more modest is the scientific attitude that places man's attainment of knowledge in an indefinitely removed future. Nor is personal experience apt to be reassuring. If everyone has some acquaintance with the spirit of inquiry and reflection, few think of making it the effective centre of their lives; and of that few, still fewer make sufficient progress to be able to withstand other attractions and persevere in their high purpose.

The lag of intellectual development, its difficulty and its apparently

meagre returns bear in an especial manner on common sense. It is concerned with the concrete and the particular. It entertains no aspirations about reaching abstract and universal laws. It is easily led to rationalize its limitations by engendering a conviction that other forms of human knowledge are useless or doubtfully valid. Every specialist runs the risk of turning his specialty into a bias by failing to recognize and appreciate the significance of other fields. Common sense almost invariably makes that mistake; for it is incapable of analyzing itself, incapable of making the discovery that it too is a specialized development of human knowledge, incapable of coming to grasp that its peculiar danger is to extend its legitimate concern for the concrete and the immediately practical into disregard of larger issues and indifference to long-term results.

### 8.1 *The Longer Cycle*

This general bias of common sense combines with group bias to account for certain features of the distorted dialectic of community. As has been noted, at each turn of the wheel of insight, proposal, action, new situation, and fresh insight, the tendency of group bias is to exclude some fruitful ideas and to mutilate others by compromise. Now fruitful ideas are of several kinds. They may lead to technical and material improvements, to adjustments of economic arrangements, or to modifications of political structure. As one might expect, technical and material improvements are less subject to the veto of dominant groups than are changes in economic and political institutions. Again, when we shift to the second phase of the distorted dialectic, the resonant demands of the unsuccessful are for material well-being; and when the clamour goes up for economic or political change, such change is apt to be viewed simply as a necessary means for attaining more palpably beneficial ends.

Accordingly, there arises a distinction between the shorter cycle, due to group bias, and the longer cycle, originated by the general bias of common sense. The shorter cycle turns upon ideas that are neglected by dominant groups only to be championed later by depressed groups. The longer cycle is characterized by the neglect of ideas to which all groups are rendered indifferent by the general bias of common sense. Still, this account of the longer cycle is mainly negative; to grasp its nature and its implications, we must turn to fundamental notions.

Generically, the course of human history is in accord with emergent probability; it is the cumulative realization of concretely possible schemes of recurrence in accord with successive schedules of probabilities. The specific difference of human history is that among the probable possibilities is a sequence of operative insights by which men grasp possible schemes of recurrence and take the initiative in bringing about the material and social conditions that make these schemes concretely possible, probable, and actual. In this fashion man becomes for man the executor of the emergent probability of human affairs. Instead of being developed by his environment, man turns to transforming his environment in his own self-development. He remains under emergent probability, inasmuch as his insights and decisions remain probable realizations of concrete possibilities, and inasmuch as earlier insights and decisions determine later possibilities and probabilities of insight and decision. Still, this subjection to emergent probability differs from the subjection of electrons or of evolving species. For, in the first place, insight is an anticipation of possible schemes, and decision brings about the concrete conditions of their functioning instead of merely waiting for such conditions to happen; moreover, the greater man's development, the greater his dominion over circumstances and so the greater his capacity to realize possible schemes by deciding to realize their conditions. But there is also a second and profounder difference. For man can discover emergent probability; he can work out the manner in which prior insights and decisions determine the possibilities and probabilities of later insights and decisions; he can guide his present decisions in the light of their influence on future insights and decisions; finally, this control of the emergent probability of the future can be exercised not only by the individual in choosing his career and in forming his character, not only by adults in educating the younger generation, but also by mankind in its consciousness of its responsibility to the future of mankind. Just as technical, economic, and political development gives man a dominion over nature, so also the advance of knowledge creates and demands a human contribution to the control of human history.



So far from granting common sense a hegemony in practical affairs, the foregoing analysis leads to the strange conclusion that common sense has to aim at being subordinated to a human science that is concerned, to adapt a phrase from Marx, not only with knowing history but also with directing it. For common sense is unequal to the task of thinking on the level of history. It stands above the scotosis of the

dramatic subject, above the egoism of the individual, above the bias of dominant and of depressed but militant groups that realize only the ideas they see to be to their immediate advantage. But the general bias of common sense prevents it from being effective in realizing ideas, however appropriate and reasonable, that suppose a long view or that set up higher integrations or that involve the solution of intricate and disputed issues. The challenge of history is for man progressively to restrict the realm of chance or fate or destiny and progressively to enlarge the realm of conscious grasp and deliberate choice. Common sense accepts the challenge, but it does so only partially. It needs to be guided but it is incompetent to choose its guide It becomes involved in incoherent enterprises. It is subjected to disasters that no one expects, that remain unexplained even after their occurrence, that can be explained only on the level of scientific or philosophic thought, that even when explained can be prevented from recurring only by subordinating common sense to a higher specialization of human intelligence.

This is not the whole story. The general bias of common sense involves sins of refusal as well as of mere omission. Its complacent practicality easily twists to the view that, as insistent desires and contracting fears necessitate and justify the realization of ideas, so ideas without that warrant are a matter of indifference. The long view, the higher integration, the disputed theoretical issue fall outside the realm of the practical; it may or may not be too bad that they do; but there is no use worrying about the matter; nothing can be done about it; indeed, what could be done about it, probably would not be done. Now I am far from suggesting that such practical realism cannot adduce impressive arguments in its favour. Like the characters in Damon Runyon's stories, politicians and statesmen are confined to doing what they can. None the less, if we are to understand the implications of the longer cycle, we must work out the consequences of such apparently hard-headed practicality and realism.

### 8.2 *Implications of the Longer Cycle*

Already we have explained the nature of the succession of higher viewpoints that characterize the development of mathematics and of empirical science. Now we must attend to the inverse phenomenon in which each successive viewpoint is less comprehensive than its predecessor. In each stage of the historical process, the facts are the social situation produced by the practical intelligence of the previous situation. Again, in each stage, practical intelligence is engaged in grasping the concrete intelligibility and the immediate potentialities immanent in the facts. Finally, at each stage of the process, the general bias of common sense involves the disregard of timely and fruitful ideas; and this disregard not only excludes their implementation but also deprives subsequent stages both of the further ideas, to which they would give rise, and of the correction that they and their retinue would bring to the ideas that are implemented. Such is the basic scheme, and it has three consequences.



In the first place, the social situation deteriorates cumulatively. For just as progress consists in a realization of some ideas that leads to the realization of others until a whole coherent set is concretely operative, so the repeated exclusion of timely and fruitful ideas involves a cumulative departure from coherence. The objective social situation possesses the intelligibility put into it by those that brought it about. But what is put in, less and less is some part of a coherent whole that will ask for its completion, and more and more it is some arbitrary fragment that can be rounded off only by giving up the attempt to complete the other arbitrary fragments that have preceded or will follow it. In this fashion social functions and enterprises begin to conflict; some atrophy and others grow like tumours; the objective situation becomes penetrated with anomalies; it loses its power to suggest new ideas and, once they are implemented, to respond with still further and better suggestions. The dynamic of progress is replaced by sluggishness and then by stagnation. In the limit, the only discernible intelligibility in the objective facts is an equilibrium of economic pressures and a balance of national powers.

The second consequence is the mounting irrelevance of detached and disinterested intelligence. Culture retreats into an ivory tower. Religion becomes an inward affair of the heart. Philosophy glitters like a gem with endless facets and no practical purpose. For man cannot serve two masters. If one is to be true to intellectual detachment and disinterestedness, to what can be intelligently grasped and reasonably affirmed, then one seems constrained to acknowledge that the busy world of practical affairs offers little scope to one's vocation. Intelligence can easily link culture, religion, philosophy to the realm of concrete living only if the latter is intelligible. But concrete living has become the function of a complex variable; like the real component of such a function, its intelligibility is only part of the whole. Already we

have spoken of an empirical residue from which understanding always abstracts; but the general bias of common sense generates an increasingly significant residue that

(1) is immanent in the social facts,
(2) is not intelligible, yet
(3) cannot be abstracted from if one is to consider the facts as in fact they are.

Let us name this residue the social surd.

The third consequence is the surrender of detached and disinterested intelligence. There is the minor surrender on the level of common sense. It is an incomplete surrender, for common sense always finds a profoundly satisfying escape from the grim realities of daily living by turning to men of culture, to representatives of religion, to spokesmen for philosophy. Still the business of common sense is daily life. Its reality has to be faced. The insights that accumulate have to be exactly in tune with the reality to be confronted and in some measure controlled. The fragmentary and incoherent intelligibility of the objective situation sets the standard to which common-sense intelligence must conform. Nor is this conformity merely passive. Intelligence is dynamic. Just as the biased intelligence of the psychoneurotic sets up an ingenious, plausible, self-adapting resistance to the efforts of the analyst, so men of practical common sense become warped by the situation in which they live and regard as starry-eyed idealism and silly unpracticality any proposal that would lay the axe to the root of the social surd.

Besides this minor surrender on the level of common sense, there is the major surrender on the speculative level. The function of human intelligence, it is claimed, is not to set up independent norms that make thought irrelevant to fact but to study the data as they are, to grasp the intelligibility that is immanent in them, to acknowledge as principle or norm only what can be reached by generalization from the data. There follow the need and the development of a new culture, a new religion, a new philosophy; and the new differs radically from the old. The new is not apriorist, wishful thinking. It is empirical, scientific, realistic. It takes its stand on things as they are. In brief, its many excellences cover its single defect. For its rejection of the normative significance of detached and disinterested intelligence makes it radically uncritical. It possesses no standpoint from which it can distinguish between social



achievement and the social surd. It fails to grasp that an excellent method for the study of electrons is bound to prove naïve and inept in the study of man. For the data on man are largely the product of man's own thinking; and the subordination of human science to the data on man is the subordination of human science to the biased intelligence of those that produce the data. From this critical incapacity, there follow the insecurity and the instability of the new culture, religion, philosophy. Each new arrival has to keep bolstering its convictions by attacking and denouncing its predecessors. Nor is there any lack of new arrivals, for in the cumulative deterioration of the social situation there is a continuous expansion of the surd and so there is an increasing demand for further contractions of the claims of intelligence, for further dropping of old principles and norms, for closer conformity to an ever growing man-made incoherence immanent in man-made facts.

It is in this major surrender of intellectual detachment that the succession of ever less comprehensive viewpoints comes to light. The development of our western civilization, from the schools founded by Charlemagne to the universities of today, has witnessed an extraordinary flowering of human intelligence in every department of its activity. But this course of human progress has not been along a smooth and mounting curve. It has taken place through the oscillations of the shorter cycle in which social groups become factions, in which nations go to war, in which the hegemony passes from one centre to another to leave its former holders with proud memories and impotent dreams. No less does it exhibit the successive lower viewpoints of the longer cycle. The medieval synthesis through the conflict of Church and State shattered into the several religions of the reformation. The wars of religion provided the evidence that man has to live not by revelation but by reason. The disagreement of reason's representatives made it clear that, while each must follow the dictates of reason as he sees them, he also must practise the virtue of tolerance to the equally reasonable views and actions of others. The helplessness of tolerance to provide coherent solutions to social problems called forth the totalitarian who takes the narrow and complacent practicality of common sense and elevates it to the role of a complete and exclusive viewpoint. On the totalitarian view, every type of intellectual independence whether personal, cultural, scientific, philosophic, or religious, has no better basis than non-conscious myth. The time has come for the conscious myth that will secure man's total subordination to the require-

ments of reality. Reality is the economic development, the military equipment, and the political dominance of the all-inclusive State. Its ends justify all means. Its means include not merely every technique of indoctrination and propaganda, every tactic of economic and diplomatic pressure, every device for breaking down the moral conscience and exploiting the secret affects of civilized man, but also the terrorism of a political police, of prisons and torture, of concentration camps, of transported or extirpated minorities, and of total war. The succession of less comprehensive viewpoints has been a succession of adaptations of theory to practice. In the limit, practice becomes a theoretically unified whole, and theory is reduced to the status of a myth that lingers on to represent the frustrated aspirations of detached and disinterested intelligence.

### 8.3 *Alternatives of the Longer Cycle*

What is the subsequent course of the longer cycle generated by the general bias of common sense? In so far as the bias remains effective, there would seem to be only one answer. The totalitarian has uncovered a secret of power. To defeat him is not to eliminate a permanent temptation to try once more his methods. Those not subjected to the temptation by their ambitions or their needs, will be subjected to it by their fears of danger and by their insistence on self-protection. So in an uneasy peace, in the unbroken tension of a prolonged emergency, one totalitarianism calls forth another. On an earth made small by a vast human population, by limited natural resources, by rapid and easy communications, by extraordinary powers of destruction, there will arise sooner or later the moment when the unstable equilibrium will seem threatened and the gamble of war will appear the lesser risk to some of the parties involved. If the war is indecisive, the basic situation is unchanged. If it is totally destructive, the longer cycle has come to its end. If there results a single world empire, then it inherits both the objective stagnation of the social surd and the warped mentality of totalitarian practicality; but it cannot whip up the feverish energy of fear or of ambition; it has no enemy to fight; it has no intelligible goal to attain.

Common sense, on the other hand, has no use for any theoretical integration, even for the totalitarian integration of common-sense practicality. It will desert the new empire for the individual or group interests that it understands. This centrifugal tendency will be augmented by the prepossessions and prejudices, the resentments and hatreds, that have been accumulating over the ages; for every reform, every revolution, every lower viewpoint overstates both the case in its own favour and the case against those it would supersede; from each generation to the next there are transmitted not only sound ideas, but also incomplete ideas, mutilated ideas, enthusiasms, passions, bitter memories, and terrifying bogies. In this fashion, the objective social surd will be matched by a disunity of minds all warped but each in its private way. The most difficult of enterprises will have to be undertaken under the most adverse circumstances and, under the present hypothesis that the general bias of common sense remains effective, one cannot but expect the great crises that end in complete disintegration and decay.



Still, on the assumption of emergent probability, nothing is inevitable. Indeed, the essential logic of the distorted dialectic is a reversal. For dialectic rests on the concrete unity of opposed principles; the dominance of either principle results in a distortion, and the distortion both weakens the dominance and strengthens the opposed principle to restore an equilibrium. Why, then, is it that the longer cycle is so long? Why is the havoc it wreaks so deep, so extensive, so complete? The obvious answer is the difficulty of the lesson that the longer cycle has to teach. Nor are we quite without hints or clues on the nature of that lesson. On the contrary, there is a convergence of evidence for the assertion that the longer cycle is to be met, not by any idea or set of ideas on the level of technology, economics, or politics, but only by the attainment of a higher viewpoint in man's understanding and making of man.

In the first place, the general bias of common sense cannot be corrected by common sense, for the bias is abstruse and general, and common sense deals with the particular. In the second place, man can discover how present insights and decisions influence through emergent probability the occurrence of future insights and decisions; as he can make this discovery, so he can use it, not only in shaping individual biographies and educating children in the image of their parents and of the state authorities, but also in the vastly more ambitious task of directing and in some measure controlling his future history. In the third place, the longer cycle of western civilization has been drawing attention repeatedly to the notion of a practical theory of history. It was conceived in one manner or another by Vico in his *Scienza nuova*, by Hegel, and

by Marx. It has exercised a conspicuous influence on events through the liberal doctrine of automatic progress, through the Marxian doctrine of class war, through the myths of nationalist totalitarianism. In the fourth place, a remedy has to be on the level of the disease; but the disease is a succession of lower viewpoints that heads towards an ultimate nihilism; and so the remedy has to be the attainment of a higher viewpoint.

As there is evidence for the necessity of a higher viewpoint, so also there is some evidence on its nature. Inquiry and insight are facts that underlie mathematics, empirical science, and common sense. The refusal of insight is a fact that accounts for individual and group egoism, for the psychoneuroses, and for the ruin of nations and civilizations. The needed higher viewpoint is the discovery, the logical expansion and the recognition of the principle that intelligence contains its own immanent norms and that these norms are equipped with sanctions which man does not have to invent or impose. Even in the sphere of practice, the last word does not lie with common sense and its panoply of technology, economy and polity; for unless common sense can learn to overcome its bias by acknowledging and submitting to a higher principle, unless common sense can be taught to resist its perpetual temptation to adopt the easy, obvious, practical compromise, then one must expect the succession of ever less comprehensive viewpoints and in the limit the destruction of all that has been achieved.

### 8.4 *Reversal of the Longer Cycle*

What is the higher principle? Since we have not as yet discussed such notions as truth and error, right and wrong, human science and philosophy, culture and religion, our immediate answer can be no more than a series of notes.

In the first place, there is such a thing as progress and its principle is liberty. There is progress, because practical intelligence grasps ideas in data, guides activity by the ideas, and reaches fuller and more accurate ideas through the situations produced by the activity. The principle of progress is liberty, for the ideas occur to the man on the spot, their only satisfactory expression is their implementation, their only adequate correction is the emergence of further insights; on the other hand, one might as well declare openly that all new ideas are taboo, as require that they be examined, evaluated, and approved by some hierarchy of officials and bureaucrats; for members of this hierarchy possess authority and power in inverse ratio to their familiarity with the concrete situations in which the new ideas emerge; they never know whether or not the new idea will work; much less can they divine how it might be corrected or developed; and since the one thing they dread is making a mistake, they devote their energies to paper work and postpone decisions.



However, while there is progress and while its principle is liberty, there also is decline and its principle is bias. There is the minor principle of group bias which tends to generate its own corrective. There is the major principle of general bias and, though it too generates its own corrective, it does so only by confronting human intelligence with the alternative of adopting a higher viewpoint or perishing. To ignore the fact of decline was the error of the old liberal views of automatic progress. The far more confusing error of Marx was to lump together both progress and the two principles of decline under the impressive name of dialectical materialism, to grasp that the minor principle of decline would correct itself more rapidly through class war, and then to leap gaily to the sweeping conclusion that class war would accelerate progress. What, in fact, was accelerated was major decline which in Russia and Germany leaped to fairly thorough brands of totalitarianism. The basic service of the higher viewpoint will be a liberation from confusion through clear distinctions. Progress is not to be confused with decline; the corrective mechanism of the minor principle of decline is not to be thought capable of meeting the issues set by the major principle.

Secondly, as there are sciences of nature, so also there is a science of man. As the sciences of nature are empirical, so also the science of man is empirical; for science is the resultant of an accumulation of related insights, and scientific insights grasp ideas that are immanent not in what is imagined but in what is given. If the sciences of nature can be led astray by the blunder that the objective is, not the verified, but the 'out there', so also can the human sciences; but while this blunder in physics yields no more than the ineptitude of Galileo's primary qualities and Newton's true motion, it leads zealous practitioners of scientific method in the human field to rule out of court a major portion of the data and so deny the empirical principle. Durkheimian sociology and behaviourist psychology may have excuses for barring the data of consciousness, for there exist notable difficulties in determining such data; but the business of the scientist is not to allege difficulties as excuses but

to overcome them, and neither objectivity in the sense of verification nor the principle of empiricism can be advanced as reasons for ignoring the data of consciousness. Further, as mathematics has to deal not only with direct intelligibilities but also with such inverse instances as primes, surds, imaginaries, continua, and infinities, as the physicist has to employ not only the classical procedures and techniques that deal with the systematic but also the statistical procedures and techniques that take into account the non-systematic, so also human science has to be critical. It can afford to drop the nineteenth-century scientific outlook of mechanist determinism in favour of an emergent probability. It can profit by the distinction between the intelligible emergent probability of prehuman process and the intelligent emergent probability that arises in the measure that man succeeds in understanding himself and in implementing that understanding. Finally, it can be of inestimable value in aiding man to understand himself and in guiding him in the implementation of that understanding, if, and only if, it can learn to distinguish between progress and decline, between the liberty that generates progress and the bias that generates decline. In other words, human science cannot be merely empirical; it has to be critical; to reach a critical standpoint, it has to be normative. This is a tall order for human science as hitherto it has existed. But people looking for easy tasks had best renounce any ambition to be scientists; and if mathematicians and physicists can surmount their surds, the human scientist can learn to master his.

### 8.5 *Culture and Reversal*

In the third place, there is culture. The dramatic subject, as practical, originates and develops capital and technology, the economy and the state. By his intelligence he progresses, and by his bias he declines. Still, this whole unfolding of practicality constitutes no more than the setting and the incidents of the drama. Delight and suffering, laughter and tears, joy and sorrow, aspiration and frustration, achievement and failure, wit and humour, stand not within practicality but above it. Man can pause and with a smile or a forced grin ask what the drama, what he himself is about. His culture is his capacity to ask, to reflect, to reach an answer that at once satisfies his intelligence and speaks to his heart.

Now if men are to meet the challenge set by major decline and its longer cycle, it will be through their culture that they do so. Were man a pure intelligence, the products of philosophy and human science would be enough to sway him. But as the dialectic in the individual and in society reveals, man is a compound-in-tension of intelligence and intersubjectivity, and it is only through the parallel compound of a culture that his tendencies to aberration can be offset proximately and effectively.



The difficulty is, of course, that human aberration makes an uncritical culture its captive. Mario Praz in *The Romantic Agony* has found that depth psychology throws an unpleasantly penetrating light upon romanticism. Nor is the ooze of abnormality anything more than a secondary symptom, for the expanding social surd of the longer cycle is not matched by a succession of less comprehensive viewpoints without the services of a parallel series of cultural transformations. Opinions and attitudes that once were the oddity of a minority gradually spread through society to become the platitudes of politicians and journalists, the assumptions of legislators and educators, the uncontroverted nucleus of the common sense of a people. Eventually, they too become antiquated; they are regarded as the obstinacy of an old guard that will not learn; their influence is restricted to backwaters immune to the renewing force of the main current of human thought and feeling. Change succeeds change. Indiscriminately, each of the new arrivals rests upon the good it brings, upon the opposite defects of the old, and upon a closer harmony with the fact of the social surd. In the limit, culture ceases to be an independent factor that passes a detached yet effective judgment upon capital formation and technology, upon economy and polity. To justify its existence, it had to become more and more practical, more and more a factor within the technological, economic, political process, more and more a tool that served palpably useful ends. The actors in the drama of living become stage-hands; the setting is magnificent; the lighting superb; the costumes gorgeous; but there is no play.

Clearly, by becoming practical, culture renounces its one essential function and, by that renunciation, condemns practicality to ruin. The general bias of common sense has to be counterbalanced by a representative of detached intelligence that both appreciates and criticizes, that identifies the good neither with the new nor with the old, that, above all else, neither will be forced into an ivory tower of ineffectualness by the social surd nor, on the other hand, will capitulate to its absurdity.

Marx looked forward to a classless society and to the withering of the state. But as long as there will be practical intelligence, there will be technology and capital, economy and polity. There will be a division of labour and a differentiation of functions. There will be the adaptation of human intersubjectivity to that division and differentiation. There will be common decisions to be reached and to be implemented. Practical intelligence necessitates classes and states, and no dialectic can promise their permanent disappearance. What is both unnecessary and disastrous is the exaltation of the practical, the supremacy of the state, the cult of the class. What is necessary is a cosmopolis that is neither class nor state, that stands above all their claims, that cuts them down to size, that is founded on the native detachment and disinterestedness of every intelligence, that commands man's first allegiance, that implements itself primarily through that allegiance, that is too universal to be bribed, too impalpable to be forced, too effective to be ignored.

### 8.6 *Cosmopolis*

Still, what is cosmopolis? Like every other object of human intelligence, it is in the first instance an *X*, what is to be known when one understands. Like every other *X*, it possesses some known properties and aspects that lead to its fuller determination. For the present, we must be content to indicate a few of these aspects and to leave until later the task of reaching conclusions.

First, cosmopolis is not a police force. Before such a force can be organized, equipped, and applied, there is needed a notable measure of agreement among a preponderant group of men. In other words, ideas have to come first and, at best, force is instrumental. In the practical order of the economy and polity, it is possible, often enough, to perform the juggling act of using some ideas to ground the use of force in favour of others and, then, using the other ideas to ground the use of force in favour of the first. The trouble with this procedure is that there is always another juggler that believes himself expert enough to play the same game the other way by using the malcontents, held down by the first use of force, to upset the second set of ideas and, as well, using malcontents, held down by the second use of force, to upset the first set of ideas. Accordingly, if ideas are not to be merely a façade, if the reality is not to be merely a balance of power, then the use of force can be no more than residual and incidental. But cosmopolis is not concerned with the residual and incidental. It is concerned with the fundamental issue of the historical process. Its business is to prevent practicality from being short-sightedly practical and so destroying itself. The notion that cosmopolis employs a police force is just an instance of the short-sighted practicality that cosmopolis has to correct. However, I am not saying that there should not be a United Nations or a World Government; I am not saying that such political entities should not have a police force; I am saying that such political entities are not what is meant by cosmopolis. Cosmopolis is above all politics. So far from being rendered superfluous by a successful World Government, it would be all the more obviously needed to offset the tendencies of that and any other government to be short-sightedly practical.



Secondly, cosmopolis is concerned to make operative the timely and fruitful ideas that otherwise are inoperative. So far from employing power or pressure or force, it has to witness to the possibility of ideas being operative without such backing. Unless it can provide that witness, then it is useless. For at the root of the general bias of common sense and at the permanent source of the longer cycle of decline, there stands the notion that only ideas backed by some sort of force can be operative. The business of cosmopolis is to make operative the ideas that, in the light of the general bias of common sense, are inoperative. In other words, its business is to break the vicious circle of an illusion: men will not venture on ideas that they grant to be correct, because they hold that such ideas will not work unless sustained by desires or fears; and, inversely, men hold that such ideas will not work, because they will not venture on them and so have no empirical evidence that such ideas can work and would work.

Thirdly, cosmopolis is not a busybody. It is supremely practical by ignoring what is thought to be really practical. It does not waste its time and energy condemning the individual egoism that is in revolt against society and already condemned by society. It is not excited by group egoism which, in the short run, generates the principles that involve its reversal. But it is very determined to prevent dominant groups from deluding mankind by the rationalization of their sins; if the sins of dominant groups are bad enough, still the erection of their sinning into universal principles is indefinitely worse; it is the universalization of the sin by rationalization that contributes to the longer cycle of decline; it is the rationalization that cosmopolis has to ridicule, explode, destroy. Again, cosmopolis is little interested in the shifts of power between

classes and nations; it is quite aware that the dialectic sooner or later upsets the short-sighted calculations of dominant groups; and it is quite free from the nonsense that the rising star of another class or nation is going to put a different human nature in the saddle. However, while shifts of power in themselves are incidental, they commonly are accompanied by another phenomenon of quite a different character. There is the creation of myths. The old regime is depicted as monstrous; the new envisages itself as the immaculate embodiment of ideal human aspiration. Catchwords that carried the new group to power assume the status of unquestionable verities. On the band-wagon of the new vision of truth there ride the adventurers in ideas that otherwise could not attain a hearing. Inversely, ideas that merit attention are ignored unless they put on the trappings of the current fashion, unless they pretend to result from alien but commonly acceptable premises, unless they disclaim implications that are true but unwanted. It is the business of cosmopolis to prevent the formation of the screening memories by which an ascent to power hides its nastiness; it is its business to prevent the falsification of history with which the new group overstates its case; it is its business to satirize the catchwords and the claptrap and thereby to prevent the notions they express from coalescing with passions and resentments to engender obsessive nonsense for future generations; it is its business to encourage and support those that would speak the simple truth though simple truth has gone out of fashion. Unless cosmopolis undertakes this essential task, it fails in its mission. One shift of power is followed by another, and if the myths of the first survive, the myths of the second will take their stand on earlier nonsense to bring forth worse nonsense still.

Fourthly, as cosmopolis has to protect the future against the rationalization of abuses and the creation of myths, so it itself must be purged of the rationalizations and myths that became part of the human heritage before it came on the scene. If the analyst suffers from a scotoma, he will communicate it to the analysand; similarly, if cosmopolis itself suffers from the general bias of common sense in any of its manifestations, then the blind will be leading the blind and both will head for a ditch. There is needed, then, a critique of history before there can be any intelligent direction of history. There is needed an exploration of the movements, the changes, the epochs of a civilization's genesis, development, and vicissitudes. The opinions and attitudes of the present have to be traced to their origins, and the origins have to be



criticized in the light of dialectic. The liberal believer in automatic progress could praise all that survives; the Marxist could denounce all that was and praise all that would be; but anyone that recognizes the existence both of intelligence and of bias, both of progress and of decline, has to be critical and his criticism will rest on the dialectic that simply affirms the presuppositions of possible criticism.

Perhaps enough has been said on the properties and aspects of our *X*, named cosmopolis, for a synthetic view to be attempted. It is not a group denouncing other groups, it is not a super-state ruling states; it is not an organization that enrols members, nor an academy that endorses opinions, nor a court that administers a legal code. It is a withdrawal from practicality to save practicality. It is a dimension of consciousness, a heightened grasp of historical origins, a discovery of historical responsibilities. It is not something altogether new, for the Marxist has been busy activating the class-consciousness of the masses and, before him, the liberal had succeeded in indoctrinating men with the notion of progress. Still, it possesses its novelty, for it is not *simpliste*. It does not leap from a fact of development to a belief in automatic progress nor from a fact of abuse to an expectation of an apocalyptic utopia reached through an accelerated decline. It is the higher synthesis of the liberal thesis and the Marxist antithesis. It comes to minds prepared for it by these earlier views, for they have taught man to think historically. It comes at a time when the totalitarian fact and threat have refuted the liberals and discredited the Marxists. It stands on a basic analysis of the compound-in-tension that is man; it confronts problems of which men are aware; it invites the vast potentialities and pent-up energies of our time to contribute to their solution by developing an art and a literature, a theatre and a broadcasting, a journalism and a history, a school and a university, a personal depth and a public opinion, that through appreciation and criticism give men of common sense the opportunity and help they need and desire to correct the general bias of their common sense.

Finally, it would be unfair not to stress the chief characteristic of cosmopolis. It is not easy. It is not a dissemination of sweetness and light, where sweetness means sweet to me, and light means light to me. Were that so, cosmopolis would be superfluous. Every scotosis puts forth a plausible, ingenious, adaptive, untiring resistance. The general bias of common sense is no exception. It is by moving with that bias rather then against it, by differing from it slightly rather than opposing

it thoroughly, that one has the best prospect of selling books and newspapers, entertainment and education. Moreover, this is only the superficial difficulty. Beneath it lies the almost insoluble problem of setting clearly and exactly what the general bias is. It is not a culture but only a compromise that results from taking the highest common factor of an aggregate of cultures. It is not a compromise that will check and reverse the longer cycle of decline. Nor is it unbiased intelligence that yields a welter of conflicting opinions. This is the problem. So far from solving it in this chapter, we do not hope to reach a full solution in this volume. But, at least, two allies can be acknowledged. On the one hand, there is common sense and in its judgments, which as yet have not been treated, common sense tends to be profoundly sane. On the other hand, there is dialectical analysis; the refusal of insight betrays itself; the Babel of our day is the cumulative product of a series of refusals to understand; and dialectical analysis can discover and expose both the series of past refusals and the tactics of contemporary resistance to enlightenment.

## 9. CONCLUSION

It is time to end this study of common sense. In the first section of Chapter VI there was worked out the parallel between common sense and empirical science; both are developments of intelligence. Next, attention centred on the differences between empirical science which relates things to one another, and common sense which relates things to us. It was seen that the relations grasped by common sense stand between two variables; on the one hand, common sense is a development of the subject to which things are related; on the other hand, common sense effects a development in the things to which we are related. Moreover, both developments are subject to aberration; besides the progressive accumulation of related insights, there is the cumulative effect of refusing insights. In the subjective field, such refusal tends to be preconscious; it heads towards psychoneurotic conflict; it is opposite to the subject's rational judgment and deliberate choice, which, accordingly, can provide the analyst with his opportunity. In the objective field, the refusal is rationalized by a distinction between theory and practice; it heads both to social conflict and to social disintegration; it is to be opposed both by the common-sense view that practicality is for man and not man for practicality and, on a more recondite level, by the principle, implicit in dialectic, that practice succeeds in diverging



from theory by taking the short view and refusing to raise and face further relevant questions.

Our account of common sense has led us to touch on many issues, but our concern is not these issues, which function illustratively, but the fact and the nature of insight. Within the perspectives of the present work, there is no point to a full and accurate account of the fields of psychology and of sociology. The topic is insight. To exhibit its nature and its implications, one has to venture into every department in which human intelligence plays a significant role. Still that venture is essentially a limited venture. For it is enough for our purpose to show that the notion of insight is indispensable in an adequate view, that it explains both the high esteem in which common sense is commonly held and the limitations to which it is subject, that this explanation can begin from independent and apparently disparate premises and, within the larger context that they yield, succeed in hitting off the thought of the average man, the problem of his affects, and the dialectic of his history.

Further, though our topic is common sense, still it has not been the whole of common sense. Besides intelligence, there are operative in common sense both judgment and choice with their implications of truth and error and of right and wrong. These higher components of common sense will receive some attention later. The foregoing study has been concerned with common sense as an accumulation of related insights.

A final observation has to do with method. From the beginning we have been directing attention to an event that occurs within consciousness. Accordingly, our method has not been the method of empirical science, which draws its data from the field of sensible presentations. However, we have had occasion to speak of a generalized empirical method that stands to the data of consciousness as empirical method stands to the data of sense. In the present chapter, the nature of this generalized method has come to light. As applied solely to the data of consciousness, it consists in determining patterns of intelligible relations that unite the data explanatorily. Such are the biological, artistic, dramatic, and intellectual forms of experience; moreover, our previous studies of mathematical and of scientific thought would regard particular cases of the intellectual form of experience; and similar differentiations could be multiplied. However, generalized method has to be able to deal, at least comprehensively, not only with the data within a single consciousness but also with the relations between different con-

scious subjects, between conscious subjects and their milieu or environment, and between consciousness and its neural basis. From this viewpoint, dialectic stands to generalized method, as the differential equation to classical physics, or the operator equation to the more recent physics. For dialectic is a pure form with general implications; it is applicable to any concrete unfolding of linked but opposed principles that are modified cumulatively by the unfolding; it can envisage at once the conscious and the non-conscious either in a single subject or in an aggregate and succession of subjects; it is adjustable to any course of events, from an ideal line of pure progress resulting from the harmonious working of the opposed principles, to any degree of conflict, aberration, break-down, and disintegration; it constitutes a principle of integration for specialized studies that concentrate on this or that aspect of human living and it can integrate not only theoretical work but also factual reports; finally, by its distinction between insight and bias, progress and decline, it contains in a general form the combination of the empirical and the critical attitudes essential to human science.

It is perhaps unnecessary to insist that dialectic provides no more than the general form of a critical attitude. Each department has to work out its own specialized criteria, but it will be able to do so by distinguishing between the purely intellectual element in its field and, on the other hand, the inertial effects and the interference of human sensibility and human nerves. Moreover, just as our study of insight has enabled us to formulate on a basis of principle a large number of directives that already had been established through mathematical and scientific development (I am thinking of higher viewpoints, the significance of symbolism, of functions, of differential equations, of invariance, of equivalence, of probability), so we may hope that a fuller study of man's mind will provide us with further general elements relevant to determining a far more nuanced yet general critical viewpoint.

To this end, the present chapters on common sense are contributory. May we note before concluding that, while common sense relates things to us, our account of common sense relates it to its neural basis and relates aggregates and successions of instances of common sense to one another.

CHAPTER VIII

# THINGS

So far we have been dodging the question, What is a thing? Now that question must be faced. The first two sections will be devoted to determining what in general a thing is and what a thing commonly but mistakenly is supposed to be. In the third section, we tackle the problem of the differentiation of things on the generic level and from an explanatory viewpoint. In the fourth, we ask whether there are things within things. In the fifth, we extend emergent probability to include an account, not of the origin of things, but of the immanent intelligibility of their numbers, differences, distributions, concentrations, developments, and break-downs. In the sixth, we attempt an explanatory formulation of the notion of species.

## 1. THE GENERAL NOTION OF THE THING

Since the notion of a thing involves a new type of insight, we had best begin by recalling the main features of the old and now familiar type. It rested upon the presence or absence of laws governing the relations between data. Thus, experiential conjugates were reached by grasping the correlation between such terms as 'red as seen' and 'seeing red', or 'heat as felt' and 'feeling heat'. Similarly, explanatory conjugates were reached by grasping the higher and more remote correlations that link and implicitly define, say, masses or the electromagnetic field vectors On the other hand, probabilities were reached by arguing from the absence of system in the relations between data.

This attention to law and system led to a consideration of data, not in the totality of their concrete aspects, but only from some abstractive viewpoint. To employ an experiential conjugate is to prescind from all aspects of data except some single quality such as 'red' or 'hot'. To employ an explanatory conjugate is to turn attention away from all directly perceptible aspects and direct it to a non-imaginable term that can be reached only through a series of correlations of correlations of correlations. To speak of a probability is to suppose a process of reasoning that rests, not directly on what is given, nor positively on what can be under-

stood in the given, but indirectly and negatively on what follows from a lack of system in the given.

Now the notion of a thing is grounded in an insight that grasps, not relations between data, but a unity, identity, whole in data; and this unity is grasped, not by considering data from any abstractive viewpoint, but by taking them in their concrete individuality and in the totality of their aspects. For if the reader will turn his mind to any object he names a thing, he will find that object to be a unity to which belongs every aspect of every datum within the unity. Thus, the dog, Fido, is a unity and to Fido is ascribed a totality of data whether of colour or shape, sound or odour, feeling or movement. Moreover, from this grasp of unity in a concrete totality of data there follow the various characteristics of things.

Thus, things are conceived as extended in space, permanent in time, and yet subject to change. They are extended in space, inasmuch as spatially distinct data pertain to the unity at any given instant. They are permanent in time, inasmuch as temporally distinct data pertain to the same unity. They are subject to change, inasmuch as there is some difference between the aggregate of data at one instant and the aggregate of data on the same unity at another instant.

Again, things possess properties and are subject to laws and to probabilities. For the very data that, taken concretely, are understood as pertaining to a single thing, may also be taken abstractly and so may lead to a grasp of experiential conjugates, explanatory conjugates, and probabilities. Because the data are the same, there results an obvious relation between the insights and between the consequent concepts. This relation is expressed by saying that the conjugates are properties of the thing and that the probabilities regard the occurrence of changes in the thing.

Again, the same relation is involved in what is named attribution. The concrete unity embraces a totality of aspects. From various abstractive viewpoints, other notions apart from the notion of the thing are to be reached. But because the same set of aspects yields both the notion of the thing and the other notions, the latter are related to the former and the relation, considered logically, is named attribution. Thus, to say that Fido is black or that he is a nuisance, is to conceive both a unity in a totality of aspects and some aspect out of the totality and then to attribute the latter to the former.

Again, Aristotle's syllogism aimed at putting an intelligible order into



the attributes of things. In a given totality of data there is grasped a unity named the moon. In the same totality there is grasped a regular series of luminous shapes named the phases of the moon. In the regular series of phases one may grasp that the surface of the moon cannot be flat and must be spherical. Aristotle would name the moon the subject, its phases the middle term, and its sphericity the predicate. He would note that the middle term accounts for the attribution of the predicate to the subject. He would draw attention to the difference between a *causa essendi* and a *causa cognoscendi:* the phases are the reason why we know the moon is spherical; but the sphericity is the reason why the borrowed light of the sun is reflected from the moon in the regular series of shapes named phases.

Again, without the notion of the thing, there can be no notion of change. For a change is not just a newly observed datum, nor the substitution of one datum for another, nor the creation of a datum that previously did not exist. Moreover, there are no changes in the realm of abstractions, for every abstraction is eternally whatever it is defined to be. If there is change, there has to be a concrete unity of concrete data extending over some interval of time, there has to be some difference between the data at the beginning and at the end of the interval, and this difference can be only partial for otherwise there would occur not a change but an annihilation and a new creation.

As the notion of the thing is necessary for the notion of change, so also is it necessary for the continuity of scientific thought and development. For scientific development involves a succession of explanatory systems. Each of such systems serves to define implicitly a set of conjugate terms that through a series of correlations of correlations can be linked with concrete data. Still this succession of systems with their implications does not suffice to constitute scientific thought. For the systems have to be discovered in data and verified in data; they cannot be discovered and verified in any data whatever; neither can they be discovered and verified in the data which they themselves select, for then a number of incompatible systems would be equally verifiable for each would satisfy equally well the data it selected. Accordingly, scientific thought needs, not only explanatory systems, but also descriptions that determine the data which explanations must satisfy. Moreover, scientific thought needs the notion of the thing, which has as its properties both experiential and explanatory conjugates, which remains identical whether it is described or explained, which by its identity de-

mands a coherent explanation or set of explanations that is verifiable in the easily ascertainable data of the thing as described.

Thus, the thing is the basic synthetic construct of scientific thought and development. It embraces in a concrete unity a totality of spatially and temporally distinct data. It possesses as its qualities and properties the experiential conjugates that can be determined by observation. It is subject to change and variation inasmuch as its data at one time differ from its data at another. Through observations of qualities, things are classified by their sensible similarities. Through measurements of changes there are reached classical laws and statistical frequencies. Such laws and frequencies are subject to revision, and the revision is effected by showing that the earlier view does not satisfy completely the data on the thing as described. Finally, not only experiential conjugates, explanatory conjugates, and probabilities of events are verifiable; the construct of the thing is itself verifiable; for the ancient list of four elements, earth, water, fire, and air, has been rejected and the new list of the periodic table has been established on the scientific ground of hypothesis and verification; both the old list and the new are lists of kinds of things.

Further, things are said to exist. Earlier we distinguished between questions that admit the simple answers, 'Yes' and 'No', and questions that do not. It is meaningless to answer either 'Yes' or 'No' to the question, What is a thing? On the other hand, that answer is quite appropriate when one asks whether there are any things. Now existence may be defined as what is known inasmuch as an affirmative answer is given to the question, Are there things? Accordingly, existence stands to the thing, as event or occurrence stands to the conjugate. For the existence of the thing is known by verifying the notion of the thing, as the occurrence is known by verifying the conjugate. Moreover, general knowledge of things like knowledge of conjugates is reached by classical procedures; but general knowledge of existence like knowledge of occurrence is obtained through statistical laws. Thus, the definitions of chemical elements and compounds are of the classical type; but predictions of successful analysis or synthesis in nature or in the laboratory have to be based on probabilities.

May it be noted, once and for all, that regularly the foregoing meaning is to be attributed to our use of the terms, 'exist', 'existence', 'existential'? Only when the context demands it, are these words to be given the meaning they bear in existentialist philosophy.



Again, all existing things are particular, but we may think of them in general, and then we abstract from their particularity. One reaches the notion of the thing by grasping a unity in individual data; but once the notion is reached, one can think and speak both of things in general and of things of determinate kinds specified by their conjugates or properties. Moreover, from such general considerations, one can revert to the particular, and that reversal may occur in any of three manners. The simplest case arises when one reverts to a particular thing whose data here and now are given; then by a simple shift of attention one moves from 'thing' or 'things' to 'this thing' or 'these things'. The second case occurs when the particular thing, to which one reverts, does not lie in the field of observation; then a spatio-temporal frame of reference has to be invoked to provide the link between the data, given here and now, and the data relevant to the particular thing in question; through the use of such a frame of reference, one comes to think and speak of 'that thing' or 'those things'. The third case arises within the confines of fully explanatory science, which deals with things, not as related to our senses, but as related to one another. Clearly, there are data on things only inasmuch as they are related to our senses; it follows that there can be no appeal to data as long as one considers things themselves, things as explained, things as related to one another, things as equivalent for all observers inasmuch as one prescinds from all observers. None the less, we think and speak of things themselves as existing; and only particulars exist. What, then, is the ground of the individuality of the thing itself? The Aristotelian solution to this problem would be to posit a prime matter that stands to the intelligible unity or form of the thing, as data stand to insight; just as data as given are prior to all insight and so prior to all distinction and relation or unification, so prime matter is conceived as a constituent of reality that is presupposed by form and so, of itself, is not a thing nor a quantity nor a quality nor a relation nor a place nor a time nor any other positively conceivable object.

As yet, however, we cannot attempt to say what possible meaning could be assigned the phrase, 'constituent of reality.' But it is worth noting that the problem of the individuality of things themselves is neither unique nor isolated. As has been seen, when there is no possibility of observation, there is no possibility of a verifiable image; for the imagined as imagined can be verified only when what is imagined also can be sensed. Accordingly, there are no verifiable images for sub-

atomic elements. But if subatomic elements cannot be imagined, then atoms cannot be imagined, for one cannot imagine a whole as made up of non-imaginable parts. It follows that no thing itself, no thing as explained, can be imagined. If atoms cannot be imagined, then by parity of reasoning, molecules cannot be imagined. If molecules cannot be imagined, then neither can cells. If cells cannot be imagined, then neither can plants. Once one enters upon the way of explanation by relating things to one another, one has stepped out of the path that yields valid representative images. No doubt, I can imagine the plant as seen, as related to my senses, as described. But if I apply the full principle of equivalence and prescind from all observers, then I also prescind from all observables. As the electron, so also the tree, in so far as it is considered as a thing itself, stands within a pattern of intelligible relations and offers no foothold for imagination. The difference between the tree and the electron is simply that the tree, besides being explained, also can be observed and described, while the electron, though it can be explained, cannot be directly observed and can be described adequately only in terms of observables that involve other things as well. For the present, however, we must be content to note that the thing itself sets problems which, as yet, we are not prepared to tackle.

## 2. BODIES

The name, thing, has been employed in a very precise meaning. It denotes a unity, identity, whole; initially it is grasped in data as individual; inasmuch as it unifies spatially and temporally distinct data, it is extended and permanent; inasmuch as the data it unifies also are understood through laws, conjugates become its properties and probabilities govern its changes; finally, things exist and only particulars exist, though the particularity and, indeed, the reality of things themselves give rise to disconcerting problems.

Now there may be men that employ the name, body, in exactly the same meaning as we have assigned to the name, thing. But men are not pure intelligences. They are animals; they live largely under the influence of their intersubjectivity; they are guided by a common sense that does not bother to ask nice questions on the meaning of familiar names. Accordingly, it would not be rash to suspect that their usage of the name, thing, does not quite coincide with the account we have given; and it is to follow up this suspicion that in the present section we turn our attention to the notion of a body or, rather, of a 'body', where the



quotation marks denote some divergence from the notion to be reached by intelligence and reasonableness.

To begin from a clear-cut instance, in which there is no need to suppose either intelligence or reasonableness, let us consider a kitten. It is awake and its stream of consciousness flows in the biological pattern. Such consciousness is a higher technique for attaining biological ends. It may be described as orientated toward such ends and as anticipating means to the ends. Moreover, the means lie in external situations, and so the anticipation is extroverted. The kitten's consciousness is directed outwards towards possible opportunities to satisfy appetites. This extroversion is spatial: as it is by the spatial manoeuvres of moving its head and limbs that the kitten deals with means to its end, so the means also must be spatial, for otherwise spatial manoeuvres would be inept and useless. The extroversion is also temporal: present data are distinct from the memories that enrich them; they are no less distinct from the imagined courses of future action to which they lead. Finally, the extroversion is concerned with the 'real': a realistic painting of a saucer of milk might attract a kitten's attention, make it investigate, sniff, perhaps try to lap; but it could not lead to lapping and, still less, to feeling replete; for the kitten, painted milk is not real.

Let us now characterize a 'body' as an 'already out there now real'. 'Already' refers to the orientation and dynamic anticipation of biological consciousness; such consciousness does not create but finds its environment; it finds it as already constituted, already offering opportunities, already issuing challenges. 'Out' refers to the extroversion of a consciousness that is aware, not of its own ground, but of objects distinct from itself. 'There' and 'now' indicate the spatial and temporal determinations of extroverted consciousness. 'Real', finally, is a subdivision within the field of the 'already out there now': part of that is mere appearance; but part is real; and its reality consists in its relevance to biological success or failure, pleasure or pain.

As the reader will have surmised, the terms, 'body', 'already', 'out', 'there', 'now', 'real', stand for concepts uttered by an intelligence that is grasping, not intelligent procedure, but a merely biological and non-intelligent response to stimulus. In other words, the point to the preceding paragraphs is not to suggest that a kitten can understand and describe its spontaneity but, on the contrary, to indicate through human concepts the elements in a non-conceptual 'knowing'.

Again, as the reader once more will have surmised, our interest in

kittens is rather limited. For the point we wish to make is that not a few men mean by 'thing' or 'body', not simply an intelligible unity grasped in data as individual, but also an 'already out there now real' which is as accessible to human animals as to kittens. When Galileo pronounced secondary qualities to be merely subjective, he meant that they were not 'already out there now real'. When the decadent Aristotelians and, generally, people that rely on good common sense, insist that secondary qualities obviously are objective, they mean that they are 'already out there now real'. When Descartes maintained that material substance must be identical with spatial extension, his material substance was the 'already out there now real'. When Kant argued that primary and secondary qualities are merely phenomenal, he meant that for him the reality of the 'already out there now real' was mere appearance. Our own position, as contained in the canon of parsimony, was that the real is the verified; it is what is to be known by the knowing constituted by experience and inquiry, insight and hypothesis, reflection and verification. Our present point is that, besides knowing in that rather complex sense, there is also 'knowing' in the elementary sense in which kittens know the 'reality' of milk.

It is not difficult to set forth the differences between the two types of knowing. The elementary type is constituted completely on the level of experience; neither questions for intelligence nor questions for reflection have any part in its genesis; and as questions do not give rise to it, neither can they undo it; essentially, it is unquestionable. On the other hand, in fully human knowing experience supplies no more than materials for questions; questions are essential to its genesis; through questions for intelligence it moves to accumulations of related insights which are expressed or formulated in concepts, suppositions, definitions, postulates, hypotheses, theories; through questions for reflection it attains a further component which hitherto has been referred to as verification and presently will have to be examined more closely in a series of chapters on judgment, its suppositions, and its implications.

Both types of knowing possess their validity. One cannot claim that one is concerned with mere appearance while the other is concerned with reality. For elementary knowing vindicates its validity by the survival, not to mention the evolution, of animal species. On the other hand, any attempt to dispute the validity of fully human knowing involves the use of that knowing and so, if the attempt is not to be frustrated by its own assumptions, it must presuppose that validity.



The problem set by the two types of knowing is, then, not a problem of elimination but a problem of critical distinction. For the difficulty lies, not in either type of knowing by itself, but in the confusion that arises when one shifts unconsciously from one type to the other. Animals have no epistemological problems. Neither do scientists as long as they stick to their task of observing, forming hypotheses, and verifying. The perennial source of nonsense is that, after the scientist has verified his hypothesis, he is likely to go a little further and tell the layman what, approximately, scientific reality looks like! Already, we have attacked the unverifiable image; but now we can see the origin of the strange urge to foist upon mankind unverifiable images. For both the scientist and the layman, besides being intelligent and reasonable, also are animals. To them as animals, a verified hypothesis is just a jumble of words or symbols. What they want is an elementary knowing of the 'really real', if not through sense, at least by imagination.

As is apparent, we are back at the notion of dialectic. There are two types of knowing. Each is modified by its own development. They are opposed, for one arises through intelligent and reasonable questions and answers, and the other does not. They are linked together in man who, at once, is an animal, intelligent, and reasonable. Unless they are distinguished sharply by a critical theory of knowledge, they become confused to generate aberrations that afflict not only scientific thought but far more conspicuously the thought of philosophers. Further development of this point must be left to the chapter on the *Method of Metaphysics* but, perhaps, enough has been said to justify the following conclusions:

(1) By a thing is meant an intelligible, concrete unity. As differentiated by experiential conjugates and common-sense expectations, it is a thing for us, a thing as described. As differentiated by explanatory conjugates and scientifically determined probabilities, it is a thing itself, a thing as explained.

(2) The notion of thing satisfies the canon of parsimony. For it adds to data only what is grasped by intelligence and reasonably is affirmed. Indeed, not only does it satisfy the canon of parsimony but it seems necessary to scientific thought, both because it is presupposed by the necessary notion of change, and because the scientist has to possess a construct that combines both descriptive and explanatory knowledge.

(3) By a 'body' is meant primarily a focal point of extroverted biological anticipation and attention. It is an 'already out there now real', where these terms have their meaning fixed solely by elements within sensitive experience and so without any use of intelligent and reasonable questions and answers.

(4) By a 'body' is meant secondarily any confusion or mixture of elements taken both from the notion of a thing and from the notion of a 'body' in its primary meaning.

(5) As Newton and Kant, so we also speak of things themselves. But for us the thing itself has the meaning defined above. For Newton it seems to have been a 'body'. For Kant it also seems to have been a 'body' though with the difference that it was inaccessible to scientific knowledge.

(6) Ernst Cassirer's work, *Substance and Function*, contains a polemic against the notion of the thing. I would say that his strictures are valid against the notion of 'body' but would claim his argument to be inefficacious against the notion of thing. It is true that the development of explanatory science tends to eliminate the notion of 'body'; on the other hand, if explanatory science were to eliminate the notion of thing, it would cut its communications with the data in which it has to be discovered and verified.*

### 3. GENUS AS EXPLANATORY

Mechanist determinism is bound to conceive all things as of a single kind. For mechanism posits things as instances of the 'already out there now real'. Determinism makes every event completely determined by laws of the classical type. And the combination of the two views leaves no room for a succession of ever higher systems; for mechanism would require the higher component to be a 'body', and determinism would exclude the possibility of the higher component modifying lower activities.

On the other hand, the notion of the thing as an intelligible, concrete unity differentiated by experiential and explanatory conjugates, clearly implies the possibility of different kinds of things. Moreover, since explanatory conjugates are defined by their relations to one another, there is the possibility of distinct sets of such conjugates. There follows the notion of the explanatory genus. Consider a genus of things, $T_i$, with explanatory conjugates, $C_i$, and a second genus of things, $T_j$, with explanatory conjugates, $C_i$ and $C_j$, such that all conjugates of the type, $C_i$, are defined by their relations to one another and, similarly, all conjugates of the type $C_j$, are defined by their relations to one another. Then, since $C_i$ and $C_j$ differ, there will be two different systems of terms and relations; as the basic terms and relations differ, all logically derived terms and relations will differ, so that by logical operations alone there is no transition from one system to the other.

Now it seems that such explanatory genera exist. The laws of physics hold for subatomic elements; the laws of physics and chemistry hold for chemical elements and compounds; the laws of physics, chemistry, and biology hold for plants; the laws of physics, chemistry, biology, and sensitive psychology hold for animals; the laws of physics, chemistry, biology, sensitive psychology, and rational psychology hold for men. As one moves from one genus to the next, there is added a new set of laws which defines its own basic terms by its own empirically established correlations. When one turns from physics and chemistry to astronomy, one employs the same basic terms and correlations; but when one turns from physics and chemistry to biology, one is confronted with an entirely new set of basic concepts and laws.

No doubt, a mechanist would have to claim that biology does not differ essentially from astronomy. He would argue that biology introduces its special terms and laws merely as a matter of convenience, that biology deals not with a new genus of things but with extremely complex macroscopic products of the same old things. Already we have stated the case against mechanism and determinism, and so we have only to indicate how the possibility of new genera arises.

Consider, then, a genus of things, $T_i$, with explanatory conjugates, $C_i$, and a consequent list of possible schemes of recurrence, $S_i$. Suppose there occurs an aggregate of events, $E_{ij}$, that is merely coincidental

* There exists a parallel duality and ambiguity in the notion of oneself. Corresponding to 'body' there is 'my body'. Corresponding to things that are understood and verified, there is the intelligently and rationally conscious subject to be considered in Chapter XI. According to H. S. Sullivan (*The Interpersonal Theory of Psychiatry*. New York 1953. Pp. 136–41), the notion of 'my body' has its origins in the infantile activity of thumb sucking. This activity meets a need, since the energy available for taking in food exceeds the need for food. It meets a need in an exceptional manner: both mouth and thumb are both sensing and sensed. Finally, it occurs at the infant's own behest without the labour of crying for mother's help. In such a fashion, clearly, there can arise an empirical consciousness of a centre of power and self-satisfaction. No less clearly, the empirically conscious self is just as intractable within a field theory of interpersonal relations as the old-style atom within modern physical theory; and so, as Cassirer attacked the notion of thing, Sullivan attacks the delusion of unique individuality. Both views have the same merit and, I suggest, the same defect.

when considered in the light of the laws of the things, $T_i$, and of all their possible schemes of recurrence, $S_i$. Then, if the aggregate of events, $E_{ij}$, occurs regularly, it is necessary to advance to the higher viewpoint of some genus of things, $T_j$, with conjugates, $C_i$ and $C_j$, and with schemes of recurrence, $S_j$. The lower viewpoint is insufficient for it has to regard as merely coincidental what in fact is regular. The higher viewpoint is justified, for the conjugates, $C_j$, and the schemes, $S_j$, constitute a higher system that makes regular what otherwise would be merely coincidental.

Accordingly, if the laws of subatomic elements have to regard the regular behaviour of atoms as mere patterns of happy coincidences, then there is an autonomous science of chemistry. If the laws of chemistry have to regard the metabolism and division of cells as mere patterns of happy coincidences, then there is an autonomous science of biology. If the laws of biology have to regard the behaviour of animals as mere patterns of happy coincidences, then there is an autonomous science of sensitive psychology. If the laws of sensitive psychology have to regard the operations of mathematicians and scientists as mere patterns of happy coincidences, then there is an autonomous science of rational psychology. Nor does the introduction of the higher autonomous science interfere with the autonomy of the lower; for the higher enters into the field of the lower only in so far as it makes systematic on the lower level what otherwise would be merely coincidental.

As has been remarked, the succession of sciences, corresponding to the succession of higher genera, does not admit any purely logical transition. Each of these main departments has its own basic terms defined implicitly by its own empirically established correlations. Still, this negation of a logical transition must not be interpreted as a negation of any transition whatever. For logical operations are confined to the field of concepts and definitions, hypotheses and theories, affirmations and negations. This field is only part of the larger domain that includes as well sensitive presentations and imaginative representations, inquiry and insight, reflection and critical understanding. Within this larger domain, the successive departments of science are related, for the laws of the lower order yield images in which insight grasps clues to laws of the higher order. In this fashion, the Bohr model of the atom is an image that is based on subatomic physics yet leads to insights into the nature of atoms. Again, the chemistry of the cell can yield an image of catalytic process in which insight can grasp biological laws. Again,



an image of the eye, optic nerve, and cerebrum can lead to insights that grasp properties of the psychic event, seeing, and so the oculist can make one see better and, more generally, the surgeon can make one feel better. Finally, it is with respect to sensed and imagined objects that the higher level of inquiry, insight, reflection, and judgment function.

This linking of the main departments of science runs parallel to the notion of successive higher viewpoints outlined in our first chapter. Just as elementary arithmetic and elementary algebra are distinct systems with different rules yielding different operations and different operations yielding different numbers, so the main departments of science are distinct systems without logical transitions from one to the other. Just as the image of 'doing arithmetic' leads to the insights that ground algebra, so images based on the lower science lead to insights that ground elements of the higher science. Finally, it is because new insights intervene that the higher science is essentially different from the lower.

Naturally, the reader will be inclined to view these images as pictures of reality. In this fashion, intelligence is reduced to a pattern of sensations; sensation is reduced to a neural pattern; neural patterns are reduced to chemical processes; and chemical processes to subatomic movements. The force of this reductionism, however, is proportionate to the tendency to conceive the real as a subdivision of the 'already out there now'. When that tendency is rejected, reductionism vanishes. The real becomes the verified, and one can argue in the opposite direction that, since there is no verifiable image of the subatomic, there can be no verifiable image of objects composed of subatomic elements. The verifiably imagined is restricted to the sensibly given. One has to be content with reasonable affirmations of intelligently conceived terms and relations. On that showing, the function of the transition images is simply heuristic; such images represent, perhaps only symbolically, the coincidental manifold that becomes systematic when subsumed within the higher genus.

To conclude, let us remark that we have been concerned merely to reveal the possibility of genera of things and their compatibility with the sciences as they exist. A much longer investigation would be needed to prove that, in fact, there are such genera. We are convinced that the longer inquiry can be omitted safely enough, for the contention that things are all of one kind has rested, not on concrete evidence, but on mechanist assumption.

### 4. THINGS WITHIN THINGS

Once things are recognized to be of different kinds, there arises the obvious question whether there are things within things Are electrons things within atoms, atoms things within compounds, compounds things within cells, cells things within animals, animals things within men?

The difficulty against an affirmative answer is that the thing is an intelligible unity grasped in some totality of data. It follows that if any datum pertains to a thing, every aspect of the datum pertains to that thing. Hence, no datum can pertain to two or more things, for if in all its aspects it pertains to one thing, there is no respect in which it can pertain to any other.

The difficulty against a negative answer is that the laws of the lower science can be verified in things pertaining to a higher genus. If the laws of the electron are observed in the atom, it would seem that electrons exist, not only in a free state, but also within atoms. If the laws of the chemical compound are observed within the living cell, it would seem that chemical compounds exist, not only in their free state, but also within cells.

Strangely, it is the argument against a negative answer that has the weak point. The fact that the laws of the lower order are verified in the higher genus proves that the conjugates of the lower order exist in things of the higher genus. But it is one thing to prove that conjugates of the lower order survive within the higher genus; it is quite another to prove that things defined solely by the lower conjugates also survive. To arrive at conjugates, abstractive procedures are normal; one considers events under some aspects and disregards other aspects of the same events. But to arrive at a thing, one must consider all data within a totality and one must take into account all their aspects. It follows that one cannot consider the aggregate of events, $E_{ij}$, in so far as they satisfy the laws of the lower order, and then conclude to the existence of things of the lower order, For this would be to abstract from the aspect of the aggregate that cannot be accounted for on the lower viewpoint and that justifies the introduction of the higher viewpoint and the higher genus. Accordingly, if there is evidence for the existence of the higher genus, there cannot be evidence for things of lower genera in the same data.

Naturally enough, the reader will be inclined to ask what happens to



the things of the lower order. But, perhaps, a moment's reflection will recall that there is quite a difference between things and 'bodies'. If the objects of the lower order were 'bodies', then it would be mere mystification to claim that they do not exist within higher genera. Our claim does not regard alleged 'bodies'. It is the simple statement of fact that in an object of a higher order, there is an intelligible, concrete unity differentiated by conjugates of both the lower and the higher order, but there is no further, intelligible, concrete unity to be discerned in the same data and to be differentiated solely by conjugates of some lower order. In other words, just as the real is what is to be known by verified hypothesis, so also change is what is to be known through correct, successive, and opposed affirmations.

### 5. THINGS AND EMERGENT PROBABILITY

Our account of the objective implications of the use of both classical and statistical procedures was cast in the form of a world view. There now arises the question, previously omitted, whether there is an emergent probability of things as well as of schemes of recurrence. Our answer will consist in a discussion of the suppositions or postulates of an affirmative answer.

A first, logical postulate will be that, if there exist conjugates, $C_j$, of a higher order, then there will exist things, $T_j$, of the same higher order. This postulate is named logical because it follows necessarily from our account of the notion of a thing. For the evidence for the conjugates, $C_j$, will be found in concrete data; in the same data there will be evidence for some thing that is to be differentiated by the conjugates verified in the same data; hence there cannot be conjugates of a higher order without things of the same order.

A second, probability postulate will be that, if there exist things, $T_t$, differentiated by conjugates, $C_t$, and functioning in schemes, $S_t$, then there exists the possibility and, as well, there will be some probability of a non-systematic occurrence of the aggregate of events, $E_{ij}$, that would occur regularly only if things of a higher order existed. There exists this possibility, for none of the events in the aggregate exceeds the capacity of the things, $T_t$. There exists some probability for an isolated occurrence of each of the events in the aggregate, for each is concretely possible. From the theory of probability it follows necessarily that there will be some probability for a non-systematic occurrence of the combination of all the events in the aggregate.

A third, evolutionary postulate will be that if non-systematically there occur suitable aggregates of events, $E_{i+1}$, then there will emerge conjugates, $C_j$, of a higher order to make the recurrence of the aggregates systematic. By the first, logical postulate, there will follow the existence of things, $T_j$, of the higher order. By emergent probability there will arise schemes of recurrence, $S_j$, that depend upon the classical laws that define the new conjugates, $C_j$.

It must be noted that this evolutionary postulate is to be understood within the limits of possible empirical science. It states what happens on the fulfilment of determinate conditions. It is relevant to an understanding of the generic, immanent intelligibility of the order of this universe. It is relevant only to an account of such immanent intelligibility; as empirical science, it prescinds from efficient, instrumental, and final causes, which refer to distinct types of intelligibility and lie beyond the qualifications of empirical method either to affirm or to deny.

Further, it may be observed that the evolutionary postulate, as stated, is equivalent to the old axiom, *Materiae dispositae advenit forma*. In the postulate and in the old axiom, there are involved exactly the same components, namely, a lower order of things, the occurrence of a suitable disposition in the lower order, and the emergence of a component that pertains to a higher order. It follows that the evidence for the axiom, which consists in certain obvious facts of transformation, generation, and nutrition, is also evidence for the postulate. Finally, while there are differences between the context of the axiom and the context of the postulate, these differences do not appear to be significant. For the context of the axiom involves efficient and final causes to which we can attend in due course, and the context of the postulate involves probabilities whose scientific import was not grasped until recently.

The fourth, sequential postulate would effect the extension of emergent probability to things. It affirms the possibility of a conditioned series of both things and schemes of recurrence realized cumulatively in accord with successive schedules of probabilities. Thus, the sequential postulate presupposes the other three; it adds an affirmation of the possibility of applying the other three postulates over and over so that one could begin from the simplest things and proceed to the most complex. On the other hand, the sequential postulate affirms no more than a possibility. It does not claim that human science has reached the stage of complete and definitive knowledge that would be necessary to state fully the total sequence of emerging things and schemes. Accordingly, the sequential postulate is methodological; it is not some hypothesis of empirical science but rather an assumption that can generate an almost endless stream of hypotheses; it is not a scientific theory that can be verified or refuted, for it is far too general to be tested in that fashion; it is an approach, a heuristic assumption, that can be worked out in an enormous number of different manners and that can be tested empirically only through such specific determinations and applications.



It follows that the validity of the sequential postulate rests simply on the validity of inquiring intelligence. Just as we endeavour to understand smaller aggregates of data, so also we seek the intelligibility immanent in the universe of data. Just as the rejection of all inquiry is a total obscurantism, so the rejection of this or that inquiry is a partial obscurantism. For all data are equally data; all are materials for understanding; and as it is impossible to exclude all understanding, so it is incoherent to attempt insight in some cases and to refuse to attempt it in others that do not significantly differ. Now if there is to be known an intelligibility immanent in the universe of data, then it will regard things no less than events and schemes of recurrence, for things are to be grasped in data; their numbers and differentiation, their distribution and concentrations, their emergence and survival, give rise to questions that require an answer. One does not escape that requirement by appealing to divine wisdom and divine providence, for that appeal reinforces the rejection of obscurantism and provides another argument for affirming an intelligible order immanent in the visible universe. Nor can a satisfactory answer be given by the necessity of determinists, for statistical residues are a fact, or by the chance of indeterminists, for chance is a residual defect of intelligibility, or by the eternally recurrent cycles of the Aristotelians, for these cycles are based on a mistaken over-estimate of the influence of the celestial spheres. In a word, the sequential postulate seems to stand without a serious competitor in the field.

Four postulates have been outlined. Together they effect the extension of emergent probability, so that it regards the differentiation, numbers, distribution, development, survival, and disintegration of things as well as of schemes of recurrence. Moreover, the extended affirmation, no less than the original, is generic and methodological. It rests on the principle that data are to be understood, that understanding grasps concrete unities, systematic relations, and non-systematic probabilities of existence and occurrence. It affirms that inquiry moves in a determinate direction and that this direction implies an emergent pro-

bability of things and schemes. At that point it stops, for it leaves to those competent in specialized departments the task of working out precise statements on the unfolding of generalized emergent probability.

### 6. SPECIES AS EXPLANATORY

As there are classifications based on the relations of things to our senses, so also there are classifications based on the relations of things to one another. The latter classifications are explanatory, and they imply not only explanatory genera but also explanatory species.

The key notion in the explanatory species is that any lower species of things, $T_s$, with their conjugates, $C_s$, and their schemes, $S_s$, admit a series of coincidental aggregates of events, say $E_{tsm}$, $E_{tsn}$, $E_{tso}$, . . . , which stand in correspondence with a series of conjugates, $C_{sm}$, $C_{sn}$, $C_{so}$, . . . , of a higher genus of things, $T_t$.

For example, let $T_t$ stand for the subatomic elements, $C_t$ for the terms implicitly defined by the laws governing such elements, $S_t$ for all the combinations of laws that yield schemes of recurrence for subatomic events. Then, the terms of the series, $E_{tsx}$, stand for a sequence of aggregates of subatomic events, where each aggregate is merely coincidental from the viewpoint of subatomic laws and schemes. Such coincidental aggregates can be represented by symbolic images, and in such images there are clues leading to insights that pertain to the higher viewpoint of chemistry Such insights form two levels. A first level yields the series of relations constitutive of the periodic table; these relations define implicitly the conjugates, $C_{sx}$; such conjugates both differentiate the chemical elements, which are the things, $T_s$, and stand as the higher system that makes systematic the coincidental aggregates, $E_{tsx}$. A second level yields the multitudinous series of chemical compounds, where combinations of aggregates, $E_{tsx}$, yield new and larger aggregates, $E_{tsy}$, that become systematic under the conjugates, $C_{sy}$.

Again, let $T_t$ now stand for the chemical elements and compounds, $C_t$ for the conjugates implicitly defined by their laws, $S_t$ for the schemes of recurrence that can be explained by chemical laws. Let the terms of the series, $E_{tsx}$, stand for aggregates of chemical processes, where each aggregate is merely coincidental from the chemical viewpoint. Such coincidental manifolds can be imagined symbolically, and in them there will be clues leading to insights that pertain to the higher viewpoint of biology. Again, the insights occur on two levels. Aggregates, $E_{tsx}$, vary with different kinds of cell; aggregates of aggregates, say $E_{tsy}$, vary with different kinds of multicellular living things. The things, $T_s$, are the series of biological species. They are higher systems that make systematic the coincidental aggregates, $E_{tsx}$, $E_{tsy}$. The terms defined by the relations of the higher systems are the conjugates, $C_{sx}$, $C_{sy}$, which vary with variations in the type of the aggregates of processes, $E_{tsx}$, $E_{tsy}$.



Though the same formal structure yields both the chemical and the biological species, the greater complexity of the latter necessitates their markedly dynamic characteristics. An inspection of the periodic table reveals some elements to be extremely inert, others to be highly unstable, some to possess fewer and others more numerous capacities for combination. It follows that chemical elements and compounds will not be all equally suitable for the aggregates of processes to be systematized biologically. Moreover, in a universe in which concrete events are never more than probable, the higher biological system will have the function not merely of systematizing what otherwise would be coincidental but also of extruding what has become inept and intussuscepting fresh materials. Again, the fulfilment of the twofold function will be only probable, and so there follows a third function of reproduction, of starting up a new instance of the system in fresh materials. Again, the system can shift its ground; instead of maintaining and reproducing a single cell, it can maintain and reproduce an ordered manifold of cells; and this shift involves a new dimension of growth and differentiation in the functions of the system. Thus, the biological species are a series of solutions to the problem of systematizing coincidental aggregates of chemical processes. Minor changes in the underlying aggregates yield variations within the species; major changes that are surmounted successfully yield new types of solution and so new species. The existence of a series of such major changes is the biological content of the sequential postulate of generalized emergent probability.

The third application of the key notion takes the biological organism as its lower level and animal sensitivity as its higher system. Already something has been said of the biological pattern of experience and of its correspondence with underlying neural demand functions. The higher conjugates, $C_{sx}$, now are defined implicitly by the laws of psychic stimulus and psychic response, and these conjugates make systematic otherwise merely coincidental aggregates of neural events, $E_{tsx}$. However, these neural events occur within an already constituted

nervous system which, in great part, would have no function if the higher psychic system did not exist to inform it.

In this fashion, we are confronted with a basic fact which a mechanistic viewpoint has tended to overlook and to obscure, namely, that immanent intelligibility or constitutive design increases in significance as one mounts from higher to still higher systems. The periodic table of chemical elements is dominated by atomic numbers and atomic weights that are explained by underlying subatomic entities. A first degree of freedom appears in the vast diversity of chemical compounds in which patterned aggregates of aggregates render subatomic limitations indirect. A second degree of freedom appears in the multicellular plant; each cell is an aggregate of aggregates; and the plant not only is an aggregate of cells but also it is the aggregate determined by its own laws of development and growth. A third degree of freedom appears in the animal, in which the second degree is exploited to provide the materials for the higher system of biological consciousness. In other words, because the multicellular structure is an immanently controlled aggregate of aggregates of aggregates of aggregates, there is the possibility of an organic nervous system that stands in correspondence with a still higher psychic system. Hence, while the chemical elements appear as dominated by the manifolds that they systematize, a multicellular structure is dominated by an idea that unfolds in the process of growth, and this idea can itself be subordinated to the higher idea of conscious stimulus and conscious response. While chemical compounds and unicellular entities systematize aggregates that, at least initially, are put together non-systematically, multicellular formations systematize aggregates that they themselves assemble in systematic fashion. There follows an enormous shift of emphasis and significance from the materials to be systematized to the conditioned series of things and schemes that represents possibilities of systematizing. No doubt, plants and animals cannot emerge without the initial aggregation of chemicals in their initial cell or without an environment in which there are the possible and probable schemes of recurrence in which they function. Yet the fulfilment of these necessary conditions seems to differ enormously from the developed plant or animal; and the ground of the difference is that development has its ultimate basis, not in outer conditions or events, but in the realm of intelligible possibility.

Accordingly, emergent probability has quite different implications from the gradual accumulation of small variations that is associated with the name of Darwin. The fundamental element in emergent probability is the conditioned series of things and schemes; that series is realized cumulatively in accord with successive schedules of probabilities; but a species is not conceived as an accumulated aggregate of theoretically observable variations; on the contrary, it is an intelligible solution to a problem of living in a given environment, where the living is a higher systematization of a controlled aggregation of aggregates of aggregates of aggregates, and the environment tends to be constituted more and more by other living things. This notion of the intelligibility of species differs greatly from Plato's eternal Forms or even from Aristotle's alleged transference of Forms from their noetic heaven into things. Still, it does not take the notion of species out of the realm of the intelligible and place it in some aggregation of sensible qualities. Though later species are solutions to concrete problems in concrete circumstances, though they are solutions that take into account and, as it were, rise upon previous solutions, still a solution is the sort of thing that insight hits upon and not the sort that results from accumulated, observable differences.



There is a further point to be made. An explanatory account of animal species will differentiate animals not by their organic but by their psychic differences. No doubt, there are many reasons for considering the study of animals to pertain not to psychology but to biology. In the first place, animal consciousness is not accessible to us. Secondly, an indirect study of an animal's psyche through its behaviour is difficult, for what is significant is not any instance of behaviour but the range of different modes of behaviour relative to another range of significantly different circumstances. Thirdly, an indirect study of the psyche through its neural basis is blocked by the peculiar difficulty of a correspondence that relates, not conjugates defined by a single system of laws, but distinct higher and lower systems of conjugates. Fourthly, it is far easier to describe organs and functions. Fifthly, such descriptive work may be reconciled more easily with the notion that science deals with 'bodies'. Still, science deals not with 'bodies' but with the intelligible unities of things; it describes, but it does so in order to move on towards explanation; and its business is not to follow some line of least resistance but to triumph in surmounting apparently insoluble difficulties. In brief, the alleged reasons are excuses. Against them stands a fact: the animal pertains to an explanatory genus beyond that of the plant; that explanatory genus turns on sensibility; its specific differences

are differences of sensibility; and it is in differences of sensibility that are to be found the basis for differences of organic structure, since that structure, as we have seen, possesses a degree of freedom that is limited but not controlled by underlying materials and outer circumstances.

The fourth application of the key notion brings us to man. As sensitive appetite and perception are a higher system of the organic, so inquiry and insight, reflection and judgment, deliberation and choice, are a higher system of sensitive process. The content of images provides the materials of mathematical understanding and thought, the content of sensible data provides the materials of empirical method; the tension between incompletely developed intelligence and imperfectly adapted sensibility grounds the dialectics of individual and social history.

Already we have noted the aesthetic liberation of human experience from the confinement of the biological pattern and the further practical liberation of human living that is brought about inasmuch as man grasps possible schemes of recurrence and fulfils by his own action the conditions for their realization. Now we must proceed to the root of these liberations. They rest on two facts. On the one hand, inquiry and insight are not so much a higher system as a perennial source of higher systems, so that human living has its basic task in reflecting on systems and judging them, deliberating on their implementation and choosing between possibilities. On the other hand, there can be in man a perennial source of higher systems because the materials of such systematization are not built in his constitution. For an animal to begin a new mode of living, there would be needed not only a new sensibility but also a new organism. An animal species is a solution to the problem of living, so that a new solution would be a new species; for an animal to begin to live in quite a new fashion, there would be required not only a modification of its sensibility but also a modification of the organism that the sensibility systematizes. But in man a new department of mathematics, a new viewpoint in science, a new civilization, a new philosophy, has its basis, not in a new sensibility but simply in a new manner of attending to data and of forming combinations of combinations of combinations of data. Seeing and hearing, tasting and smelling, imagining and feeling, are events with a corresponding neural basis; but inquiring and understanding have their basis, not in a neural structure, but in a structure of psychic contents. Sensation supposes sense organs; but understanding is not another type of sensation with another sense organ; it operates with respect to the content of sensation and



imagination; it represents a still further degree of freedom. A multicellular formation is an immanently directed aggregation of aggregates of aggregates of aggregates. Sensibility is a higher system of otherwise coincidental events in the immanently directed aggregation. Intelligence is the source of a sequence of systems that unify and relate otherwise coincidental aggregates of sensible contents. Just as the famous experiments on sea urchins reveal the immanent direction of the aggregation of aggregates of aggregates of aggregates, so the constructive and repressive censorship exercised preconsciously by intelligence reveals a still higher immanent direction that controls the sensible and imaginative contents that are to emerge into consciousness.

Man, then, is at once explanatory genus and explanatory species. He is explanatory genus, for he represents a higher system beyond sensibility. But that genus is coincident with species, for it is not just a higher system but a source of higher systems. In man there occurs the transition from the intelligible to the intelligent.

## 7. CONCLUDING SUMMARY

Frequently in the course of reading earlier chapters, the reader may have wondered, to the point of impatience and annoyance, why we did not begin from the simple and obvious notion of the thing. Now, perhaps, he will grant not only that that notion is neither as simple nor as obvious as it seems but also that things, since they are concrete syntheses both of the object and of the subject, cannot be treated until there are assembled the elements to be synthesized.

The basic difficulty is from the side of the subject. He is involved in a dialectical tension, and he can be made aware of the fact only after he has grasped what is meant and what is not meant by inquiry, insight, and conception as opposed to sensible data and schematic images. Accordingly, our first task was to clarify the nature of insight, and to it we devoted our first five chapters. On that foundation, we constructed, first, a pure theory of common sense and, secondly, an account of its dialectical involvement. Only then could we hope to distinguish effectively between things and 'bodies', between the intelligible unities to be grasped when one is within the intellectual pattern of experience and, on the other hand, the highly convincing instances of the 'already out there now real' that are unquestioned and unquestionable not only for animals but also for the general bias of common sense.

If that distinction has been drawn effectively, still it does not follow

that the reader will always find it convincing. For the distinction is a work of intelligence operating in the intellectual pattern of experience. No one can hope to live exclusively in that pattern. As soon as anyone moves from that pattern to the dramatic pattern of his intercourse with others or the practical pattern of his daily tasks, things as intelligible unities once more will take on for him the appearance of unreal speculation while 'bodies' or instances of the 'already out there now real' will resume the ascendency that they acquired without opposition in his infancy. Accordingly, the attainment of a critical position means not merely that one distinguishes clearly between things and 'bodies' but also that one distinguishes between the different patterns of one's own experience and refuses to commit oneself intellectually unless one is operating within the intellectual pattern of experience. Inversely, it is the failure to reach the full critical position that accounts for the endless variety of philosophic positions so rightly lamented by Kant; and it is by a dialectical analysis, based on the full critical position, that one can hope to set up a philosophy of philosophies in the fully reflective manner that at least imperfectly was initiated by Hegel and still is demanded by modern needs. But, clearly enough, these points can be developed only after we have answered questions on the nature of rational consciousness, of critical reflection, of judgment, of the notions of being and objectivity.

To revert from these high matters, which belong to later chapters, we turn from the dialectical involvement of the thing as subject to the thing as object. Things are concrete, intelligible unities. As such, all are alike. Still they are of different kinds, not merely when described in terms of their relations to us, but still more so when explained in terms of their relations to one another. For there is a succession of higher viewpoints; each is expressed in its own system of correlations and implicitly defined conjugates; and each successive system makes systematic what otherwise would be merely coincidental on the preceding viewpoint. In this fashion, one proceeds from the subatomic to the chemical, from the chemical to the biological, from the biological to the sensitive, and from the sensitive to the intelligent. Moreover, emergent probability is extended to realize cumulatively, in accord with successive schedules of probabilities, a conditioned series not only of schemes of recurrence but also of things. The conditioned series reveals not only an increasing systematization of events but also an increasing liberation of serial possibilities from limitations and restrictions imposed by previous realizations. Plants and, still more so, animals function, not in this or that scheme of recurrence, but in any of ever increasing ranges of schemes. Man invents his own schemes and produces by his labour and his conventions the conditions for their actuality. Again, there is an immanent direction in the aggregation of aggregates in the multicellular formations that is exploited by plants and animals; there is a similar immanent direction exercised by the censorship over contents to emerge into consciousness; and so, in the limiting case of man, the intelligible yields to the intelligent, and the higher system is replaced by a perennial source of higher systems.



This view of the thing is opposed by other views. The uncritical mechanist supposes that things are 'bodies' and that the unities and systems, grasped by intelligence, are merely subjective contents of merely subjective activities. No doubt, if subjectivity is simply the opposite of 'body', then what is grasped by intelligence is merely subjective. But it is not quite so clear that 'objectivity' and 'body' are convertible terms. The uncritical realist would dispute our account of explanatory genera and species; on his view the empirical scientist understands, not reality but phenomena; beyond the unities and relations, grasped by the scientist, there is a deeper reality, a metaphysical essence, apprehended by philosophic intuition. But what is this philosophic intuition? I have looked for it and failed to find it. I know no reason for affirming its occurrence, and I know no reason for refusing to identify the alleged metaphysical essence with the already quite precisely defined notion of 'body'.

Besides the uncritical mechanists and the uncritical realists, there is a variety of more or less critical positions. Before we tackle them, let us ask ourselves a rather pertinent question. All along we have been concerned with insight, with what it is to understand. But among the more conspicuous properties of understanding is its liability to incompleteness, inadequacy, error. What we have ventured to say about mathematics, empirical science, common sense, things, may be quite coherent and intelligible. Still, that is not enough. Is it correct? Are things so? Have we been offering mere airy speculations?

Our answer is threefold. With regard to what has been put forward, it is quite enough for our purpose that what has been said is coherent and intelligible; for our purpose has been to reveal the nature of insight and to indicate its basic role in human knowledge; the fact that there are other views more coherent and more intelligible as well as more satis-

factory than our own on mathematics and empirical method, on common sense and things, will not change our account of insight but confirm it. Secondly, there has been raised the second type of question, Is it so? Such are the questions, not of intelligent inquiry, but of critical reflection. It is to such questions and to the possibility of answering them that the following chapters are devoted. Thirdly, just as an account of insight is an account of method and so an account of what method cannot but yield at the term of inquiry, so also an account of critical reflection and the possibility of judgment will reveal unavoidable judgments. Those unavoidable judgments will be our answer to the question whether we are indulging in airy speculation or not.

CHAPTER IX

## THE NOTION OF JUDGMENT

A first determination of the notion of judgment is reached by relating it to propositions.

For present purposes it will suffice to distinguish

(1) utterance,
(2) sentence, and
(3) proposition,

in the following summary manner.

If you say 'The king is dead' and I say 'The king is dead', then there are two utterances but only one sentence.

If you say '*Der König ist tot*' and I say 'The king is dead', then there are two utterances and two sentences but only one proposition.

Similarly, if you write in decimal notation '2+2=4' and I write in binary notation '10+10=100', again there are two utterances and two sentences but only one proposition.

Further, it will be supposed that utterances may be spoken, written, or merely imagined, and that the imagining may be visual, auditory, or motor; again, grammarians distinguish declarative, interrogative, optative, and exclamatory sentences, but of these only the declarative corresponds to the proposition.

Now with regard to propositions there are two distinct mental attitudes; one may merely consider them; or, one may agree or disagree with them. Thus, what I write, I also affirm; but what you are reading, you may neither affirm nor deny but merely consider.

A proposition, then, may be simply an object of thought, the content of an act of conceiving, defining, thinking, supposing, considering.

But a proposition, also, may be the content of an act of judging; and then it is the content of an affirming or denying, an agreeing or disagreeing, an assenting or dissenting.

A second determination of the notion of judgment is reached by relating it to questions.

Questions fall into two main classes. There are questions for reflection, and they may be met by answering 'Yes' or 'No'. There are ques-

tions for intelligence, and they may not be met by answering 'Yes' or 'No'.

Thus, one may ask, 'Is there a logarithm of the square root of minus one?' This is a question for reflection. It is answered correctly by saying 'Yes'. On the other hand, though it would be a mistake to answer 'No', still that answer would make sense. But if one asks, 'What is the logarithm of the square root of minus one?' there is no sense in answering either 'Yes' or 'No'. The question is not for reflection but for intelligence. The only appropriate answer is to show that the square root of minus one results from raising a given base to a certain power.

Our second determination of the notion of judgment is, then, that judging is answering 'Yes' or 'No' to a question for reflection.

A third determination of the notion of judgment is that it involves a personal commitment. As de la Rochefoucauld remarked, 'Everyone complains of his memory but no one of his judgment.' One is ready to confess to a poor memory because one believes that memory is not within one's power. One is not ready to confess to poor judgment because the question for reflection can be answered not only by 'Yes' or 'No' but also by 'I don't know'; it can be answered assertorically or modally, with certitude or only probability; finally, the question as presented can be dismissed, distinctions introduced, and new questions substituted. The variety of possible answers makes full allowance for the misfortunes and shortcomings of the person answering, and by the same stroke it closes the door on possible excuses for mistakes. A judgment is the responsibility of the one that judges. It is a personal commitment.

However, just what a person is, or what responsibility is, or why the person is responsible for his judgments, are further questions that cannot be considered as yet. We now observe the fact and leave explanation to more appropriate occasions.

On the basis of the foregoing determinations we next attempt to relate judgment to the general structure of our cognitional process. We distinguish a direct and an introspective process, and in both of these we distinguish three levels: a level of presentations, a level of intelligence, and a level of reflection.

Hitherto, our inquiry has centred on the level of intelligence. It consists in acts of inquiry, understanding, and formulation. Thus, the question, 'What is it?' leads to a grasp and formulation of an intelligible unity-identity-whole in data as individual. The question, 'Why?' leads



to a grasp and formulation of a law, a correlation, a system. The question, 'How often?' leads to a grasp and formulation of an ideal frequency from which actual frequencies non-systematically diverge.

Our account of the classical and statistical phases of empirical method, of the notion of the thing, of explanatory abstraction and system, has been concerned with the level of intelligence in cognitional process.

However, this level of intelligence presupposes and complements another level. Inquiry presupposes elements in knowledge about which inquiry is made. Understanding presupposes presentations to be understood. Formulation expresses not only what is grasped by understanding but also what is essential to the understanding in the understood. This prior level was described in the chapter on common sense. It is the level of presentations. Its defining characteristic is the fact that it is presupposed and complemented by the level of intelligence, that it supplies, as it were, the raw materials on which intelligence operates, that, in a word, it is empirical, given indeed but merely given, open to understanding and formulation but by itself not understood and in itself ineffable.

Thirdly, the level of intelligence, besides presupposing and complementing an initial level, is itself presupposed and complemented by a further level of reflection.

The formulations of understanding yield concepts, definitions, objects of thought, suppositions, considerations. But man demands more. Every answer to a question for intelligence raises a further question for reflection. There is an ulterior motive to conceiving and defining, thinking and considering, forming suppositions, hypotheses, theories, systems. That motive appears when such activities are followed by the question, 'Is it so?' We conceive in order to judge. As questions for intelligence, What? and Why? and How often?, stand to insights and formulations, so questions for reflection stand to a further kind of insight and to judgment. It is on this third level that there emerge the notions of truth and falsity, of certitude and the probability that is not a frequency but a quality of judgment. It is within this third level that there is involved the personal commitment that makes one responsible for one's judgments. It is from this third level that come utterances to express one's affirming or denying, assenting or dissenting, agreeing or disagreeing.

It will be useful to represent schematically the three levels of cognitional process.

| | | | |
|---|---|---|---|
| I. | Data. Perceptual Images. | Free Images. | Utterances. |
| II. | Questions for Intelligence. | Insights. | Formulations. |
| III. | Questions for Reflection. | Reflection. | Judgment. |

The second level presupposes and complements the first. The third level presupposes and complements the second. The exception lies in free images and utterances which commonly are under the influence of the higher levels before they provide a basis for inquiry and reflection. Further, by questions for intelligence and reflection are not meant utterances or even conceptual formulations; by the question is meant the attitude of the inquiring mind that effects the transition from the first level to the second and, again, the attitude of the critical mind that effects the transition from the second level to the third. Finally the scheme is anticipatory inasmuch as the nature of reflection comes up for discussion only in the next chapter.

Now, as has been remarked, the three levels of the cognitional process operate in two modes. Data include data of sense and data of consciousness. Data of sense include colours, shapes, sounds, odours, tastes, the hard and soft, rough and smooth, hot and cold, wet and dry, and so forth. The direct mode of cognitional process begins from data of sense, advances through insights and formulations to reach reflection and judgment. Thus, empirical science pertains to the direct mode of cognitional process. On the other hand, the data of consciousness consist of acts of seeing, hearing, tasting, smelling, touching, perceiving, imagining, inquiring, understanding, formulating, reflecting, judging, and so forth. As data, such acts are experienced; but, as experienced, they are not described, distinguished, compared, related, defined, for all such activities are the work of inquiry, insight, and formulation. Finally, such formulations are, of themselves, just hypotheses; they may be accurate or inaccurate, correct or mistaken; and to pronounce upon them is the work of reflection and judgment. Thus, the three levels of the direct mode of cognitional process provide the data for the introspective mode; and as the direct mode, so also the introspective unfolds on the three levels, an initial level of data, a second level of understanding and formulation, and a third level of reflection and judgment.

The foregoing offers an analysis of cognitional process. A whole is divided into different levels; on each level different kinds of operation are distinguished and related; each level is related to the others; and two



modes of the whole process are contrasted. But analysis prepares the way for synthesis. Accordingly, we have now to ask how the various elements come together to constitute knowing. As yet, we are unprepared to answer the Kantian question that regards the constitution of the relation of knowing subject and known object. Our concern is the more elementary question of the unification of the contents of several acts into a single known content.

To this the general answer has already been indicated. Contents of different acts come together inasmuch as the earlier are incomplete without the later while the later have nothing to complete without the earlier. Questions for intelligence presuppose something to be understood, and that something is supplied by the initial level. Understanding grasps in given or imagined presentations an intelligible form emergent in the presentations. Conception formulates the grasped idea along with what is essential to the idea in the presentations. Reflection asks whether such understanding and formulation are correct. Judgment answers that they are or are not.

The cognitional process is thus a cumulative process; later steps presuppose earlier contributions and add to them. However, not all additions have the same significance. Some are merely provisional, as are free images. Some put together in a new mode the contributions of previous acts; thus, abstract formulation puts generally what insight grasps in a particular presentation. Finally, some constitute, as it were, the addition of new dimensions in the construction of the full cognitional content; and it is this addition of a new dimension that forms the basis of the distinction between the three levels of presentation, intelligence, and reflection.

From this viewpoint one may distinguish between the proper and the borrowed content of judgment.

The proper content of a judgment is its specific contribution to cognitional process. This consists in the answers 'Yes' or 'No'.

The borrowed content of a judgment is twofold. There is the direct borrowed content that is found in the question to which one answers 'Yes' or 'No'; and there is the indirect borrowed content that emerges in the reflective act linking question and answer, that claims the 'Yes' or 'No' to be true and, indeed, either certainly or only probably true.

Thus, the direct borrowed content of the judgment, 'I am writing' is the question, 'Am I writing?' The proper content of that judgment is the answer, 'Yes', 'I am'. The indirect borrowed content of the same

judgment is the implicit meaning, 'It certainly is true that I am writing.'

Again, from the same viewpoint, the judgment may be described as the total increment in cognitional process.

Every element in that process is at least a partial increment. It makes some contribution to knowing. But the judgment is the last act in the series that begins from presentations and advances through understanding and formulation ultimately to reach reflection and affirmation or denial. Thus, the proper content of judgment, the 'Yes' or 'No', is the final partial increment in the process. But this proper content is meaningless apart from the question it answers. With the question it forms an integrated whole. But the question takes over a formulation from the level of intelligence, and that formulation draws upon both insight and presentation. It follows that the judgment as a whole is a total increment in cognitional process, that it brings to a close one whole step in the development of knowledge.

Finally, there is the contextual aspect of judgment. Though single judgments bring single steps in inquiries to their conclusion, still the single steps are related to one another in a highly complex fashion.

The most general aspects of cognitional context are represented by logic and dialectic. Logic is the effort of knowledge to attain the coherence and organization proper to any stage of its development. Dialectic, on the other hand, rests on the break-down of efforts to attain coherence and organization at a given stage and consists in bringing to birth a new stage in which logic again will endeavour to attain coherence and organization.

From the viewpoint of the logical ideal, every term has one and only one precise meaning, every relation of every term to every other term is set down in an unequivocal proposition, the totality of propositions is neatly divided into primitive and derived, the derived may all be obtained by the rules of inference from a minimum number of primitive propositions, no proposition contradicts any other and, finally, the employment of the principle of excluded middle does not introduce undefined or false suppositions as does the question, 'Have you or have you not stopped beating your wife?'

Now the pursuit of the logical ideal, so far from favouring a static immobility, serves to reveal the inadequacy of any intermediate stage in the development of knowledge. The more deeply it probes, the more effectively it forces the cognitional process to undergo a radical revision of its terms and postulates and so to pursue the logical ideal



from a new base of operations. However, such revision has its limits, for there is no revision of revisers themselves. They are subject to the general conditions of beginning from presentations, advancing through insights and formulations, to terminate with reflections and judgments. Their insights are acts of grasping concrete unities, systematic regularities, or ideal frequencies. Their judgments are personal commitments to a 'Yes' or 'No'; both answers cannot be given to the same question; and, under ideal conditions, either one of the two answers has to be given. The simple fact of the uniformity of nature in revisers provides both logic and dialectic with an immutable ultimacy.

Within the general schemes of logic and dialectic, the contextual aspect of judgment appears in three manners.

There is the relation of the present to the past. Thus, past judgments remain with us. They form a habitual orientation, present and operative, but only from behind the scenes. They govern the direction of attention, evaluate insights, guide formulations, and influence the acceptance or rejection of new judgments. Previous insights remain with us. They facilitate the occurrence of fresh insights, exert their influence on new formulations, provide presuppositions that underlie new judgments whether in the same or in connected or in merely analogous fields of inquiry. Hence, when a new judgment is made, there is within us a habitual context of insights and other judgments, and it stands ready to elucidate the judgment just made, to complement it, to balance it, to draw distinctions, to add qualifications, to provide defence, to offer evidence or proof, to attempt persuasion.

Secondly, there are the relations within the present. Existing judgments may be found to conflict, and so they release the dialectical process. Again, though they do not conflict, they may not be completely independent of each other, and so they stimulate the logical effort for organized coherence.

Thirdly, there are the relations of the present to the future. The questions we answer are few compared to the questions that await an answer. Knowing is a dynamic structure. If each judgment is a total increment consisting of many parts, still it is only a minute contribution towards the whole of knowledge. But, further, our knowing is dynamic in another sense. It is irretrievably habitual. For we can make but one judgment at a time, and one judgment cannot bring all we know into the full light of actual knowing. A judgment may be very comprehensive and so bear witness to the depth and breadth of our

perspectives. It may be very concrete and so reveal our grasp of nuance and detail. But it cannot be both comprehensive and concrete. All we know is somehow with us; it is present and operative within our knowing; but it lurks behind the scenes and it reveals itself only in the exactitude with which each minor increment to our knowing is effected. The business of the human mind in this life seems to be, not contemplation of what we know, but relentless devotion to the task of adding increments to a merely habitual knowledge.

CHAPTER X

# REFLECTIVE UNDERSTANDING

Like the acts of direct and introspective understanding, the act of reflective understanding is an insight. As they meet questions for intelligence, it meets questions for reflection. As they lead to definitions and formulations, it leads to judgments. As they grasp unity, or system, or ideal frequency, it grasps the sufficiency of the evidence for a prospective judgment.

When Archimedes shouted his 'Eureka!', he was aware of a significant addition to his knowledge, but it is not likely that he would have been able to formulate explicitly just what a direct insight is. Similarly, we perform acts of reflective understanding, we know that we have grasped the sufficiency of the evidence for a judgment on which we have been deliberating, but without prolonged efforts at introspective analysis we could not say just what occurs in the reflective insight. What we know is that to pronounce judgment without that reflective grasp is merely to guess; again, what we know is that, once that grasp has occurred, then to refuse to judge is just silly.

Accordingly, the present section will be an effort to determine what precisely is meant by the sufficiency of the evidence for a prospective judgment. There is presupposed a question for reflection, 'Is it so?' There follows a judgment, 'It is so.' Between the two there is a marshalling and weighing of evidence. But what are the scales on which evidence is weighed? What weight must evidence have, if one is to pronounce a 'Yes' or a 'No'?

Unfortunately, the more complex judgments become, the more complex is the analysis of the grounding act of reflective understanding. The whole answer cannot be given at once and partial answers are incomplete. Hence, we shall begin from a very general statement and then illustrate its meaning from the form of deductive inference. Next, we shall turn to the concrete judgments of everyday life, and consider in turn concrete judgments of fact, judgments on the correctness of insights into concrete situations, and finally the occurrence of analogies and generalizations. In the third place there will be considered the judgments of empirical science, the radical difference of such judg-

ments from those of ordinary living, the nature of scientific generalization and verification, and what is meant by the probability of scientific opinions. Fourthly, analytic propositions and principles are distinguished and their criteria investigated. Fifthly, the nature of mathematical judgments is considered. Finally, we may add that philosophic judgments are not treated in this chapter, for they can be examined satisfactorily only after further elements in the problem have been set forth.

### 1. THE GENERAL FORM OF REFLECTIVE INSIGHT

To grasp evidence as sufficient for a prospective judgment is to grasp the prospective judgment as virtually unconditioned.

Distinguish then between the formally and the virtually unconditioned. The formally unconditioned has no conditions whatever. The virtually unconditioned has conditions indeed but they are fulfilled.

Accordingly, a virtually unconditioned involves three elements, namely:

(1) a conditioned,
(2) a link between the conditioned and its conditions, and
(3) the fulfilment of the conditions.

Hence, a prospective judgment will be virtually unconditioned if

(1) it is the conditioned,
(2) its conditions are known, and
(3) the conditions are fulfilled.

By the mere fact that a question for reflection has been put, the prospective judgment is a conditioned; it stands in need of evidence sufficient for reasonable pronouncement. The function of reflective understanding is to meet the question for reflection by transforming the prospective judgment from the status of a conditioned to the status of a virtually unconditioned; and reflective understanding effects this transformation by grasping the conditions of the conditioned and their fulfilment.

Such is the general scheme and we proceed to illustrate it from the form of deductive inference. Where *A* and *B* each stand for one or more propositions, the deductive form is:

If *A*, then B.
But *A*.
Therefore *B*.



For instance:

If *X* is material and alive, *X* is mortal.
But men are material and alive.
Therefore, men are mortal.

Now the conclusion is a conditioned, for an argument is needed to support it. The major premise links this conditioned to its conditions, for it affirms, If *A*, then *B*. The minor premise presents the fulfilment of the conditions, for it affirms the antecedent, *A*. The function, then, of the form of deductive inference is to exhibit a conclusion as virtually unconditioned. Reflective insight grasps the pattern, and by rational compulsion there follows the judgment.

However, deductive inference cannot be the basic case of judgment, for it presupposes other judgments to be true For that reason we have said that the form of deductive inference is merely a clear illustration of what is meant by grasping a prospective judgment as virtually unconditioned. Far more general than the form of deductive inference is the form of reflective insight itself. If there is to be a deduction, the link between the conditioned and its conditions must be a judgment, and the fulfilment of the conditions must be a further judgment. But judgments are the final products of cognitional process. Before the link between conditioned and conditions appears in the act of judgment, it existed in a more rudimentary state within cognitional process itself. Before the fulfilment of conditions appears in another act of judgment, it too was present in a more rudimentary state within cognitional process. The remarkable fact about reflective insight is that it can make use of those more rudimentary elements in cognitional process to reach the virtually unconditioned. Let us now see how this is done in various cases.

### 2. CONCRETE JUDGMENTS OF FACT

Suppose a man to return from work to his tidy home and to find the windows smashed, smoke in the air, and water on the floor. Suppose him to make the extremely restrained judgment of fact, Something happened. The question is, not whether he was right, but how he reached his affirmation.

The conditioned will be the judgment that something happened.

The fulfilling conditions will be two sets of data: the remembered data of his home as he left it in the morning; the present data of his home as he finds it in the evening. Observe that the fulfilling condi-

tions are found on the level of presentations. They are not judgments, as is the minor premise of syllogisms. They involve no questions for intelligence nor insights nor concepts. They lie simply on the level of past and present experience, of the occurrence of acts of seeing and smelling.

The link between the conditioned and the fulfilling conditions is a structure immanent and operative within cognitional process. It is not a judgment. It is not a formulated set of concepts, such as a definition. It is simply a way of doing things, a procedure within the cognitional field.

The general form of all such structures and procedures has already been outlined in terms of the three levels of presentations, intelligence, and reflection. Specializations of the general form may be exemplified by the classical and statistical phases of empirical method, by the notion of the thing, and by the differences between description and explanation. However, such accounts of the general form and its specializations pertain to introspective analysis. Prior to such an investigation and formulation, the structures and procedures exist and operate; nor, in general, do they operate any better because the analysis has been effected.

Now, in the particular instance under consideration, the weary worker not only experiences present data and recalls different data but by direct insights he refers both sets of data to the same set of things which he calls his home. The direct insight, however, fulfils a double function. Not merely are two fields of individual data referred to one identical set of things but a second level of cognitional process is added to a first. The two together contain a specific structure of that process, which we may name the notion of knowing change. Just as knowing a thing consists in grasping an intelligible unity-identity-whole in individual data, so knowing change consists in grasping the same identity or identities at different times in different individual data. If the same thing exhibits different individual data at different times, it has changed. If there occurs a change, something has happened. But these are statements. If they are affirmed, they are judgments. But prior to being either statements or judgments, they exist as unanalysed structures or procedures immanent and operative within cognitional process. It is such a structure that links the conditioned with the fulfilling conditions in the concrete judgment of fact.

The three elements have been assembled. On the level of presentations there are two sets of data. On the level of intelligence there is an insight referring both sets to the same things. When both levels are taken together, there is involved the notion of knowing change. Reflective understanding grasps all three as a virtually unconditioned to ground the judgment, Something happened.



While our illustrative instance was as simple as it could be, still it provides the model for the analysis of more complex instances of the concrete judgment of fact. The fulfilling conditions may be any combination of data from the memories of a long life, and their acquisition may have involved exceptional powers of observation. The cognitional structure may suppose the cumulative development of understanding exemplified by the man of experience, the specialist, the expert. Both complex data and a complex structure may combine to yield a virtually unconditioned that introspective analysis could hardly hope to reproduce accurately and convincingly. But the general nature of the concrete judgment of fact would remain the same as in the simple case we considered.

However, the reader is probably asking how we know whether the insights that constitute the pivot of such structures are themselves correct. To this point we have now to turn.

### 3. Insights into Concrete Situations

Direct and introspective insights arise in response to an inquiring attitude. There are data to be understood; inquiry seeks understanding; and the insight arises as the relevant understanding. But a mere bright idea is one thing, and a correct idea is another. How do we distinguish between the two?

The question is asked not in its full generality but with respect to concrete situations that diverge from our expectations and by that divergence set us a problem. Thus, to retain our former illustration, the man on returning home might have said, There has been a fire. Since there no longer was any fire, that judgment would suppose an insight that put two and two together. Our question is on what grounds such an insight could be pronounced correct.

First, then, observe that insights not only arise in answer to questions but also are followed by further questions. Observe, moreover, that such further questions are of two kinds. They may stick to the initial issue, or they may go on to raise distinct issues. What started the fire? Where is my wife? Observe, thirdly, that the transition to distinct issues may result from very different reasons; it may be because different

interests supervene to draw attention elsewhere; but it may also be because the initial issue is exhausted, because about it there are no further questions to be asked.

Let us now distinguish between vulnerable and invulnerable insights. Insights are vulnerable when there are further questions to be asked on the same issue. For the further questions lead to further insights that certainly complement the initial insight, that to a greater or less extent modify its expression and implications, that perhaps lead to an entirely new slant on the issue. But when there are no further questions, the insight is invulnerable. For it is only through further questions that there arise the further insights that complement, modify, or revise the initial approach and explanation.

Now this reveals a law immanent and operative in cognitional process. Prior to our conceptual distinction between correct and mistaken insights, there is an operational distinction between invulnerable and vulnerable insights. When an insight meets the issue squarely, when it hits the bull's eye, when it settles the matter, there are no futher questions to be asked and so there are no further insights to challenge the initial position. But when the issue is not met squarely, there are further questions that would reveal the unsatisfactoriness of the insight and would evoke the further insights that put a new light on the matter.

Such, then, is the basic element in our solution The link between the conditioned and its conditions is a law immanent and operative in cognitional process. The conditioned is the prospective judgment, This or that direct or introspective insight is correct. The immanent law of cognitional process may be formulated from our analysis Such an insight is correct, if there are no further, pertinent questions.

At once it follows that the conditions for the prospective judgment are fulfilled when there are no further, pertinent questions.

Note that it is not enough to say that the conditions are fulfilled when no further questions occur to me. The mere absence of further questions in my mind can have other causes. My intellectual curiosity may be stifled by other interests. My eagerness to satisfy other drives may refuse the further questions a chance to emerge To pass judgment in that case is to be rash, to leap before one looks.

As there is rash judgment, so also there is mere indecision. As the mere absence of further questions in my mind is not enough, so it is too much to demand that the very possibility of further questions has to be excluded. If, in fact, there are no further questions, then, in fact, the insight is invulnerable; if, in fact, the insight is invulnerable, then, in fact, the judgment approving it will be correct.



But how is one to strike this happy balance between rashness and indecision? How is one to know when it is reached? Were there some simple formula or recipe in answer to such questions, then men of good judgment could be produced at will and indefinitely. All we can attempt is an analysis of the main factors in the problem and an outline of the general nature of their solution.

In the first place, then, one has to give the further questions a chance to arise. The seed of intellectual curiosity has to grow into a rugged tree to hold its own against the desires and fears, conations and appetites, drives and interests, that inhabit the heart of man. Moreover, every insight has its retinue of presuppositions, implications, and applications. One has to take the steps needed for that retinue to come to light. The presuppositions and implications of a given insight have to knit coherently with the presuppositions and implications of other insights. Its possibilities of concrete application have to enter into the field of operations and undergo the test of success or failure. I do not mean, of course, that concrete living is to pursue this logical and operational expansion in the explicit, deliberate, and elaborate manner of the scientific investigator. But I do mean that something equivalent is to be sought by intellectual alertness, by taking one's time, by talking things over, by putting viewpoints to the test of action.

In the second place, the prior issue is to be noted. Behind the theory of correct insights, there is a theory of correct problems. It was to dodge this prior issue that we supposed a concrete situation that diverges from our expectations and by that divergence defines a problem. In other words, there has been postulated an inquirer that understands the background of the situation and so knows what is to be expected; there also has been postulated a problem that exists, that is accurately defined by the divergence of the situation from current expectations, that in turn provides a definition of the pertinence of any further questions.

Now this amounts to saying that good judgment about any insight has to rest on the previous acquisition of a large number of other, connected, and correct insights. But before attempting to break this vicious circle, let us assure ourselves of the fact of its existence. Children ask endless questions; we have no doubt about their intellectual curiosity; but so far from crediting them with good judgment, we do not suppose them to reach the age of reason before their seventh year. Young men and

women have the alertness of mind that justifies their crowding into schools and universities, but the law doubts the soundness of their judgment and regards them as minors, while Aristotle denied they had enough experience to study ethics with profit. Nor is there merely the initial difficulty of acquisition but, as well, there is the subsequent necessity of keeping in touch. The man that returns to a field of commerce or industry, to a profession or a milieu, in which once he was completely at home, may try to carry on from where he left off. But unless from mistakes and minor ineptitudes he learns to be more wary, he is merely inviting blunders and disaster. Good judgment about concrete insights presupposes the prior acquisition of an organized set of complementary insights.

In the third place, then, there is the process of learning. It is the gradual acquisition and accumulation of insights bearing on a single domain. During that process one's own judgment is in abeyance. It is being developed and formed but it has not yet reached the maturity needed for its independent exercise. For the gradual acquisition and accumulation of insights are not merely a matter of advancing in direct or introspective understanding. At the same time, intellectual curiosity is asserting itself against other desires. At the same time, the logical retinues of presuppositions and implications of each insight are being expanded either to conflict and provoke further questions or else to mesh into coherence. At the same time, operational possibilities are envisaged to be tested in thought experiments, to be contrasted with actual practice, to be executed in ventures that gradually increase in moment and scope to enlighten us by failures and to generate confidence through success.

So it is the process of learning that breaks the vicious circle. Judgment on the correctness of insights supposes the prior acquisition of a large number of correct insights. But the prior insights are not correct because we judge them to be correct. They occur within a self-correcting process in which the shortcomings of each insight provoke further questions to yield complementary insights. Moreover, this self-correcting process tends to a limit. We become familiar with concrete situations; we know what to expect; when the unexpected occurs, we can spot just what happened and why and what can be done to favour or to prevent such a recurrence; or, if the unexpected is quite novel, we know enough to recommence the process of learning and we can recognize when, once more, that self-correcting process reaches its limit in familiarity with the concrete situation and in easy mastery of it.



In the fourth place, rashness and indecision commonly have a basis in temperament. Apart from occasional outbursts, that we view as out of character, the rash man nearly always is quite sure and the indecisive man regularly is unable to make up his mind. In such cases it is not enough to point out that learning is a self-correcting process that tends to a limit or that, while the limit is not marked with a label, still its attainment is revealed by a habitual ability to know just what is up. For unless a special effort is made to cope with temperament itself, the rash man continues to presume too quickly that he has nothing more to learn, and the indecisive man continues to suspect that deeper depths of shadowy possibilities threaten to invalidate what he knows quite well.

Finally, we note that we leave to another occasion a discussion of the philosophic opinions that no one ever can be certain. Our immediate purpose is to explain the facts. Human judgments and refusals to judge oscillate about a central mean. If the precise locus of that divide can hardly be defined, at least there are many points on which even the rash would not venture to pronounce and many others on which even the indecisive would not doubt. What, then, is the general form of such certitude of ignorance and such certitude of knowledge?

Our answer is in terms of the virtually unconditioned. There occurs a reflective insight in which at once one grasps

(1) a conditioned, the prospective judgment that a given direct or introspective insight is correct,

(2) a link between the conditioned and its conditions, and this on introspective analysis proves to be that an insight is correct if it is invulnerable and it is invulnerable if there are no further, pertinent questions, and

(3) the fulfilment of the conditions, namely, that the given insight does put an end to further, pertinent questioning and that this occurs in a mind that is alert, familiar with the concrete situation, and intellectually master of it.

### 4. CONCRETE ANALOGIES AND GENERALIZATIONS

Two brief corollaries have to be drawn.

An argument from analogy assumes that some concrete situation, *A*, is correctly understood. It argues that some other similar situation, *B*, is to be understood in the same fashion.

A generalization makes the same assumption to argue that any other similar situation, *X*, is to be understood in the same fashion.

In both cases what is at work is the law, immanent and operative in cognitional process, that similars are similarly understood. Unless there is a significant difference in the data, there cannot be a difference in understanding the data. This point has already been made in discussing the heuristic procedure of the classical phase of empirical method. Clearly enough, it holds not merely for regularities, rules, laws, correlations but also for ideal frequencies and for things. A second look does not necessarily mean one is looking at a second thing. A second actual frequency does not necessarily mean that one will establish a second ideal frequency. If there is to be a second thing or a second ideal frequency an appropriate difference in the data has to be supposed.

In the simplest possible manner, then, our analysis resolves the so-called problem of induction. It makes the transition from one particular case to another or from a particular case to the general case an almost automatic procedure of intelligence. We appeal to analogies and we generalize because we cannot help understanding similars similarly. This solution, be it noted, squares with the broad fact that there is no problem of teaching men to generalize. There is a problem of teaching them to frame their generalizations accurately, indeed, the whole point of the analogy is that it absolves one from that conceptual task and the complexities it involves. There is, above all, a problem of preventing men from generalizing on insufficient grounds, and very easily such grounds are merely putative.

For if our view makes generalization an easy matter, it also clips the generalizer's wings. There must be a correct insight with respect to the basic situation. Before similars can be similarly understood, there is needed an act of understanding; and if that act is mistaken in the first instance, it will be equally mistaken in the second. But, as we have seen, to know one's insights are correct presupposes a process of learning and the attainment of familiarity and mastery. Further, the analogous or the general situation must be similar. If there is any significant dissimilarity, then further, pertinent questions arise to complement, to modify, perhaps to revise the basic insight. Finally, and this is the real catch, what differences are significant? My familiarity and mastery of the initial situation enables me to tell whether further questions there are pertinent. Another's familiarity and mastery of the analogous situation would enable him to tell whether further questions are pertinent in that situation. But unless the two situations are similar in all respects, my familiarity with one does not enable me to tell whether or not further questions arise when my insight is transferred to the other.



To conclude, analogy and generalization are essentially valid procedures. But when their basis is an insight into a concrete situation, the conditions of their proper use can become so stringent as to render them almost useless. It is this fact that grounds the suspicion with which men greet arguments from analogy and generalizations. But, at the same time, there is a compensating factor that arises from human collaboration in the process of learning. To this we have now to turn our attention.

## 5. COMMON-SENSE JUDGMENTS

Common sense is that vague name given to the unknown source of a large and floating population of elementary judgments which everyone makes, everyone relies on, and almost everyone regards as obvious and indisputable. Though some repetition will be involved, three points, I think, call for our attention:

(1) the source of these judgments,
(2) their proper object or field, and
(3) their relation to empirical science.

### 5.1 *The Source of Common-Sense Judgments*

The proximate ground and source of common-sense judgments lies in the procedures, just described, of concrete judgments of fact, judgments on the correctness of insights into concrete situations, and concrete analogies and generalizations. The remote source is more complex. One has to envisage these procedures carried out, not by isolated individuals, but by members of families, of tribes, of nations, over the face of the earth for generation after generation. One has to take into account the diffusion of judgments by communication and their transmission by tradition. Finally, one has to note that there results not merely an enlargement but also a unification and transformation of the self-correcting process of learning.

If I may repeat myself, besides the hard way of finding things out for oneself, there is the comparatively easy way of learning from others. Archimedes had to rack his brains to discover what every schoolboy can be taught. For teaching is a vast acceleration of the process of learning. It throws out the clues, the pointed hints, that lead to insights; it

cajoles attention to remove the distracting images that obstruct them; it puts the further questions that reveal the need of further insights to complement and modify and transform the acquired store; it grasps the seriation of acts of understanding to begin from the simple and work towards the more complex. But what is done explicitly and deliberately by professional teachers, also is done implicitly and unconsciously by parents with their children and by equals among themselves. Talking is a basic human art; by it each reveals what he knows and provokes from others the further questions that direct his attention to what he had overlooked. More general and more impressive than talking is doing: deeds excite our admiration and stir us to emulation; we watch to see how things are done; we experiment to see if we can do them ourselves; we watch again to discover the oversights that led to our failures. Thus it is that what anyone discovers passes into the possession of many, to be checked against their experience and to be confronted with the test of their further questions. Thus too it is that the discoveries of different individuals enter into single, cumulative series; that the later presuppose and improve upon the earlier; and that the starting-point of each generation is where its predecessor left off.

The remote source then of common-sense judgments is a collaboration. The self-correcting process of learning goes on in the minds of individuals, but the individual minds are in communication. The results reached by one are checked by many, and new results are added to old to form a common fund from which each draws his variable share measured by his interests and his energy.

There is another side to the story. It is human to err, and common-sense judgments are very human. They rest upon the self-correcting process of learning as transformed by communication and collaboration. But men share not only in intellectual curiosity but also in more earthy passions and prejudices. The mixed character of human drives can generate a common deviation from the pure product of intelligence and even a common dishonesty in refusing to acknowledge the effective pertinence of further, pertinent questions. So it is that we find each tribe and nation, each group and class, prone to develop its own brand of common sense and to strengthen its convictions by pouring ridicule upon the common nonsense of others. From the contradictory varieties of common sense, men have appealed to the common consent of the human race. But one may well doubt that such a procedure goes quite



to the root of the matter. If one must suspect the collaboration of groups and classes, of tribes and nations, it does not follow that one cannot suspect the collaboration of mankind. Error is not primarily a class product or a national product. It is human. The group or class, the tribe or nation, only gives a more specific twist to the mixed motives of human effort. Undertake to select the judgments on which all men agree, and you have no guarantee either that when all men agree, they will do so from the pure and detached motives of intelligence and reason or, indeed, that you yourself in your investigation and selection have operated exclusively from that unmixed drive.

The collaboration named common sense not only offers enormous benefits and advantages but it also intertwines them with more than a danger of deviation and aberration. Nor do we ourselves stand outside this collaboration as spectators. We were born into it. We had no choice but to become participants, to profit by its benefits, and to share in its errors. We have no choice about withdrawing from it, for the past development of one's own intellect can no more easily be blotted out than the past growth of one's body, and future development will have to take place under essentially the same conditions and limitations as that of the past. There is, then, a fundamental problem, and how it is to be met, we cannot discuss at once. Our immediate objective has to be confined to discerning the field or domain within which common sense might be expected to operate successfully. This brings us to our second topic.

### 5.2 *The Object of Common-Sense Judgments*

Already a distinction has been drawn between description and explanation. Description deals with things as related to us. Explanation deals with the same things as related among themselves. The two are not totally independent, for they deal with the same things and, as we have seen, description supplies, as it were, the tweezers by which we hold things while explanations are being discovered or verified, applied or revised. But despite their intimate connection, it remains that description and explanation envisage things in fundamentally different manners. The relations of things among themselves are, in general, a different field from the relations of things to us. There is an apparent overlapping only when we consider the relations of men among themselves; and then the different procedures of description and explanation prevent the overlapping from being more than apparent, for descrip-

tion is in terms of the given while explanation is in terms of the ultimates reached by analysis.

Not only are description and explanation distinct, but there are two main varieties of description. There are the ordinary descriptions that can be cast in ordinary language. There are also scientific descriptions for which ordinary language quickly proves inadequate and so is forced to yield its place to a special, technical terminology. Nor is it difficult to discern behind these linguistic differences a more fundamental difference. Both ordinary and scientific description are concerned with things as related to us, but both are not concerned with the same relations to us. The scientist selects the relations of things to us that lead more directly to knowledge of the relations between things themselves. Ordinary description is free from this ulterior preoccupation. As it begins, so also it ends with human apprehensions and interests as its centre.

There exists then a determinate field or domain of ordinary description. Its defining or formal viewpoint is the thing as related to us, as it enters into the concerns of man. Its object is what is to be known by concrete judgments of fact, by judgments on the correctness of insights into concrete situations, by concrete analogies and generalizations, and by the collaboration of common sense. It is as much an object of knowledge as any other, for it is reached by beginning from the level of presentations, by advancing through inquiry, insights, and formulation, by culminating in the critical inquiry of reflective understanding, the grasp of the unconditioned, and the rationally compelled pronouncement of judgment. To anticipate a later vocabulary, the domain of ordinary description is a section of the universe of being, of what intelligently is grasped and reasonably is affirmed. How much of that section really is reached by ordinary description, is of course a further question. At least, it is something to know the goal at which it aims, and that has been our restricted topic.

But before going on to our third topic, it may be well to preclude possible misconceptions. First, then, the human collaboration that results in a common sense involves belief. The analysis of belief cannot as yet be undertaken. But the type of belief that is essential in this collaboration resembles that of the pupil, who believes his teacher only that later he himself may understand and be able to judge for himself. It resembles that of the scientist who does not insist on exploring for himself all the blind alleys down which his predecessors wandered but is content to test their final results either directly by repeating experiments or, more commonly, by operating on the principle that, if those results were erroneous, the error would be revealed indirectly in the experiments he himself does perform. Hence it is that a man pronouncing a common-sense judgment is convinced that he is uttering, not what someone else told him, but what he himself knows.



Secondly, the human collaboration that results in a common sense is under the dominance of practical considerations and pragmatic sanctions. The further questions that arise and are considered pertinent do not come from any theoretical realm, and the tests that are employed move within the orbit of human success and failure. Still that dominance, so far from vitiating the results, is dictated by the object to be known, by the thing as it is related to us and as it enters into the concerns of men. It was a philosophic school that invented the notion that ideas are true because they happen to work. Despite its practicality, common sense is convinced that ideas work only if they are true. Nor is this surprising, for the practical further question is a further question that leads to the modification or revision of an insight; and the pragmatic criterion of success is the absence of the failure that would reveal the necessity of thinking things out afresh.

Thirdly, the human collaboration that results in a common sense is subject to the deviations and aberrations that have their root in the mixed motives of man. But it is only in so far as I myself share in those mixed motives that my understanding and my judgment will suffer the same bias and fall in line with the same deviations and aberrations. As long as I share in them, my efforts at correction and selection will be just as suspect as the judgments I wish to eliminate. It is only when I go to the root of the matter and become efficaciously critical of myself that I can begin to become a reliable judge, and then that becoming will consist in the self-correcting process of learning which has already been described.

### 5.3 *Common-Sense Judgment and Empirical Science*

Our third main topic was the relation of common sense to science, and our fundamental assertion is that the two regard distinct and separate fields. Common sense is concerned with things as related to us. Science is concerned with things as related among themselves. In principle, they cannot conflict, for if they speak about the same things, they do so from radically different viewpoints.

When I say that in principle they cannot conflict, I mean of course

tion is in terms of the given while explanation is in terms of the ultimates reached by analysis.

Not only are description and explanation distinct, but there are two main varieties of description. There are the ordinary descriptions that can be cast in ordinary language. There are also scientific descriptions for which ordinary language quickly proves inadequate and so is forced to yield its place to a special, technical terminology. Nor is it difficult to discern behind these linguistic differences a more fundamental difference. Both ordinary and scientific description are concerned with things as related to us, but both are not concerned with the same relations to us. The scientist selects the relations of things to us that lead more directly to knowledge of the relations between things themselves. Ordinary description is free from this ulterior preoccupation. As it begins, so also it ends with human apprehensions and interests as its centre.

There exists then a determinate field or domain of ordinary description. Its defining or formal viewpoint is the thing as related to us, as it enters into the concerns of man. Its object is what is to be known by concrete judgments of fact, by judgments on the correctness of insights into concrete situations, by concrete analogies and generalizations, and by the collaboration of common sense. It is as much an object of knowledge as any other, for it is reached by beginning from the level of presentations, by advancing through inquiry, insights, and formulation, by culminating in the critical inquiry of reflective understanding, the grasp of the unconditioned, and the rationally compelled pronouncement of judgment. To anticipate a later vocabulary, the domain of ordinary description is a section of the universe of being, of what intelligently is grasped and reasonably is affirmed. How much of that section really is reached by ordinary description, is of course a further question. At least, it is something to know the goal at which it aims, and that has been our restricted topic.

But before going on to our third topic, it may be well to preclude possible misconceptions. First, then, the human collaboration that results in a common sense involves belief. The analysis of belief cannot as yet be undertaken. But the type of belief that is essential in this collaboration resembles that of the pupil, who believes his teacher only that later he himself may understand and be able to judge for himself. It resembles that of the scientist who does not insist on exploring for himself all the blind alleys down which his predecessors wandered but is content to test their final results either directly by repeating experiments or, more commonly, by operating on the principle that, if those results were erroneous, the error would be revealed indirectly in the experiments he himself does perform. Hence it is that a man pronouncing a common-sense judgment is convinced that he is uttering, not what someone else told him, but what he himself knows.



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that in fact they can and do. To eliminate actual conflict, it is necessary to grasp the principle and to apply it accurately.

The basic difficulty has been to grasp the principle. The scientists of the Renaissance were quite aware that there was some difference in principle, but they expressed it by a distinction between primary and secondary qualities. Science is concerned with things and their primary qualities, that is, with things as they really are. Common sense is concerned with things, with their primary qualities, and most of all with their secondary qualities, that is, mainly with things as they merely appear. On this showing, knowledge is science, and where common sense diverges from science, partly it is the darkness of ignorance and error, partly it is the twilight soon to be replaced by a scientific dawn. Naturally enough such exclusive pretensions were met by opposite pretensions equally exclusive, and the debate raged on a mistaken issue. Today, I think, we can be not only cooler but also wiser about the whole matter. As has been argued in earlier chapters, it is necessary to distinguish within knowledge between separate yet complementary domains. There is a comprehensive, universal, invariant, non-imaginable domain; its object is the thing-itself, with differences in kind defined by explanatory conjugates, and with differences in state defined by ideal frequencies. There is also an experiential, particular, relative, imaginable domain; its object is the thing-for-us, with differences in kind defined by experiential conjugates, and with differences in state defined by expectations of the normal. The former field of empirical science is to be reached only by abstracting from the empirical residue. The latter field includes the empirical residue; it views things in their individuality, their accidental determinations, their arbitrariness, their continuity.

The significance of this distinction appears in logic as the separation of two universes of discourse. To put the matter concretely, let us take illustrative propositions and consider the three cases of

(1) ignoring the distinction of the domains,
(2) denying the distinction of the domains, and
(3) accepting the distinction of the domains.

First, if one ignores the distinction of the domains, then one has the problem of choosing between the propositions:

The planets move in approximately elliptical orbits with the sun at their focus.

The earth is at rest, and the sun rises and sets.



Secondly, if one denies the distinction of the domains, one is committed to the more rigorous choice between the propositions:

From every viewpoint, the planets move in elliptical orbits with the sun at their focus.

From every viewpoint, the earth is at rest and the sun rises and sets.

Thirdly, if one affirms the distinction of the domains, then one will reject all four of the preceding propositions to assert both of the following:

From the viewpoint of explanation, the planets move in approximately elliptical orbits with the sun at their focus.

From the viewpoint of ordinary description, the earth is at rest and the sun rises and sets.

On this third position there result two separate universes of discourse. All the affirmations of empirical science contain the qualifying reservation, 'from the viewpoint of explanation.' Similarly, all the affirmations of common sense contain the qualifying reservation, 'from the viewpoint of ordinary description.' Automatically, all logical conflict is eliminated, for the qualifying reservations prevent the propositions of one universe from contradicting the propositions of the other.

Underlying this logical separation, there will be more fundamental methodological differences. Both ordinary description and empirical science reach their conclusions through the self-correcting process of learning. Still they reach very different conclusions because, though they use essentially the same process, they operate with different standards and criteria. What is a further, pertinent question for empirical science is not necessarily a further, pertinent question for ordinary description. Inversely, what is a further, pertinent question for ordinary description is not necessarily a further, pertinent question for empirical science. It is this fundamental difference in the criterion of the relevance of further questions that marks the great divide between a scientific attitude and a common-sense attitude. Because he aims at ultimate explanation, the scientist has to keep asking 'Why?', until ultimate explanation is reached. Because the layman aims at knowing things as related to us, as entering into the domain of human concerns, his questioning ceases as soon as further inquiry would lead to no immediate, appreciable difference in the daily life of man. Hence it is that the layman is attempting to impose his criteria on the scientist when he asks him what he is doing and follows that up with the further question,

'What is the good of it?' For if the practical question can be put to engineers and technologists and medical doctors, its only effect upon pure science would be to eliminate all further progress. Inversely, the pure scientist is attempting to impose his criteria upon common sense, when he interprets a practical attitude as a lack of interest in truth; it is, indeed, a lack of interest in the truth that the scientist seeks, but that is not the sole domain in which truth is to be learned. Reflective understanding can reach the virtually unconditioned to pronounce correct judgments of concrete fact and to discern correct insights into concrete situations. Without those basic judgments, science has no starting-point and, equally, the glorious achievements of applied science cannot be truly affirmed.

The difference of the domains appears not only in different criteria of the pertinence of further questions but also in the difference of the terms employed and in the possibilities they respectively offer for logical deduction. Because ordinary description is concerned with things-for-us, it derives its terms from everyday experience; because the elements of daily experience are constant, the terms of ordinary description are constant; visible shapes and the spectrum of colours, the volume, pitch, and tone of sounds, the hot and cold, wet and dry, hard and soft, slow and swift, now and then, here and there, do not shift in meaning with the successive revisions of scientific theories; the concrete unities that are men and animals and plants, the regularities of nature and the expectations of a normal course of events form a necessary and unchanged basis and context into which applied science introduces its improvements.

Inversely, because science seeks knowledge of the things as related among themselves, because such relations lie outside our immediate experience, because the ultimates in such relations are to be reached only when ultimate explanation is reached, each great forward step of scientific knowledge involves a more or less profound revision of its fundamental terms. Again, because science is analytic and abstractive, its terms are exact; because its correlations purport to be generally valid, they must be determined with utmost precision; because its terms are exact and its correlations general, it must be ready to bear the weight of a vast superstructure of logical deductions in which each conclusion must be equally exact and valid generally.

On the other hand, as we have seen, ordinary description must be perpetually on its guard against analogies and generalizations; for



though similars are similarly understood, still concrete situations rarely are similar, and the synthesis of an aggregate of concrete situations is not itself a concrete situation. Because things fall away from the Pole Star in the northern hemisphere, it does not follow that they will do so in the southern. Because within the range of human vision the earth is approximately flat, it does not follow that the integration of all such views will be a flat surface. The procedure of sound common sense is not to generalize nor to argue from analogy, but to retain the insights gained in former experience and to add the complementary insights needed in fresh situations. The collaboration of common sense aims, not at establishing general truths, but at building up a core of habitual understanding that is to be adjusted by further learning in each new situation that arises.

Common sense, then, has its own specialized field or domain. It has its own criteria on the relevance of further questions. It has its own basically constant vocabulary, its proper universe of discourse, and its own methodological precepts of keeping to the concrete, of speaking in human terms, of avoiding analogies and generalizations and deductions, of acknowledging that it does not know the abstract, the universal, the ultimate. Precisely because it is so confined, common sense cannot explicitly formulate its own nature, its own domain, its own logic and methodology. These it has to learn, if it would limit properly its pronouncements, but it has to learn them in its own shrewd fashion through instances and examples, fables and lessons, paradigms and proverbs, that will function in future judgments not as premises for deductions but as possibly relevant rules of procedure. Finally, because common sense has to be acquired, it is not possessed equally by all It has its adept pupils who make mistakes, indeed, but also learn by them. Within their familiar field they are masters, and as well they know that their mastery ends when they step beyond its limits. Above all they know that they must master their own hearts, that the pull of desire, the push of fear, the deeper currents of passion are poor counsellors, for they rob a man of that full, untroubled, unhurried view demanded by sure and balanced judgment.

If the domains of science and common sense are distinct, so also they are complementary. If one must recognize the differences in their objects, their criteria, their universes of discourse, their methodological precepts, one must also insist that they are the functionally related parts within a single knowledge of a single world. The intelligibility that

science grasps comprehensively is the intelligibility of the concrete with which common sense deals effectively. To regard them as rivals or competitors is a mistake, for essentially they are partners and it is their successful co-operation that constitutes applied science and technology, that adds inventions to scientific discoveries, that supplements inventions with organizations, know-how, and specialized skills.

But if common sense itself, once it is supplied with its appropriate evidence, has little difficulty in recognizing this fact, theorists of science can hardly be credited with an equal perspicacity. Misled by a confusion between the heuristic and the representative functions of imagination, they assumed that the business of science was to paint a picture of the really real. If, as we have argued, such a picture is essentially unverifiable and gratuitous, it cannot coincide with the verifiable pictures of common sense. If from this conflict the theorists of science proceeded to conclude that common sense must be some brutish survival, that it was in need of being instructed in lofty tones on the far superior virtues and techniques of the scientist, one cannot be surprised that common sense retaliated with its jokes on the ineptitude of the theorists and professors and with its quietly imperious demand that, if they were to justify their existence, they had best continue to provide palpable evidence of their usefulness. But such opposition, I would contend, does justice neither to common sense nor to science; it has no better basis than a mistaken theory; and it had best be written off as an error incidental to an age of transition. During the past four centuries, empirical science has emerged and developed, to set us the twofold problem both of determining its nature and of working out the proper adjustment of the complementary functions of common sense. If such large problems cannot be solved in short order, one should not infer that they cannot be solved at all.

To conclude, common sense is one thing and common-sense judgments are another. Common sense is common and specific. It is a specialized domain of knowledge with a proper universe of discourse, proper criteria on the pertinence of further questions, and proper methodological precepts. Operation within that domain is basically and fundamentally a communal collaboration in the self-correcting process of learning. The fruit of that collaboration is a habitual core of accumulated insights into concrete situations and into the procedures needed to complement and adjust that core before one can pass judgment on further, concrete situations. Hence it is that common-sense



judgments are issued, not by some public authority named common sense, but only by individual judges in their own individual situations. Further, they can be known to be correct only by the individual judges in the individual situations, for no one else is in possession of the evidence as it is given and no one else is informed with the familiarity and mastery that result from the self-correcting process of learning within that situation. I can be certain that I am writing this, and you can be certain that you are reading it. But it is quite another matter for you to be certain that I am correct in affirming that I am writing, as it will be quite another matter for me to be certain that you are correct in affirming that you are reading. The common element in common sense is not some list of general truths about which all men can agree; it is not some list of particular truths about which all men can agree; but it is a collaboration in the erection of a basic structure by which, with appropriate adjustments, each individual is enabled to fill out his individual list of particular truths. Finally, each of those particular pronouncements occurs inasmuch as reflective understanding grasps the virtually unconditioned in the manner described in the sections on concrete judgments of fact and on judgments on the correctness of insights into concrete situations.

### 6. PROBABLE JUDGMENTS

When the virtually unconditioned is grasped by reflective understanding, we affirm or deny absolutely. When there is no preponderance of evidence in favour of either affirmation or denial, we can only acknowledge our ignorance. But between these extremes there is a series of intermediate positions, and probable judgments are their outcome.

This probability of judgment differs from the probability investigated in studying statistical method. As has been seen, the probable expectation answers a question for intelligence by assigning an ideal frequency from which actual events non-systematically diverge. But the probable judgment answers a question for reflection and, though it anticipates a divergence between the judgment and actual fact, still the ground of this anticipation lies, not in a non-systematic element in the facts, but in the incompleteness of our knowledge. Hence, judgments about things, about correlations, and about probability expectations, may be certain and may be only probable.

Probable judgments differ from guesses. In both cases knowledge is

parallel investigation of the tension of wires, and a certain amount of dabbling with gravitation. Further questions arise. Not only do they arise from the concrete problems set by tension and gravitation. What is far more significant is the presence of the highly abstract theorems and procedures. Can every force be represented by a vector? Are all forces applied at a point? Did Euclid have the last word? The initial abstraction allows one to return to the concrete only after the exploration of successively widening circles of inquiry. Statics is mastered only to raise the problems of kinetics. Kinetics is mastered only to reveal that thermal and electromagnetic phenomena may be the antecedents or the consequents of local movements. One begins to get the lot in line and to feel that the future of physics is a matter of determining accurately a few more decimal points when along come a Planck and an Einstein with their further questions.

The generalization of classical laws, then, is no more than probable because the application of single laws raises further questions that head towards the systematization of a whole field. In turn, such systematization is no more than probable until the limit of no further, pertinent questions is reached. But that limit is not reached, first, if there may be further, unknown facts that would raise further questions to force a revision or, secondly, if there may be further, known facts whose capacity to raise such further questions is not grasped.

Similar considerations render the generalization of statistical laws no more than probable. For statistical laws presuppose some classification of events. One is not going to advance quantum theory by investigating baseball averages. Hence definitive statistical laws suppose definitive classifications. The future discovery of new kinds or of new subdivisions of subatomic elements will invite a revision of the statistical laws. Similarly, more accurate investigations may lead to the discernment within the statistical law of a systematic element that can be abstracted in classical form to leave a new statistical residue.

If empirical generalizations are no more than probable, what about the particular facts that ground them? Here a distinction seems necessary. In so far as such facts are expressed in the terms of ordinary description, they fall under the criteria of the concrete judgment of fact. In so far as they are relevant to be the establishment of a scientific theory, they come under the control of empirical method. What has to be observed is, not the percept with its spontaneous integration into the processes of sensitive living, but the sheer datum that is stripped of non-scientific



memories, associations, and anticipations. Again, measurements must conform to the best available rules and utilize the best available instruments. Finally, the observables have to be the terms defined by the theoretical structure and, as this structure is subject to revision, so also are its definitions. Hence, one may say that empirical science is solidly grounded in fact in virtue of its concrete judgments and, at the same time, one may add that technical developments and theoretical advance can render such facts more or less obsolescent.

But if empirical science is no more than probable, still it truly is probable. If it does not attain definitive truth, still it converges upon truth. This convergence, this increasing approximation, is what is meant by the familiar phrase, the advance of science. Questions yield insights that are expressed in hypotheses; the testing of hypotheses raises further questions that generate complementary insights and more satisfactory hypotheses. For a while the process advances in widening circles; then the coherence of system begins to close in; investigation turns from fresh ventures in new fields to the labour of consolidation, of working out implications fully, of settling issues that leave the general view unchanged. The self-correcting process of learning is palpably approaching a limit.

An ulterior question may be raised. Is scientific progress indefinite? Does the self-correcting process of learning reach one limit only to discover, sooner or later, that there are further developments to be effected? If I am unable to answer this question directly, still certain observations seem relevant.

First, the advance of science through increasing accuracy would seem to head towards a limit. A measurement is not a point but an interval, not simply a number but a number plus or minus some quantity determined by a theory of errors. Hence increasing accuracy has to result from the invention of new techniques and instruments and, while such inventions may go well beyond our present anticipations, still we have no reason to expect an infinite series of them. Once such possibilities become exhausted, the canon of selection comes into play. Empirical method settles only the theoretical differences that imply sensible differences. If a second theory supplants a first by advancing from the second decimal place to the fourth, and a third supplants the second by advancing from the fourth decimal place to the sixth, it does not follow that there can be some $n$th theory established by advancing from $2n$ decimals to $(2n+2)$, where $n$ is as large a number as you please.

Secondly, as the advance of science has a lower limit in the field of presentations, so also it has an upper limit in the basic structure of the human mind. Theories can be revised if there is a reviser. But to talk about revising the revisers is to enter a field of empty speculation in which the name, revision, loses its determinate meaning. Moreover, theorists take advantage of this fact. Thus, the foundations of logic are placed in the inevitabilities of our processes of thought. Nor is logic a unique example. As we have already indicated, the theory of relativity in its basic postulate rests upon a structural feature of our cognitional process. Now if the invariants governing mental process imply invariants in our theoretical constructions, there will follow an upper limit to the variation of theoretical constructions and a possibility of mapping out in advance the alternatives between which theoretical effort has to choose. To this topic we return in investigating what will be named the elements or categories of the range of proportionate being.

In conclusion, it may be noted that these considerations confirm the positive probability of the conclusions of empirical science. For those conclusions are probable inasmuch as the self-correcting process of learning is approaching a limit. Our argument was based upon the immanent tendency of the process itself to a limit, inasmuch as each great stage of scientific development heads for the closed coherence of system, and each successive system grips the facts with greater nuance and accuracy over wider expanses of data. Still this immanent tendency receives confirmation if there exist external limitations to the process itself. For they, too, point to the possibility of some system, as yet unknown, that increasingly is determined inasmuch as it will have to meet the requirement of verification in a body of fact that is increasingly large and increasingly organized.

### 7. ANALYTIC PROPOSITIONS AND PRINCIPLES

A proposition is what is proposed either for consideration or for affirmation. An analysis of propositions is reached by distinguishing what is meant from acts of meaning and from sources of meaning. Any cognitional activity is a source of meaning. Conceiving, judging, and uttering are three quite different acts of meaning. Finally, as sources lead to acts, so acts refer to terms of meaning, to what is meant.

Terms of meaning may be divided in two ways. There is the basic distinction between what is meant when one affirms or denies and, on the other hand, what is meant when one merely considers, supposes, defines. Again, in utterances there is the obvious distinction between the incomplete meaning of a word and the complete meaning of a sentence. So one is led to distinguish

(1) partial terms of meaning,
(2) rules of meaning,
(3) formal terms of meaning, and
(4) full terms of meaning.



The full term of meaning is what is affirmed or denied.

The formal term of meaning is what could be affirmed or denied but, in fact, is merely supposed or considered.

The partial term of meaning is what is meant by a word or by a phrase.

Rules of meaning govern the coalescence of words and phrases into the complete sense that may be supposed or considered, affirmed or denied.

There results at once a particular case of the virtually unconditioned. A formal term of meaning provides the conditioned. The definitions of its partial terms provide the fulfilling conditions. And the rules of meaning provide the link between the conditions and the conditioned. Such propositions are termed analytic.

Thus, if $A$ is defined by a relation, $R$, to $B$, and $B$ is defined by the converse relation, $R'$, to $A$, then by the rules of meaning it follows that there cannot be an $A$ without the relation, $R$, to $B$, and that there cannot be a $B$ without the relation, $R'$, to $A$. Such conclusions resting on definitions and rules of meaning are analytic propositions.

Now, since the analytic proposition is an instance of the virtually unconditioned, reflective understanding will find in it its proper object and thereby ground a judgment. There then arises a further question, What precisely is the meaning or force or implication of such a judgment?

It would seem that its meaning is not assertoric but hypothetical. If there occur suppositions or judgments containing significant terms in the same sense as they are assigned in the analytic proposition, then such suppositions or judgments must be consistent with the analytic proposition; moreover, when that condition and other logical requirements are met, there follow valid inferences. On the other hand, the mere fact that a proposition is analytic offers no guarantee that its

terms in their defined sense occur in any supposition or judgment apart from the affirmation of the analytic proposition.

It follows that analytic propositions remain in sterile isolation unless there accrues to them some form of validation. This will consist in the occurrence of the same terms in their defined sense in some other supposition or judgment; and the precise nature of the validation will depend upon the nature of the added supposition or judgment.

There also follows the explanation of the fact that analytic propositions can be produced more or less at will and indefinitely. Partial terms of meaning are a vast multitude and further partial terms can be supplied by the art of definition. Rules of meaning provide a principle of selection of the partial terms that will coalesce into analytic propositions. And if this seems to require too much ingenuity, the task can be simplified by using symbols instead of words and by defining them by their relations in propositions. But significant increments of knowledge are not to be obtained by mere ingenuity and, in fact, the analytic proposition, by itself, is not a significant increment of knowledge; without the fulfilment of further conditions it remains in isolation and fails to enter fruitfully into the texture of knowing.

Hence, we are in substantial agreement with the contemporary view that mere analytic propositions are tautologies. The use of the term, tautology, would seem to be incorrect, but the general meaning of the statement is sound. However, it may not be out of place to add that the present point was made centuries ago. Aquinas advanced that conclusions depend upon principles and that principles depend upon their terms; but he was not ready to accept any terms whatever; he added that proper terms are selected by wisdom* and by wisdom he meant an accumulation of insights that stands to the universe as common sense stands to the domain of the particular, incidental, relative, and imaginable

Let us now turn from analytic propositions to analytic principles.

By an analytic principle is meant an analytic proposition of which the partial terms are existential; further, the partial terms of an analytic proposition are existential if they occur in their defined sense in judgments of fact, such as the concrete judgment of fact or the definitively established empirical generalization.

Further, since such analytic principles are hard to come by, we shall also speak of two mitigated cases.

* *Sum. Theol.*, I–II, q. 66, a. 5, ad 4m



The provisional analytic principle is an analytic proposition of which the terms are probably existential, that is, they occur in probable empirical generalizations.

The serially analytic principle is an analytic proposition of which the terms are serially existential; what is meant by the serially existential, will be clarified in our next section on mathematical judgments.

It may be remarked that the analytic principle also connotes in its terms not only an existential reference but also a basic, primitive character. I think this feature will be found to follow from the defined requirements for, as we shall proceed to argue, analytic principles lie pretty well outside the reach of common sense and empirical science.

They lie outside the reach of common sense because analytic principles are universal and common sense regards the particular. Common sense makes concrete judgments of fact and it passes judgment on the correctness of insights into concrete situations. But in neither case does it employ terms in the sense assigned them by abstract definitions. As Socrates discovered, the average man does not define; he is suspicious of the search for definitions; and when that pursuit brings out the inference that he does not know what he is talking about, he is rather resentful.

The fact would seem to be that the structure of common-sense meanings is much the same as the structure of common sense itself. There is a communal collaboration that yields a habitual core of understanding and, as well, a range of concepts and linguistic terms in ordinary use. But just as the common core of understanding has to be adjusted by complementary insights into the present, concrete situation before judgment occurs, so also common concepts and terms receive their ultimate complement of meaning from those complementary insights.

'This is a dog.' 'What do you mean by a *dog*?' The question supposes that the term *dog* has a precise meaning outside the series of statements in which it occurs. But in fact what comes first is the series of statements and what comes only later, and then only if one goes in for analysis, is the determination of the precise meaning of the single, partial term. What the average man means by a dog is

(1) what he would with certainty pronounce to be a dog in any concrete situation with which he is familiar,
(2) what he could learn to be a dog, and
(3) what he would be willing to believe is a dog.

Hence it is that a dictionary is constructed, not by the Socratic art of definition, but by the pedestrian, inductive process of listing sentences in which each word occurs in good usage.

It may be objected that one cannot make a brick house without first making bricks But one is only arguing from a false analogy if one claims that the mind develops in the same fashion as the wall of a house is built Prior to concepts there are insights. A single insight is expressed only by uttering several concepts. They are uttered in conjunction, and reflection pronounces whether the insight and so the conjunction is correct. The isolation and definition of concepts is a subsequent procedure and common sense does not undertake it.

Because we have denied that common sense reaches analytic principles, it is not to be inferred that the average-man has no principles. Analytic principles suppose analysis; analysis supposes accurate conceptualization. But prior to analysis, to concepts, to judgments, there are the native endowments of intelligence and reasonableness and the inherent structures of cognitional process These are the real principles on which the rest depend. Moreover, all understanding has its universal aspect, for similars are similarly understood. But it is one thing to exploit this universal aspect in a professional manner, it is another to exploit the intelligibility, which is by itself universal, by adding further intelligibilities until one comes to grips with concrete situations. The latter line of development we have named common sense so that, by definition, common sense deals with the particular Again, the latter line of development is conspicuous in the average man. But what else the average man knows and how he knows it, are further questions. As has been remarked already, one cannot treat all issues at the same time

Next, analytic principles lie outside the reach of empirical science It is true of course that every insight yields several concepts linked together through the insight; it also is true that the empirical scientist formulates definitions, postulates, and inferences, but the trouble is that the empirical scientist knows his insights not as certainly correct but only as probable. Hence his defined terms, in the sense they are defined, are as much subject to revision as the probable judgments of fact that contain them and validate them.

Thus, consider the assertions:

(1) water probably is $H_2O$;
(2) what I mean by water is $H_2O$;



(3) this water contains impurities;
(4) there are two kinds of water, heavy and ordinary.

The first is an empirical conclusion. The second is a definition. The third is a concrete judgment of fact; its meaning is that this sample is water in the sense of the empirical conclusion but it is not solely water in the sense of the definition. The fourth introduces a new basis of definition that has its ground in fresh experimental work. Now both the initial definition and the later definitions yield analytic propositions, namely, that what does not satisfy certain specifications is not pure water, or it is not pure water of molecular weight eighteen, or it is not pure heavy water. Moreover, none of these are merely analytic propositions; they are not the sort of thing that can be produced at will and indefinitely. On the other hand, they are not strictly analytic principles, for though their terms possess validating judgments of fact, still those judgments are subject to revision, and, indeed, the discovery of heavy water has already forced such a revision.

Generally one may say that the advance of empirical science is an instance of the advance of the self-correcting process of learning. But in this instance the previous insights yield correlations, definitions, and inferences. It is in terms of such formulations that are framed the further questions that will complement and modify the previous insights by later insights. In like manner the later insights receive their formulation which is presupposed by the further questions that lead to a still fuller understanding. Now in this process the successive formulations have three distinct aspects First, they are the expression of insights that grasp the intelligible form of data, thus, they are probable empirical conclusions Secondly, they are the presupposition of the further questions that lead to further insights, from this viewpoint they are provisional analytic principles. Thirdly, they are revised in the light of the further insights and so cease to be probable empirical conclusions and provisional analytic principles to pass into the limbo of the analytic propositions whose terms have no existential reference.

The reader interested in further illustrations of this process will find numerous examples in Arthur Pap's *The A Priori in Physical Theory*, New York, 1946.

### 8. MATHEMATICAL JUDGMENTS

In mathematical thought one may readily discern the difference

between operations on the level of intelligence and operations on the level of reflection.

The level of intelligence is the level of discovery and invention, of catching on and learning, of grasping problems and coming to grasp their solutions, of seeing the point made in each of a series of mathematical statements and then seeing how the successive points hang together.

The level of reflection is the complementary process of checking. One understands and now one wishes to know whether what is understood is also correct. One has grasped the point and one asks whether it is right. One has seen how the successive steps hang together and one is out to make sure that what hangs together is really cogent.

Now the process of checking can be developed into an elaborate technique. What is checked becomes a whole department of mathematics. Definitions are worked out. Postulates are added. From the definitions and postulates it is shown that all the conclusions of the department can be reached by the rigorous procedure of deductive inference.

But what is the goal of checking? Clearly, it is to marshall the evidence in the shape in which reflective understanding can grasp the virtually unconditioned and so ground rational judgment. In so far as the checking reduces conclusions to premises, there is the virtually unconditioned of the form of deductive inference. In so far as the definitions and postulates coalesce into a self-justifying meaning, there is the virtually unconditioned of analytic propositions. Both of these types of the virtually unconditioned have already been considered and so, for us, the problem of mathematical judgment consists in determining what else is required for such judgment.

First of all, something else is required. For if the premises of mathematical thought are analytic propositions, still not all analytic propositions are mathematical premises. Analytic propositions can be produced at will and indefinitely. But the premises of mathematical thought are to be reached only through the discoveries of genius and the labour of learning what genius has grasped. Further, it does happen that abstruse regions of mathematics are occasionally pulled out of their cold and airy regions to become the tools of empirical hypotheses and theories and to share with such formulations the probable existential reference that they possess. But prior to a probable existential reference or isomorphism there is a possible existential reference or isomorphism; before a



department of mathematics can be applied, it must possess an inherent possibility of being applied. What, then, is that inherent possibility? And what is its criterion?

Secondly, we have to undertake an examination of mathematics to determine what this further element is and what its criterion is. Let us say, then, that there is a mathematical series, that each term in the series is a department of mathematics, that each department consists

(1) of rules governing and so defining operations, and
(2) of operations proceeding from some terms to others and so relating and defining them.

Further, we may presuppose each department of mathematics to be formalized, that is, to be stated in a set of definitions, postulates, and deductions. Finally, we shall presuppose that there are other formalizations, equally rigorous, equally elegant, but in fact not members of the mathematical series. Our problem thus becomes the question, In the light of our general analysis of knowledge, how is one to recognize some formalizations as mathematical and others as not mathematical?

Our answer contains three elements, and it will be convenient to refer to them respectively as the material element, the formal element, and the actual element.

The material element is what we have named the empirical residue. There are aspects of data from which understanding always abstracts. Such have been seen to be the individual, the continuum, particular places and times, and the non-systematic divergence of actual frequency from probable expectations.

The formal element may be designated by abstraction as enriching. It has been seen that insight goes beyond images and data by adding intelligible unities and correlations and frequencies, which, indeed, contain a reference to images or data but, none the less, add a component to knowledge that does not exist actually on the level of sense or imagination.

Finally, the actual element lies in the conjunction of the material and the formal elements.

By the mathematician, the formal element commonly is viewed as dynamic. There is a laborious process named 'learning mathematics'. It consists in gradually acquiring the insights that are necessary to understand mathematical problems, to follow mathematical arguments, to work out mathematical solutions. This acquisition occurs in a suc-

cession of higher viewpoints. One department of mathematics follows upon another. Logically, they are discontinuous, for each has its own definitions, postulates, and inferences. But intellectually they are continuous, inasmuch as the symbolic representation of operations in the lower field provides the images in which intelligence grasps the idea of the new rules that govern operations in the higher field.

However, this expansion of intelligence does not seem to be completely free. Not only is there the link between higher viewpoints and preceding lower viewpoints, but also there is a bias from the particular to the general, from the part to the totality, from the approximate to the ideal. If there exist concrete instances of one, two, three, the mathematician explores the totality of positive integers, of real numbers, of complex numbers, of ordered sets. If there exist edges and surfaces, the mathematician works out not merely one geometry but the total series of possible geometries. If there are various fields in which it seems mathematics may be applied, the mathematician sets out to explore the whole of each region in which the fields occur.

Again, besides its preference for the general, the complete, the ideal, the development of mathematical thought also seems restricted by its material element. By this I do not mean that the mathematician is confined to individuals that exist, to continua that exist, to places and times that exist, to non-systematic divergences that occur, or to any other actual elements in the empirical residue that may be discovered through the introduction of new techniques of abstraction. For it is quite clear that mathematical thought in its pursuit of the general and complete and ideal reveals a profound unconcern for the existent. Still it does seem to be true that the empirical residue does supply mathematics with samples of the type of stuff on which mathematical ideas confer intelligibility and order. For unless the mathematician is investigating the pure intelligibilities that Aquinas identified with angels, there must be some mathematical matter; and since there are other sciences that deal with data as of determinate kinds, there remains for the mathematician the empirical residue of all data.

If we have succeeded in characterizing the material and formal elements of mathematics, there remains the question of the significance of their conjunction. Briefly, this may be indicated by recalling that we found the heuristic structures of empirical method to operate in a scissors-like fashion. Not only is there a lower blade that rises from data through measurements and curve-fitting to formulae, but also there is



an upper blade that moves downward from differential and operator equations and from postulates of invariance and equivalence. Moreover, it is no secret that the upper blade owes its effectiveness to the labours of mathematicians. But what is the possibility of that upper blade?

To grasp the answer to that question, two complementary tendencies have to be envisaged at once. On the one hand, there is the movement of empirical science from description to explanation, from proper domains of data to systems of laws that implicitly define the terms they relate, and at the end of this movement there is the ideal goal that is to be attained when all aspects of data, except the empirical residue, will have their intelligible counterpart in systems of explanatory conjugates and ideal frequencies. On the other hand, there is the movement of mathematical thought that begins from the empirical residue and endeavours to explore the totality of manners in which enriching abstraction can confer intelligibility upon any materials that resemble the empirical residue. Clearly, these two movements are complementary. For the mathematician begins from the empirical residue with which the empirical scientist would end; and if the mathematical exploration of intelligible systems is thorough, then it is bound to include the systems of explanatory conjugates that the empirical sciences will verify in their respective domains.

Let us now revert to our distinction between outright analytic principles, provisional analytic principles, and serially analytic principles. All are analytic propositions, i.e., instances of the virtually unconditioned in which the conditioned is linked to its conditions by syntactical rules and the conditions are fulfilled by defining terms. None are mere analytic propositions that are obtained by devising any definitions or syntactical rules that one pleases. For the terms and relations of outright analytic principles occur, in their defined sense, in certain judgments of fact. The terms and relations of provisional analytic principles occur, in their defined sense, in probable judgments of fact. Finally, the terms and relations of serially analytic principles ground the deductive expansions that explore completely, generally, and ideally, the total range of fields to which outright and provisional analytic principles give access in a particular, fragmentary, or approximate manner.

Next, it seems possible to identify the basic propositions of mathematics with serially analytic principles For there is a material element in mathematical thought, and it bears some similarity to the empirical residue in the data of the empirical sciences. Again, there is a formal

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Next, it seems possible to identify the basic propositions of mathematics with serially analytic principles. For there is a material element in mathematical thought, and it bears some similarity to the empirical residue in the data of the empirical sciences. Again, there is a formal

element in mathematical thought, and it tends towards a general, complete, and ideal account of the manners in which enriching abstraction can add intelligibility and order to the material element. But the empirical sciences are in search of the intelligibility and order that, when combined with the empirical residue in the data of their several domains, will provide a complete and definitive explanation of those data. It follows that the mathematician is concerned to establish generally, completely, and ideally, the range of possible systems that include verifiable scientific systems as particular, fragmentary, or approximate cases

Thirdly, if the basic propositions of mathematics are serially analytic principles, then we have the answer to our principal question that asked the difference between free formalizations and mathematical formalizations.

Fourthly, there readily follows an account of the possibility of isomorphism between mathematical relations and the relations of the empirical sciences. Both sets of relations are products of enriching abstraction, and both possess a relevance to the empirical residue in data.

Finally, it seems appropriate to add a note on the difference between the foregoing account of the field of mathematics and current views. Commonly, it would be agreed that mathematics is based on mere analytic propositions, and it would be explained that, if one disregards merely arbitrary definitions and syntactical rules, one can distinguish logic which deals with such relations as 'and', 'or', 'if . . . , then . . ', mathematics which deals with relations of equivalence or congruence in individuals and sets, and a more general subject, call it 'mathesis', which deals with rules common to logic and to mathematics.

The principal difference in our approach is that it goes behind concepts and affirmations to the grounding acts of direct and reflective understanding. From this feature there follows its dynamic character, for it contains an invitation to mathematicians to explore the possibility of setting up the series of deductive expansions that would do as much for other empirical sciences as has been done for physics. On the other hand, while we have emphasized a relation between mathematics and empirical science, it must be insisted that we have not done so by restricting materially the field of mathematics. The mathematician remains free to take as his materials anything that resembles the empirical residue. He is free to discover further additions to the residue that, at present, is known. He is free to explore with full generality, completeness, and ideality, the enrichments that the exercise of human intelligence can add. Yet his creations will remain serially existential, for they will exhibit the series of systems to some of which the empirical scientist will be able to say 'Yes'.



## 9. SUMMARY

Prospective judgments are propositions

(1) that are the content of an act of conceiving, thinking, defining, considering, or supposing,

(2) that are subjected to the question for reflection, to the critical attitude of intelligence, and

(3) that thereby are constituted as the conditioned.

There is sufficient evidence for a prospective judgment when it may be grasped by reflective understanding as virtually unconditioned. Hence sufficient evidence involves

(1) a link of the conditioned to its conditions, and

(2) the fulfilment of the conditions.

These two elements are supplied in different manners in different cases.

In formal inference the link is provided by the hypothetical premise, If the antecedent, then the consequent. The fulfilment is the minor premise.

In judgment on the correctness of insights, the link is that the insight is correct if there are no further, pertinent questions, and the fulfilment lies in the self-correcting process of learning reaching its limit in familiarity and mastery.

In judgments of fact the link is the correct insight or set of insights and the fulfilment lies in present and/or remembered data.

In generalizations the link is the cognitional law that similars are similarly understood and the fulfilment lies in such similarity that further, pertinent questions no more arise in the general case than in the correctly understood particular case.

In probable judgments the link is that insights are correct when there are no further pertinent questions and the fulfilment is some approximation of the self-correcting process of learning to its limit of familiarity and mastery.

In analytic propositions the link lies in rules of meaning that generate

propositions out of partial terms of meaning and the fulfilment is supplied by the meanings or definitions of the terms.

Analytic propositions become analytic principles when their terms are existential; and terms are existential when they occur in definitive, factual judgments.

Provisional analytic principles are analytic propositions whose terms are probably existential.

Serially analytic principles are the analytic propositions from which follow the ranges of systems of which some in some fashion exist.

*PART TWO*

# INSIGHT AS KNOWLEDGE

CHAPTER XI

# SELF-AFFIRMATION OF THE KNOWER

It is time to turn from theory to practice. Judgment has been analysed. Its grounds in reflective understanding have been explored. Clearly the next question is whether correct judgments occur, and the answer to it is the act of making one.

Since our study has been of cognitional process, the judgment we are best prepared to make is the self-affirmation of an instance of such a process as cognitional. By the 'self' is meant a concrete and intelligible unity-identity-whole. By 'self-affirmation' is meant that the self both affirms and is affirmed. By 'self-affirmation of the knower' is meant that the self as affirmed is characterized by such occurrences as sensing, perceiving, imagining, inquiring, understanding, formulating, reflecting, grasping the unconditioned, and affirming.

The affirmation to be made is a judgment of fact. It is not that I exist necessarily, but merely that in fact I do. It is not that I am of necessity a knower, but merely that in fact I am. It is not that an individual performing the listed acts really does know, but merely that I perform them and that by 'knowing' I mean no more than such performance.

As all judgment, self-affirmation rests upon a grasp of the unconditioned. The unconditioned is the combination of

(1) a conditioned,
(2) a link between the conditioned and its conditions, and
(3) the fulfilment of the conditions.

The relevant conditioned is the statement, I am a knower. The link between the conditioned and its conditions may be cast in the proposition, I am a knower, if I am a concrete and intelligible unity-identity-whole, characterized by acts of sensing, perceiving, imagining, inquiring, understanding, formulating, reflecting, grasping the unconditioned, and judging. The fulfilment of the conditions is given in consciousness.

The conditioned offers no difficulty. It is merely the expression of what is to be affirmed. Similarly, the link offers no difficulty; the link itself is a statement of meaning; and the conditions which it lists have become familiar in the course of this investigation. The problematic

element, then, lies in the fulfilment of the conditions and we proceed to indicate what is meant and not meant by consciousness and by the fulfilment of conditions.

### 1. THE NOTION OF CONSCIOUSNESS

First, consciousness is not to be thought of as some sort of inward look. People are apt to think of knowing by imagining a man taking a look at something and, further, they are apt to think of consciousness by imagining themselves looking into themselves. Not merely do they indulge in such imaginative opinions but also they are likely to justify them by argument. Knowing, they will say, is knowing something; it is being confronted by an object; it is the strange, mysterious, irreducible presence of one thing to another. Hence, though knowing is not exclusively a matter of ocular vision, still it is radically that sort of thing. It is gazing, intuiting, contemplating. Whatever words you care to employ, consciousness is a knowing and so it is some sort of inward looking.

Now while consciousness is a factor in knowing, and while knowing is an activity to which a problem of objectivity is annexed, still it is one thing to give an account of the activity and it is something else to tackle the problem of objectivity. For the present we are concerned simply with an account of the activity, and so we have defined the knower, not by saying that he knows something, but solely by saying that he performs certain kinds of acts. In like manner, we have not asked whether the knower knows himself; we ask solely whether he can perform the act of self-affirmation. Hence, while some of our readers may possess the rather remarkable power of looking into themselves and intuiting things quite clearly and distinctly, we shall not base our case upon their success. For, after all, there may well exist other readers that, like the writer, find looking into themselves rather unrewarding.

Secondly, by consciousness we shall mean that there is an awareness immanent in cognitional acts. Already a distinction has been drawn between act and content, for instance, between seeing and colour, hearing and sound, imagining and image, insight and idea. To affirm consciousness is to affirm that cognitional process is not merely a procession of contents but also a succession of acts. It is to affirm that the acts differ radically from such unconscious acts as the metabolism of one's cells, the maintenance of one's organs, the multitudinous biological processes that one learns about through the study of contemporary medical



science. Both kinds of acts occur, but the biological occur outside consciousness, and the cognitional occur within consciousness. Seeing is not merely a response to the stimulus of colour and shape; it is a response that consists in becoming aware of colour and shape. Hearing is not merely a response to the stimulus of sound; it is a response that consists in becoming aware of sound. As colour differs from sound, so seeing differs from hearing. Still seeing and hearing have a common feature, for in both occurrences there is not merely content but also conscious act.

By the conscious act is not meant a deliberate act; we are conscious of acts without debating whether we will perform them. By the conscious act is not meant an act to which one attends; consciousness can be heightened by shifting attention from the content to the act; but consciousness is not constituted by that shift of attention, for it is a quality immanent in acts of certain kinds, and without it the acts would be unconscious as is the growth of one's beard. By the conscious act is not meant that the act is somehow isolated for inspection, nor that one grasps its function in cognitional process, nor that one can assign it a name, nor that one can distinguish it from other acts, nor that one is certain of its occurrence.

Does then, 'conscious act' mean no more than 'cognitional act'? A distinction has to be drawn. First, I do not think that only cognitional acts are conscious. Secondly, there are those that would define 'seeing' as 'awareness of colour' and then proceed to argue that in seeing one was aware of colour but of nothing else whatever, that 'awareness of colour' occurs but that a concomitant 'awareness of awareness' is a fiction. This, I think, does not accurately reflect the facts. If seeing is an awareness of nothing but colour and hearing is an awareness of nothing but sound, why are both named 'awareness'? Is it because there is some similarity between colour and sound? Or is it that colour and sound are disparate, yet with respect to both there are acts that are similar? In the latter case, what is the similarity? Is it that both acts are occurrences, as metabolism is an occurrence? Or is it that both acts are conscious? One may quarrel with the phrase, awareness of awareness, particularly if one imagines awareness to be a looking and finds it preposterous to talk about looking at a look. But one cannot deny that, within the cognitional act as it occurs, there is a factor or element or component over and above its content, and that this factor is what differentiates cognitional acts from unconscious occurrences.

## 2. EMPIRICAL, INTELLIGENT, AND RATIONAL CONSCIOUSNESS

By consciouness is meant an awareness immanent in cognitional acts. But such acts differ in kind, and so the awareness differs in kind with the acts. There is an empirical consciousness characteristic of sensing, perceiving, imagining. As the content of these acts is merely presented or represented, so the awareness immanent in the acts is the mere givenness of the acts. But there is an intelligent consciousness characteristic of inquiry, insight, and formulation. On this level cognitional process not merely strives for and reaches the intelligible, but in doing so it exhibits its intelligence; it operates intelligently. The awareness is present but it is the awareness of intelligence, of what strives to understand, of what is satisfied by understanding, of what formulates the understood, not as a schoolboy repeating by rote a definition, but as one that defines because he grasps why that definition hits things off. Finally, on the third level of reflection, grasp of the unconditioned, and judgment, there is rational consciousness. It is the emergence and the effective operation of a single law of utmost generality, the law of sufficient reason, where the sufficient reason is the unconditioned. It emerges as a demand for the unconditioned and a refusal to assent unreservedly on any lesser ground. It advances to grasp of the unconditioned. It terminates in the rational compulsion by which grasp of the unconditioned commands assent.

Empirical consciousness needs, perhaps, no further comment, for by it we illustrated the difference between conscious and unconscious acts. Intelligent and rational consciousness, on the other hand, may be clarified by a contrast. In their different manners both common sense and positive science view the material world as subject to intelligible patterns and as governed by some law of causality. To confine our attention to what man knows best, namely, his own artefacts, there is discernible in them an intelligible design and their existence has its ground in the labour of production. But before the design is realized in things, it was invented by intelligence; before the sequence of productive operations was undertaken, it was affirmed as worth while for some sufficient or apparently sufficient reason. In the thing there is the intelligible design, but in the inventor there was not only the intelligibility on the side of the object but also intelligent consciousness on the side of the subject. In the thing there is the groundedness that consists in its existence being accounted for by a sequence of operations; but in the



entrepreneur there was not only the groundedness of his judgment in the reasons that led to it but also the rational consciousness that required reasons to reach judgment.

Intelligence and intelligibility are the obverse and reverse of the second level of knowing: intelligence looks for intelligible patterns in presentations and representations; it grasps such patterns in its moments of insight; it exploits such grasp in its formulations and in further operations equally guided by insights. In like manner, reasonableness and groundedness are the obverse and reverse of the third level of knowing. Reasonableness is reflection inasmuch as it seeks groundedness for objects of thought; reasonableness discovers groundedness in its reflective grasp of the unconditioned; reasonableness exploits groundedness when it affirms objects because they are grounded. In man's artefacts there are the reverse elements of the intelligibility and groundedness, but there are not the obverse elements of intelligence and reasonableness. The obverse elements pertain to cognitional process on its second and third levels; they do not pertain to the contents emergent on those levels, to the idea or concept, to the unconditioned or affirmed; on the contrary, they characterize the acts with which those contents are coupled and so they are specific differentiations of the awareness of consciousness. Clear and distinct conception not only reveals the intelligibility of the object but also manifests the intelligence of the subject. Exact and balanced judgment not only affirms things as they are but also testifies to the dominance of reasonableness in the subject.

Still, it may be asked, Am I really conscious of intelligence and reasonableness? The question, I think, is misleading. It suggests that there is a type of knowing in which intelligence and reasonableness come up for inspection. But what is asserted is not that you can uncover intelligence by introspection, as you can point to Calcutta on a map. The assertion is that you have conscious states and conscious acts that are intelligent and reasonable. Intelligent and rational consciousness denote characters of cognitional process, and the characters they denote pertain not to the contents but to the proceeding. It is repugnant to me to place astrology and astronomy, alchemy and chemistry, legend and history, hypothesis and fact, on exactly the same footing. I am not content with theories, however brilliantly coherent, but insist on raising the further question, Are they true? What is that repugnance, that discontent, that insistence? They are just so many variations on the

more basic expression that I am rationally conscious, that I demand sufficient reason, that I find it in the unconditioned, that I assent unreservedly to nothing less, that such demanding, finding, self-committing occur, not like the growth of my hair, but within a field of consciousness or awareness.

Again, if at moments I can slip into a lotus land in which mere presentations and representations are juxtaposed or successive, still that is not my normal state. The Humean world of mere impressions comes to me as a puzzle to be pieced together. I want to understand, to grasp intelligible unities and relations, to know what's up and where I stand. Praise of the scientific spirit that inquires, that masters, that controls, is not without an echo, a deep resonance within me, for, in my more modest way, I too inquire and catch on, see the thing to do and see that it is properly done. But what are these but variations on the more basic expression that I am intelligently conscious, that the awareness characteristic of cognitional acts on the second level is an active contributing to the intelligibility of its products? When I listen to the story of Archimedes and when I read the recital of a mystical experience, there is a marked difference. What a mystic experiences, I do not know. But, though I never enjoyed so remarkable an insight as Archimedes, still I do know what it is to miss the point and to get the point, not to have a clue and then to catch on, to see things in a new light, to grasp how they hang together, to come to know why, the reason, the explanation, the cause. After Archimedes shouted 'I've got it', he might well be puzzled by the question whether he was conscious of an insight. Still there can be no doubt that he was conscious of an increment of knowledge, an increment that he had wanted very much. Did he want the king's favour? Did he want to enhance his reputation? Perhaps, but at a deeper and more spontaneous level, he wanted to know how to do something, he wanted to solve a problem; he wanted to understand; his consciousness was on the second level where it seeks the intelligible and follows up partial insights with further questions until there comes the final crowning insight that ends questioning and satisfies intelligent consciousness.

### 3. THE UNITY OF CONSCIOUSNESS

In the fourth place, there are unities of consciousness. Besides cognitional contents there are cognitional acts; different kinds of acts have different kinds of awareness, empirical, intelligent, rational. But the



contents cumulate into unities: what is perceived is what is inquired about; what is inquired about is what is understood; what is understood is what is formulated; what is formulated is what is reflected on; what is reflected on is what is grasped as unconditioned; what is grasped as unconditioned is what is affirmed. Now, just as there are unities on the side of the object, so there are unities on the side of the subject. Conscious acts are not so many isolated, random atoms of knowing, but many acts coalesce into a single knowing. Not only is there a similarity between my seeing and your hearing, inasmuch as both acts are conscious; there also is an identity involved when my seeing and my hearing or your seeing and your hearing are compared. Moreover, this identity extends all along the line. Not only is the percept inquired about, understood, formulated, reflected on, grasped as unconditioned, and affirmed, but also there is an identity involved in perceiving, inquiring, understanding, formulating, reflecting, grasping the unconditioned, and affirming. Indeed, consciousness is much more obviously of this unity in diverse acts than of the diverse acts, for it is within the unity that the acts are found and distinguished, and it is to the unity that we appeal when we talk about a single field of consciousness and draw a distinction between conscious acts occurring within the field and unconscious acts occurring outside it.

One might go further and argue that, were the unity of consciousness not given, then it would have to be postulated. For many contents on diverse levels cumulate into a single known. But how can that occur? How can images be derived from sensations? How can inquiry be about percepts? How can insight be into images? How can definition draw upon both images and the ideas grasped in insight? How can reflecting be about formulations? How can the grasp of the unconditioned be obtained by combining the conditioned that is thought and the fulfilment that is sensed? How can each judgment emerge in a context of other judgments that determine its meaning, complement it, qualify it, defend it, so that it is but a single increment within a far vaster knowing? I cannot inquire into your experience or reflect on your thoughts. But if there were no 'I', how could there be a 'my experience' with respect to which a 'my inquiry' occurred, or 'my thoughts' with respect to which 'my reflection' occurred? If there were not one consciousness, at once empirical, intelligent, and rational, how could rational judgment proceed from an unconditioned grasped in the combination of thought and sensible experience?

### 4. THE UNITY AS GIVEN

Still, if the unity of consciousness would have to be postulated on the hypothesis that it were not given, it remains that it is given. By this, of course, I do not mean that it is the object of some inward look. What is meant is that a single agent is involved in many acts, that it is an abstraction to speak of the acts as conscious, that, concretely, consciousness pertains to the acting agent. Seeing and hearing differ inasmuch as one is an awareness of colour and the other an awareness of sound. Seeing and hearing are similar inasmuch as each is an awareness. But the similarity between my seeing and your hearing is an abstract indication of consciousness which, as it is given, is primarily an identity uniting my seeing and my hearing or your seeing and your hearing.

We have been engaged in determining what precisely is meant by consciousness. We have contended that it is not some inward look but a quality of cognitional acts, a quality that differs on the different levels of cognitional process, a quality that concretely is the identity immanent in the diversity and the multiplicity of the process. However, one cannot insist too strongly that such an account of consciousness is not itself consciousness. The account supposes consciousness as its data for inquiry, for insight, for formulation, for reflection, for grasp of the unconditioned, for judgment. But giving the account is the formulating and the judging, while the account itself is what is formulated and affirmed. Consciousness as given is neither formulated nor affirmed. Consciousness is given independently of its being formulated or affirmed. To formulate it does not make one more conscious, for the effect of formulation is to add to one's concepts. To affirm it does not make one more conscious, for the effect of affirmation is to add to one's judgments. Finally, as consciousness is not increased by affirming it, so it is not diminished by denying it, for the effect of denying it is to add to the list of one's judgments and not to subtract from the grounds on which judgments may be based.

By an experiential fulfilment, then, one does not mean the conditioned, nor the link between the conditioned and its conditions, nor the conditions as formulated, let alone as affirmed. One does mean that the conditions, which are formulated, also are to be found in a more rudimentary state within cognitional process. Just as inquiry brings about the advance from the perceived and not understood to the perceived and understood, so there is a reverse shift by which one moves



from the perceived and understood to the merely perceived. It is this reverse shift that commonly is meant by verification. If from a more general theory I obtain the formula, $PV=64$, then I can infer that when $P$ is 2, 4, 8, 16, 32, $V$ will have theoretically the values 32, 16, 8, 4, 2. By setting up suitable apparatus and securing appropriate conditions defined by the theory, I can advance from theoretical inference to an experimental check. The results of the experiment may be expressed in a series of propositions, such as the statement that, when $P$ was approximately 2, $V$ was approximately 32, but such a series of statements, however accurate, is not what was given by the experiment. The statements represent judgments of fact; the judgments rest on grasping the unconditioned; the grasp rests on formulations and visual experiences. The experiment gives neither statements nor judgments nor reflective understanding nor formulations but only visual experiences. The experiment gives not visual experiences as described but visual experiences on the level of merely seeing. That $P$ is 2 when the needle on a dial stands at a certain place, is a judgment. That $V$ is 32 when certain dimensions of an object coincide with certain dimensions of a measuring rod is another judgment. All that is seen is the needle in a position on the dial or the dimensions of an object standing in coincidence with numbered units on a rod. Nor is it this description that is seen, but only what is so described. In brief, verification is an appropriate pattern of acts of checking; acts of checking are reversals from formulations of what would be perceived to the corresponding but more rudimentary cognitional contents of acts of perceiving or sensing. In the formulation there always are elements derived from inquiry, insight, conceiving. But in virtue of the checking one can say that the formulation is not pure theory, that it is not merely supposed or merely postulated or merely inferred, that its sensible component is given.

Now just as there is reversal to what is given sensibly, so there is reversal to what is given consciously. Just as the former reversal is away from the understood as understood, the formulated as formulated, the affirmed as affirmed, and to the merely sensed, so also the latter reversal is from the understood, formulated, affirmed as such, to the merely given. Hence, in the self-affirmation of the knower, the conditioned is the statement, I am a knower. The link between the conditioned and its conditions is cast in the proposition, I am a knower if I am a unity performing certain kinds of acts. The conditions as formu-

lated are the unity-identity-whole to be grasped in data as individual and the kinds of acts to be grasped in data as similar. But the fulfilment of the conditions in consciousness is to be had by reverting from such formulations to the more rudimentary state of the formulated where there is no formulation but merely experience.

### 5. SELF-AFFIRMATION

From preliminary clarifications, we turn to the issue, Am I a knower? Each has to ask the question of himself. But anyone who asks it, is rationally conscious. For the question is a question for reflection, a question to be met with a 'Yes' or 'No'; and asking the question does not mean repeating the words but entering the dynamic state in which dissatisfaction with mere theory manifests itself in a demand for fact, for what is so. Further, the question is not any question. If I ask it, I know what it means. What do I mean by 'I'? The answer is difficult to formulate, but strangely, in some obscure fashion, I know very well what it means without formulation, and by that obscure yet familiar awareness, I find fault with various formulations of what is meant by 'I'. In other words, 'I' has a rudimentary meaning from consciousness and it envisages neither the multiplicity nor the diversity of contents and conscious acts but rather the unity that goes along with them. But if 'I' has some such rudimentary meaning from consciousness, then consciousness supplies the fulfilment of one element in the conditions for affirming that I am a knower. Does consciousness supply the fulfilment for the other conditions? Do I see, or am I blind? Do I hear, or am I deaf? Do I try to understand or is the distinction between intelligence and stupidity no more applicable to me than to a stone? Have I any experience of insight, or is the story of Archimedes as strange to me as the account of Plotinus' vision of the One? Do I conceive, think, consider, suppose, define, formulate, or is my talking like the talking of a parrot? I reflect, for I ask whether I am a knower. Do I grasp the unconditioned, if not in other instances, then in this one? If I grasped the unconditioned, would I not be under the rational compulsion of affirming that I am a knower and so, either affirm it, or else find some loop-hole, some weakness, some incoherence, in this account of the genesis of self-affirmation? As each has to ask these questions of himself, so too he has to answer them for himself. But the fact of the asking and the possibility of the answering are themselves the sufficient reason for the affirmative answer.



### 6. SELF-AFFIRMATION AS IMMANENT LAW

The foregoing account of self-affirmation stresses its positive aspect. It is a judgment of fact and so it rests heavily upon the experiential component in knowing. Still it is a singular type of judgment for it possesses a variety of overtones. I might not be yet, if I am, I am. I might be other than I am, yet, in fact, I am what I am. The contingent, if you suppose it as a fact, becomes conditionally necessary, and this piece of elementary logic places the merely factual self-affirmation in a context of necessity.

Am I a knower? The answer, Yes, is coherent, for if I am a knower, I can know that fact. But the answer, No, is incoherent, for if I am not a knower, how could the question be raised and answered by me? No less, the hedging answer, I do not know, is incoherent. For if I know that I do not know, then I am a knower; and if I do not know that I do not know, then I should not answer.

Am I a knower? If I am not, then I know nothing. My only course is silence. My only course is not the excused and explained silence of the sceptic, but the complete silence of the animal that offers neither excuse nor explanation for its complacent absorption in merely sensitive routines. For if I know nothing, I do not know excuses for not knowing. If I know nothing, then I cannot know the explanation of my ignorance.

It is this conditional necessity of contingent fact that involves the talking sceptic in contradiction. If enthusiasm for the achievement of Freud were to lead me to affirm that all thought and affirmation is just a by-product of the libido, then since I have admitted no exceptions, this very assertion of mine would have to be mere assertion from a suspect source. If second thoughts lead me to acknowledge an exception, they lead me to acknowledge the necessary presuppositions of the exception. By the time that list has been drawn up and accepted, I am no longer a sceptic.

Still the Aristotelian prescription of getting the sceptic to talk derives its efficacy not only from the conditional necessity of contingent fact but also from the nature, the natural spontaneities and natural inevitabilities, that go with that fact. Why is it that the talking sceptic does not talk gibberish? Why is it that one can count on his being nonplussed by self-contradiction? It is because he is conscious, empirically, intelligently, and rationally. It is because he has no choice in the matter. It

is because extreme ingenuity is needed for him not to betray his real nature. It is because, were his ingenuity successful, the only result would be that he had revealed himself an idiot and lost all claim to be heard.

This aspect of the matter deserves further attention. Cognitional process does not lie outside the realm of natural law. Not merely do I possess the power to elicit certain types of acts when certain conditions are fulfilled, but also with statistical regularity the conditions are fulfilled and the acts occur. I cannot escape sensations, percepts, images. All three keep occurring during my waking hours, and the images often continue during my sleep. No doubt, I can exercise a selective control over what I sense, perceive, imagine. But the choice I cannot make effective is to sense nothing, perceive nothing, imagine nothing. Not only are the contents of these acts imposed upon me, but also consciousness in some degree is inseparable from the acts. Nor is that consciousness merely an aggregate of isolated atoms; it is a unity.

If I cannot escape presentations and representations, neither can I be content with them. Spontaneously I fall victim to the wonder that Aristotle named the beginning of all science and philosophy. I try to understand. I enter, without questioning, the dynamic state that is revealed in questions for intelligence. Theoretically there is a disjunction between 'being intelligent' and 'not being intelligent'. But the theoretical disjunction is not a practical choice for me. I can deprecate intelligence; I can ridicule its aspirations; I can reduce its use to a minimum; but it does not follow that I can eliminate it. I can question everything else, but to question questioning is self-destructive. I might call upon intelligence for the conception of a plan to escape intelligence, but the effort to escape would only reveal my present involvement and, strangely enough, I would want to go about the business intelligently and I would want to claim that escaping was the intelligent thing to do.

As I cannot be content with the cinematographic flow of presentations and representations, so I cannot be content with inquiry, understanding, and formulation. I may say I want not the quarry but the chase, but I am careful to restrict my chasing to fields where the quarry lies. If, above all, I want to understand, still I want to understand the facts. Inevitably, the achievement of understanding, however stupendous, only gives rise to the further question, Is it so? Inevitably, the progress of understanding is interrupted by the check of judgment. Intelligence may be a thoroughbred exulting in the race; but there is a rider on its back; and, without the rider, the best of horses is a poor bet. The insistence that modern science envisages an indefinite future of repeated revisions does not imply an indifference to fact. On the contrary, it is fact that will force the revisions, that will toss into the wastebasket the brilliant theories of previous understanding, that will make each new theory better because it is closer to the facts. But what is fact? What is that clear, precise, definitive, irrevocable, dominant something that we name fact? The question is too large to be settled here. Each philosophy has its own view on what fact is and its consequent theory on the precise nature of our knowledge of fact. All that can be attempted now is to state what we happen to mean by knowing fact.



Clearly, then, fact is concrete as is sense or consciousness. Again, fact is intelligible: if it is independent of all doubtful theory, it is not independent of the modest insight and formulation necessary to give it its precision and its accuracy. Finally, fact is virtually unconditioned: it might not have been; it might have been other than it is; but as things stand, it possesses conditional necessity, and nothing can possibly alter it now. Fact, then, combines the concreteness of experience, the determinateness of accurate intelligence, and the absoluteness of rational judgment. It is the natural objective of human cognitional process. It is the anticipated unity to which sensation, perception, imagination, inquiry, insight, formulation, reflection, grasp of the unconditioned, and judgment make their several, complementary contributions. When Newton knew that the water in his bucket was rotating, he knew a fact, though he thought that he knew absolute space. When quantum mechanics and relativity posit the unimaginable in a four-dimensional manifold, they bring to light the not too surprising fact that scientific intelligence and verifying judgment go beyond the realm of imagination to the realm of fact. Just what that realm is, as has been said, is a difficult and complicated problem. Our present concern is that we are committed to it. We are committed, not by knowing what it is and that it is worth while, but by an inability to avoid experience, by the subtle conquest in us of the Eros that would understand, by the inevitable aftermath of that sweet adventure when a rationality identical with us demands the absolute, refuses unreserved assent to less than the unconditioned and, when that is attained, imposes upon us a commitment in which we bow to an immanent Anagke.

Confronted with the standard of the unconditioned, the sceptic despairs. Set before it, the products of human understanding are ashamed.

Great are the achievements of modern science; by far are they to be preferred to earlier guesswork; yet rational consciousness finds that they approximate indeed to the unconditioned but do not attain it; and so it assigns them the modest status of probability. Still, if rational consciousness can criticize the achievement of science, it cannot criticize itself. The critical spirit can weigh all else in the balance, only on condition that it does not criticize itself. It is a self-assertive spontaneity that demands sufficient reason for all else but offers no justification for its demanding. It arises, fact-like, to generate knowledge of fact, to push the cognitional process from the conditioned structures of intelligence to unreserved affirmation of the unconditioned. It occurs. It will recur whenever the conditions for reflection are fulfilled. With statistical regularity those conditions keep being fulfilled. Nor is that all, for I am involved, engaged, committed. The disjunction between rationality and non-rationality is an abstract alternative but not a concrete choice. Rationality is my very dignity, and so closely to it do I cling, that I would want the best of reasons for abandoning it. Indeed, I am so much one with my reasonableness that, when I lapse from its high standards, I am compelled either to repent my folly or to rationalize it.

Self-affirmation has been considered as a concrete judgment of fact. The contradiction of self-negation has been indicated. Behind that contradiction there have been discerned natural inevitabilities and spontaneities that constitute the possibility of knowing, not by demonstrating that one can know, but pragmatically by engaging one in the process. Nor in the last resort can one reach a deeper foundation than that pragmatic engagement. Even to seek it involves a vicious circle; for if one seeks such a foundation, one employs one's cognitional process; and the foundation to be reached will be no more secure or solid than the inquiry utilized to reach it. As I might not be, as I might be other than I am, so my knowing might not be and it might be other than it is. The ultimate basis of our knowing is not necessity but contingent fact, and the fact is established, not prior to our engagement in knowing, but simultaneously with it. The sceptic, then, is not involved in a conflict with absolute necessity. He might not be; he might not be a knower. Contradiction arises when he utilizes cognitional process to deny it.

### 7. DESCRIPTION AND EXPLANATION

There is a further aspect to the matter. Is the self-affirmation that has been outlined descriptive of the thing-for-us or explanatory of the thing-itself? We have spoken of natural inevitabilities and spontaneities. But did we speak of these as they are themselves or as they are for us?



Unfortunately, there is a prior question. The distinction that was drawn earlier between description and explanation was couched in terms that sufficed to cover the difference in the fields of positive science. But human science contains an element not to be found in other departments. Both the study of man and the study of nature begin from inquiry and insight into sensible data. Both the study of man and the study of nature can advance from the descriptive relations of the object to the inquirer, to the explanatory relations that obtain immediately between objects. Just as the physicist measures, correlates measurements, and implicitly defines correlatives by the correlations, so too the student of human nature can forsake the literary approach to determine economic, political, sociological, cultural, historical correlations. But the study of man also enjoys through consciousness an immediate access to man, and this access can be used in two manners.

The initial use is descriptive. In this fashion we began from an account of an event named insight. We pointed out that it was satisfying, that it came unexpectedly, that its emergence was conditioned more by a dynamic inner state of inquiry than by external circumstance, that while the first emergence was difficult, repeated occurrence was easy and spontaneous, that single acts of insight accumulate into clusters bearing on a single topic, that such clusters may remain without exact formulation, or may be worked out into a systematic doctrine. Naturally enough, this general description of insight was presupposed and utilized when we came to examine it more closely; and this closer examination was in turn presupposed in our account of explanatory abstraction and explanatory system and in our study of empirical method. Moreover, since data, percepts, and images are prior to inquiry, insight, and formulation, and since all definition is subsequent to inquiry and insight, it was necessary to define data, percepts, and images as the materials presupposed and complemented by inquiry and insight and, further, it was necessary to distinguish between them by contrasting the formulations of empirical science with those of mathematics and the formulations of both of these with the formulations of common sense. Finally, the analysis of judgment and the account of reflective understanding consisted in relating these acts to each other, and to

the formulations of understanding, and to the fulfilment provided by experience

As the reader will discern, the initial procedure of description gradually yielded to definition by relation; and the defining relations obtained immediately between different kinds of cognitional state or act. But definition by this type of relation is explanatory, and so descriptive procedure was superseded by explanatory.

There are, then, two types of description and two types of explanation. If the inquirer starts from the data of sense, he begins by describing but goes on to explain. Again, if he starts from the data of consciousness, he begins by describing and goes on to explain. Still, there is an important difference between the two types of explaining. For explanation on the basis of sense can reduce the element of hypothesis to a minimum but it cannot eliminate it entirely. But explanation on the basis of consciousness can escape entirely the merely supposed, the merely postulated, the merely inferred.

First, explanation on the basis of sense can reduce hypothesis to a minimum. This, of course, is the point of the principle of relevance. Galileo's law of falling bodies does not merely suppose or postulate distance or time or the measurements of either. It does not merely suppose or postulate the correlation between distance and time; for there is some relation between the two inasmuch as a falling body falls farther in a longer time; and the actual measurements ground a numerical determination of that relation. Moreover, what holds for the law of falling bodies, holds for the other laws of mechanics. If one pleases, one may contend that the use of inquiry, insight, formulation, and consequent generalization, is mere supposition or mere postulation; but at least it is not the type of mere supposition that the empirical scientist systematically avoids or that he seriously fears will be eliminated in some more intelligent method of inquiry to be devised and accepted in the future. To reach the element of mere supposition that makes any system of mechanics subject to future revision, one must shift attention from single laws to the set of primitive terms and relations which the system employs in formulating all its laws. In other words, one has to distinguish between, say, mass as defined by correlations between masses and, on the other hand, mass as enjoying the position of an ultimate mechanical concept. Any future system of mechanics will have to satisfy the data that now are covered by the notion of mass. But it is not necessary that every future system of mechanics will have



to satisfy the same data by employing our concept of mass. Further developments might lead to the introduction of a different set of ultimate concepts, to a consequent reformulation of all laws, and so to a dethronement of the notion of mass from its present position as an ultimate of mechanical system. Hence, while empirical method can reduce the hypothetical to a minimum, it cannot eliminate it entirely. Its concepts as concepts are not hypothetical, for they are defined implicitly by empirically established correlations. None the less, its concepts as systematically significant, as ultimate or derived, as preferred to other concepts that might be empirically reached, do involve an element of mere supposition. For the selection of certain concepts as ultimate occurs in the work of systematization, and that work is provisional. At any time a system is accepted because it provides the simplest account of all the known facts. But at the same time it is acknowledged that there may be unknown yet relevant facts, that they might give rise to further questions that would lead to further insights, and that the further insights might involve a radical revision of the accepted system.

Secondly, explanation on the basis of consciousness can escape this limitation. I do not mean, of course, that such explanation is not to be reached through the series of revisions involved in the self-correcting process of learning. Nor do I mean that, once explanation is reached, there remains no possibility of the minor revisions that leave basic lines intact but attain a greater exactitude and a greater fullness of detail. Again, I am not contending here and now that human nature and so human knowledge are immutable, that there could not arise a new nature and a new knowledge to which present theory would not be applicable. What is excluded is the radical revision that involves a shift in the fundamental terms and relations of the explanatory account of the human knowledge underlying existing common sense, mathematics and empirical science.

### 8. THE IMPOSSIBILITY OF REVISION

The impossibility of such revision appears from the very notion of revision. A revision appeals to data. It contends that previous theory does not satisfactorily account for all the data. It claims to have reached complementary insights that lead to more accurate statements. It shows that these new statements either are unconditioned or more closely approximate to the unconditioned than previous statements. Now, if in

fact revision is as described, then it presupposes that cognitional process falls on the three levels of presentation, intelligence, and reflection; it presupposes that insights are cumulative and complementary; it presupposes that they head towards a limit described by the adjective, satisfactory; it presupposes a reflective grasp of the unconditioned or of what approximates to the unconditioned. Clearly, revision cannot revise its own presuppositions. A reviser cannot appeal to data to deny data, to his new insights to deny insights, to his new formulation to deny formulation, to his reflective grasp to deny reflective grasp.

The same point may be put in another manner. Popular relativism is prone to argue that empirical science is the most reliable form of human knowledge; but empirical science is subject to indefinite revision; therefore, all human knowledge is equally subject to indefinite revision. Now such argument is necessarily fallacious. One must definitely know invariant features of human knowledge before one can assert that empirical science is subject to indefinite revision; and if one definitely knows invariant features of human knowledge, then one knows what is not subject to revision. Moreover, as is obvious, such knowledge surpasses empirical science at least in the respect that it is not subject to revision.

### 9. Self-Affirmation in the Possibility of Judgments of Fact

The same conclusion may be reached by setting forth the *a priori* conditions of any possible judgment of fact. For any such judgment can be represented by a 'Yes' or 'No' in answer to a question, Is it so? The answer will be rational, that is, it will rest on known sufficient reason. Moreover, the answer will be absolute; 'Yes' utterly excludes 'No'; and 'No' utterly excludes 'Yes'. Hence, since the known sufficient reason for an absolute answer must itself be absolute and known, the 'Yes' or 'No' must rest on some apprehension or grasp of the unconditioned. Now the judgment of fact is not to the effect that something must be so or could not be otherwise; it merely states that something is so; hence the unconditioned that grounds it will be not formally but only virtually unconditioned. The first condition, then, of any possible judgment of fact is the grasp of

(1) a conditioned,
(2) a link between the conditioned and its conditions, and
(3) the fulfilment of the conditions.



It is such a grasp that effects the transition from the question, Is it so? to a rational, absolute answer.

But this first requirement presupposes other requirements. The 'it' of the judgment of fact is not a bare 'it'. On the contrary, it is the conditioned, known as conditioned, that through the fulfilment of its conditions is grasped as virtually unconditioned. Prior to the question for reflection, there must be a level of activity that yields the conditioned as conditioned, the conditioned as linked to its conditions. But this is a level of intelligence, of positing systematic unities and systematic relations. Moreover, it will be a freely developing level; for without free development questions of fact would not arise. The only instances of the conditioned that would be envisaged would be instances with the conditions fulfilled. In that case the answer would always be an automatic 'Yes'; and if the answer were always an automatic 'Yes', there would be no need to raise any questions of fact. Still, though there is free development of systematic unities and relations, such development cannot occur in some pure isolation from the fulfilling conditions. Were there such isolation, it would be impossible to tell whether or not conditions were fulfilled; and if that were impossible, then judgments of fact could not occur. This yields the second condition of judgment of fact. It is a level of intellectual activity that posits systematic unities and relations

(1) with some independence of a field of fulfilling conditions, and
(2) with reference to such a field.

But this second requirement presupposes a third. There must be a field of fulfilling conditions. More exactly, since conditions are simultaneous with what they condition, there must be a prior field containing what can become fulfilling conditions. Of themselves, they will be neither conditioning nor conditioned; they will be merely given.

Finally, possibility is concrete. Logicians may say that a 'mountain of gold' is possible if there is no intrinsic contradiction involved in supposing such a mountain. But, in fact, a mountain of gold is possible only if the means are available for acquiring enough gold to make a mountain, for transporting it to a single place, for heaping it up in the fashion of a mountain, and for keeping it there long enough for the golden mountain to exist for some minimum interval of time. Similarly, any possible judgment of fact would be some concrete judgment

The conditions of its possibility include the conditions of bringing together its diverse components. There must be, then, a concrete unity-identity-whole that experiences the given, that inquires about the given to generate the free development of systematic unities and relations, that reflects upon such developments and demands the virtually unconditioned as its ground for answering 'Yes' or 'No'. It is this concrete unity that asks, 'Is it so?' It is this concrete unity that initiates the free development by asking about the given, What is this? Why is it? How often does it exist or happen? It is this concrete unity that grasps and formulates the conditioned as conditioned and that appeals to the given to grasp the virtually unconditioned and to affirm it rationally and absolutely.

There remains a corollary. Judgments of fact may be not only possible. They may actually occur. But if any judgment of fact occurs, there must be as well the occurrence of its conditions. Hence, if there is any judgment of fact, no matter what its content, there also is a concrete unity-identity-whole that experiences some given, that inquires, understands, and formulates, that reflects, grasps the unconditioned, and so affirms or denies. Finally, such a concrete unity-identity-whole is a thing-itself, for it is defined by an internally related set of operations, and the relations may be experientially validated in the conscious and dynamic states:

(1) of inquiry leading from the given to insight,
(2) of insight leading to formulation,
(3) of reflection leading from formulation to grasp of the unconditioned, and
(4) of that grasp leading to affirmation or denial.

From the corollary there results our prior contention. There cannot occur a revision without the occurrence of some judgment of fact. But if there occurs any judgment of fact, there occur the dynamic states in which may be validated experientially the relations that define the conjugate terms by which the thing-itself that knows is differentiated.

What is the source of this peculiarity of cognitional theory? It is that other theory reaches its thing-itself by turning away from the thing as related to us by sense or by consciousness, but cognitional theory reaches its thing-itself by understanding itself and affirming itself as concrete unity in a process that is conscious empirically, intelligently, and rationally. Moreover, since every other known becomes known through this process, no known could impugn the process without simultaneously impugning its own status as a known.



### 10. CONTRAST WITH KANTIAN ANALYSIS

We have performed something similar to what a Kantian would name a transcendental deduction. Accordingly, we shall be asked to explain the fact that our deduction yields different results from Kant's.

A first difference is that Kant asked the *a priori* conditions of the possibility of experience in the sense of knowing an object. We have distinguished two issues; there is the problem of objectivity, and from this we have carefully prescinded not only in the present section but also in all earlier sections; there also is the prior problem of determining just what activities are involved in knowing, and to this prior problem we have so far confined our efforts. Hence we asked, not for the conditions of knowing an object, but for the conditions of the possible occurrence of a judgment of fact. We have asked for the conditions of an absolute and rational 'Yes' or 'No' viewed simply as an act. We have not asked on what conditions there would be some fact that corresponded to the 'Yes'. We have not even asked what meaning such correspondence might have.

A second difference lies in the distinction between thing-for-us and thing-itself. Kant distinguished these as phenomenon and noumenon. Just what he meant is a matter of dispute but, at least, it is clear that the distinction pertained to his formulation of a theory of objectivity. Moreover, it seems to me to be probable enough that the historical origin of the Kantian distinction is to be sought in the Renaissance distinction of primary and secondary qualities where the former pertained to the real and objective things themselves while the latter pertained to the subject's apprehension of them. In any case, our distinction is neither the Renaissance nor the Kantian distinction. It is simply a distinction between description and explanation, between the kind of cognitional activities that fix contents by indicating what they resemble and, on the other hand, the kind that fix contents by assigning their experientially validated relations. A thing is a concrete unity-identity-whole grasped in data as individual. Describe it, and it is a thing-for-us. Explain it, and it is a thing-itself. Is it real? Is it objective? Is it anything more than the immanent determination of the cognitional act? These are all quite reasonable questions. But as yet we answer neither 'Yes' nor 'No'. For the moment, our answer is simply that objectivity is a highly complex

issue and that we shall handle it satisfactorily only if we begin by determining what precisely cognitional process is. No doubt, there are objections that may be urged against this procedure; but the objections too will be handled satisfactorily only after the prior questions are answered.

A third difference regards universal and necessary judgments. They stand in the forefront of the Kantian critique which was largely engaged in the problem of transcending Hume's experiential atomism. But in our analysis they play a minor role. A universal and necessary judgment may be merely the affirmation of an analytic proposition, and such analytic propositions may be mere abstract possibilities without relevance to the central context of judgments that we name knowledge. Our emphasis falls on the judgment of fact that itself is an increment of knowledge and, as well, contributes to the transition from the analytic proposition to the analytic principle, that is, to the universal and necessary judgment whose terms and relations are existential in the sense that they occur in judgments of fact.

A fourth difference regards the immediate ground of judgment. Kant formulated this ground by setting forth his schematism of the categories. There is a proper use of the category, Real, if there occurs a filling of the empty form of Time. There is a proper use of the category, Substance, if there is a permanence of the Real in Time. However, Kant's schematism is not regarded as one of his happiest inventions. In any case we have argued that by their very genesis concepts are united with data. Inquiry is about the data of sense or of consciousness. Insight is into the data of inquiry. Concepts and theories are the products of insight and have to be checked against the data. Moreover, and this is the essential difference, the process of checking reveals in human knowledge, beyond experience and understanding, a third, distinct, constitutive level that is both self-authenticating and decisive. It is self-authenticating. rational reflection demands and reflective understanding grasps a virtually unconditioned, and once that grasp has occurred, one cannot be reasonable and yet fail to pass judgment. Again, the third level is alone decisive. until I judge, I am merely thinking, once I judge, I know, as insight draws the definite object of thought from the hazy object of experience, so judgment selects the objects of thought that are objects of knowledge. Finally, as will appear in chapters XII and XIII, to know means to know being, and to know being includes knowing objects and subjects.



Now because the third level is self-authenticating, reason and its ideal, the unconditioned, cannot be left in the dubious and merely supervisory role assigned them by Kant. Because it is constitutive and alone decisive, the one criterion in our knowledge is rational judgment; and this rules out the vestigial empiricism so often denounced in Kantian thought. Still, our unconditioned is only virtual; it is only what is so in fact; and the universal relevance of fact in this sense (see p. 331) both corrects pre-Kantian rationalism and excludes post-Kantian idealism. Finally, our realism, while not intuitive, will be immediate: cognitional analysis is needed not to know being but to know knowledge.

A fifth difference has to do with consciousness. Kant acknowledged an inner sense that corresponds roughly to what we have named empirical consciousness, namely, the awareness that is immanent in acts of sensing, perceiving, imagining, desiring, fearing, and the like. Besides this acknowledgement of inner sense, Kant deduced or postulated an original synthetic unity of apperception as the *a priori* condition of the '*I think*' accompanying all cognitional acts. On the other hand, Kantian theory has no room for a consciousness of the generative principles of the categories, the categories may be inferred from the judgments in which they occur; but it is impossible to reach behind the categories to their source. It is precisely this aspect of Kantian thought that gives the categories their inflexibility and their irreducible mysteriousness. It is the same aspect that provided Fichte and Hegel with their opportunity to march into the unoccupied territory of intelligent and rational consciousness. The dynamic states named inquiry and reflection do occur. Inquiry is generative of all understanding, and understanding is generative of all concepts and systems. Reflection is generative of all reflective grasp of the unconditioned, and that grasp is generative of all judgment. If the Kantian proscribes consideration of inquiry and reflection, he lays himself open to the charge of obscurantism. If he admits such consideration, if he praises intelligent curiosity and the critical spirit, then he is on his way to acknowledge the generative principles both of the categories Kant knew and of the categories Kant did not know.

The foregoing list of differences account for the divergence between Kant's conclusion and our own. They are differences in the problem under consideration, in the viewpoint from which it is considered, in the method by which it is solved. More fundamentally there are differ-

ences about questions of fact, for our self-affirmation is, as we have insisted and may be pardoned for repeating, primarily and ultimately a judgment of fact. The orthodox Kantian would refer to our stand as mere psychologism, as an appeal to the empirical that can yield no more than a provisional probability. But our retort is simple enough. Without judgments of fact one cannot get beyond mere analytic propositions. Further, though self-affirmation is no more than a judgment of mere fact, still it is a privileged judgment. Self-negation is incoherent. One has only to inquire and reflect, to find oneself caught in the spontaneities and inevitabilities that supply the evidence for self-affirmation. One has only to make a single judgment of fact, no matter what its content, to involve oneself in a necessary self-affirmation. Finally, cognitional theory differs from other theory; for other theory reaches explanation only by venturing into the merely supposed; but cognitional theory reaches explanation without any such venture; and since it contains no merely hypothetical element, it is not subject to radical revision.

### II. CONTRAST WITH RELATIVIST ANALYSIS

From Kantian we turn to relativist thought. The initial question in the present section was whether correct judgments occur. Our account of self-affirmation directly contradicts the relativist contention that correct judgments do not occur. Though the arguments for our position have been given, it will not be amiss to indicate where the relativist would disagree and why.

First, relativist thought is largely devoted to a refutation of empiricism. Correctly it insists that human knowing cannot be accounted for by the level of presentations alone. There is, as well, the level of intelligence, of grasping and formulating intelligible unities and systematic relations. Without this second level of activities, there is, indeed, a given but there is no possibility of saying what is given.

Secondly, just as the relativist insists on the level of intelligence against the empiricist, so we insist on the level of reflection against the relativist. Human knowing is not merely theory about the given; there are also facts; and the relativist has not and cannot establish that there are no facts, for the absence of any other fact would itself be a fact.

Thirdly, just as the empiricist could have nothing to say if, in fact, he did not utilize operations on the level of intelligence, so also the relativist does not confine himself strictly to the levels of presentations and of intelligence. He is quite familiar with the notion of the unconditioned. He regards the unconditioned as the ideal towards which human knowing tends. But he supposes that this ideal is to be reached through understanding. If the universe in its every part and aspect were thoroughly understood, there could be no further questions; everything would be conceived as it ought to be, on every possible topic a man could say just what he meant and mean just what he said. On the other hand, short of this comprehensive coherence, there can be no sure footing. There is understanding, but it is partial; it is joined with incomprehension; it is open to revision when present incomprehension yields to future understanding; and so intimately are all things related that knowledge of anything can be definitive only when everything is known.



Fourthly, the relativist is able to follow up this general view by facing concrete issues. Is this a typewriter? Probably, Yes. For practical purposes, Yes. Absolutely? The relativist would prefer to be clear about the precise meaning of the name, *typewriter;* he would like to be told just what is meant by the demonstrative, *this;* he would be grateful for an explanation of the meaning of the copula, *is.* Your simple question is met by three further questions; and if you answer these three, your answers will give rise to many more. If you are quick and see that you are starting on an infinite series, you may confront the relativist with a rounded system. But the relativist is also a smart fellow. He will point out that ordinary people, quite certain that this is a typewriter, know nothing of the system on which you base their knowledge. Nor is this all. For human knowledge is limited; systems have their weak points; and the relativist will pounce upon the very issues on which a defender of the system would prefer to profess ignorance.

Fifthly, not only will the relativist make it plain that there are further questions until everything is known, but also he will explain why this is so. A relation is named internal to an object when, without the relation, the object would differ radically. Thus, we have spoken of inquiry and insight. But by inquiry we have not meant some pure wonder; we have meant a wonder about something. Similarly, by insight we have not meant a pure understanding but an understanding of something. Inquiry and insight, then, are related internally to materials about which one inquires and into which one gains insight. Now, if one supposes that the whole universe is a pattern of internal relations,

clearly it follows that no part and no aspect of the universe can be known in isolation from any other part or aspect; for every item is related internally to every other; and to prescind from such relations is to prescind from things as they are and to substitute in their place other imaginary objects that simply are not. If, then, one asks the relativist to explain why questions run off to infinity, he has a ready answer. The universe to be known by answering questions is a tissue of internal relations.

Sixthly, if the foregoing fairly represents the relativist position, it also reveals its oversights. Questions are of two kinds. There are questions for intelligence asking what this is, what that means, why this is so, how frequently it occurs or exists. There also are questions for reflection that ask whether answers to the former type of question are correct. Next, the unconditioned that is required for judgment is not the comprehensive coherence that is the ideal of understanding, that grounds answers to all questions of the first type. On the contrary, it is a virtually unconditioned that results from the combination of a conditioned with the fulfilment of its conditions. Further, a judgment is a limited commitment; so far from resting on knowledge of the universe, it is to the effect that, no matter what the rest of the universe may prove to be, at least this is so. I may not be able to settle border-line instances in which one might dispute whether the name, *typewriter*, would be appropriate. But, at least, I can settle definitely that this is a typewriter. I may not be able to clarify the meaning of *is*, but it is sufficient for present purposes to know the difference between *is* and *is not*, and that, I know. I am not very articulate when it comes to explaining the meaning of *this;* but if you prefer to use *that*, it will make no difference provided we both see what we are talking about. You warn me that I have made mistakes in the past. But your warning is meaningless, if I am making a further mistake in recognizing a past mistake as a mistake. And in any case, the sole present issue is whether or not I am mistaken in affirming this to be a typewriter. You explain to me that my notion of a typewriter would be very different, if I understood the chemistry of the materials, the mechanics of the construction, the psychology of the typist's skill, the effect on sentence structure resulting from the use of a machine in composing, the economic and sociological repercussions of the invention, its relation to commercial and political bureaucracy, and so forth. But may I not explain to you that all these further items, however interesting and



significant, are to be known through further judgments, that such further judgments, so far from shifting me from my present conviction that this is a typewriter, will only confirm me in it, that to make those further judgments would be rather difficult if, at the start, I could not be certain whether or not this is a typewriter?

Seventhly, however, the questions that are answered by a pattern of internal relations are only questions that ask for explanatory system. But besides things-themselves and prior to them in our knowing, there are things-for-us, things as described. Moreover, the existents and occurrences, in which explanatory systems are verified, diverge non-systematically from the ideal frequencies that ideally would be deduced from the explanatory systems. Again, the activity of verifying involves the use of description as an intermediary between the system defined by internal relations and, on the other hand, the presentations of sense that are the fulfilling conditions. Finally, it would be a mistake to suppose that explanation is the one true knowledge; not only does its verification rest on description but also the relations of things to us are just as much objects of knowledge as are the relations of things among themselves.

Eighthly, the relativist invents for himself a universe that consists merely of explanatory system because he conceives the unconditioned as the ideal of understanding, as the comprehensive coherence towards which understanding tends by asking what and why. But as we have seen, the criterion of judgment is the virtually unconditioned. Each judgment is a limited commitment. So far from pronouncing on the universe, it is content to affirm some single conditioned that has a finite number of conditions which, in fact, are fulfilled. No doubt, were the universe simply a vast explanatory system, knowledge of the conditions of any conditioned would be identical with knowledge of the universe. But, in fact, the universe is not simply explanatory system; its existents and its occurrences diverge non-systematically from pure intelligibility; it exhibits an empirical residue of the individual, the incidental, the continuous, the merely juxtaposed, and the merely successive; it is a universe of facts, and explanatory system has validity in the measure that it conforms to descriptive facts.

Ninthly, the relativist argument from unending further questions is more impressive than conclusive. Human knowing does not begin from previous knowing but from natural spontaneities and inevitabilities. Its basic terms are not defined for it in some knowing prior to

knowing; they are fixed by the dynamic structure of cognitional process itself. The relativist asks what is meant by the copula, *is*, and the demonstrative, *this*. But neither he nor anyone else is given to confusing *is* with *is not* or *this* with *not this;* and that basic clarity is all that is relevant to the meaning of the affirmation, This is a typewriter. A cognitional theorist would be called upon to explain such elementary terms; he would do so by saying that *is* represents the *Yes* that occurs in judgment and that is anticipated by such questions as, Is it? What is it? Similarly, a theorist would explain *this* as the return from the field of conception to the empirical residue in the field of presentations. But questions relevant to cognitional theory are not relevant to every instance of knowing. They are not universally relevant because, in fact, there is no operational obscurity about the meanings that cognitional theory elucidates. Again, they are not universally relevant, because such elementary meanings are fixed, in a manner that surpasses determination by definition, with the native immutability of the dynamic structures of cognitional process.

Tenthly, as human knowing begins from natural spontaneity, so its initial developments are inarticulate. As it asks what and why without being given the reason for its inquiry, so also it sets off on the self-correcting process of learning without the explicit formulations that rightly would be required in an explanatory system. Single insights are partial. Spontaneously they give rise to the further questions that elicit complementary insights. Were the universe purely an explanatory system, the minor clusters of insights reached by what is called common sense would not head for a limiting position of familiarity and mastery in which evidently it is silly to doubt whether or not this is a typewriter. But, in fact, the universe to be known by answering questions is not pure explanatory system. In fact, insights do head for limiting positions of familiarity and mastery. In fact, as everyone knows very well, it is silly to doubt whether or not this is a typewriter. The relativist would beg me to advert to the enormous difference in my notion of the typewriter were I to understand fully the chemistry of its materials, the mechanics of its construction, the psychology of the typist's skill, the twist given literary style by composing on a typewriter, the effect of its invention on the development of commercial and political bureaucracy, and so forth. But granted such an enrichment of my knowledge to be possible and desirable, none the less it is further knowledge to be obtained by further judgments; and since the enrichment is explanatory, since explanatory knowledge rests on descriptive knowledge, not only must I begin by knowing that this is a typewriter, not only must I advance by learning how similar other machines must be if they are to be named typewriters, but also I can attain valid explanation only in so far as my descriptions are exact.



Eleventhly, it is quite true that I can be mistaken. But that truth presupposes that I am not making a further mistake in acknowledging a past mistake as a mistake. More generally, judgments of fact are correct or incorrect, not of necessity, but merely in fact. If this is something, still it might be nothing at all. If it is a typewriter, still it might be something else. Similarly, if I am correct in affirming it to be a typewriter, it is not a pure necessity, but merely a fact that I am correct. To ask for the evidence that excludes the possibility of my being mistaken in affirming this to be a typewriter, is to ask too much. Such evidence is not available, for if I am correct, that is merely fact. But if that evidence is not available, still less is there the evidence that will exclude the possibility of error in all judgments of fact. Errors are just as much facts as are correct judgments. But the relativist is in conflict with both categories of fact. For him nothing is simply true, for that is possible only when comprehensive coherence is reached; for him, nothing is simply wrong, for every statement involves some understanding and so some part of what he names truth. In the last analysis, just as the empiricist tries to banish intelligence, so the relativist tries to banish fact and, with it, what everyone else names truth.

CHAPTER XII

# THE NOTION OF BEING

If the main lines of cognitional process have been set down, it remains that certain fundamental and pervasive notions have still to be clarified. Among them, in the first place, is the notion of being. It is a tricky topic and, perhaps, the most satisfactory procedure will be to begin from a definition.

## 1. A DEFINITION

Being, then, is the objective of the pure desire to know.

By the desire to know is meant the dynamic orientation manifested in questions for intelligence and for reflection. It is not the verbal utterance of questions. It is not the conceptual formulation of questions. It is not any insight or thought. It is not any reflective grasp or judgment. It is the prior and enveloping drive that carries cognitional process from sense and imagination to understanding, from understanding to judgment, from judgment to the complete context of correct judgments that is named knowledge. The desire to know, then, is simply the inquiring and critical spirit of man. By moving him to seek understanding, it prevents him from being content with the mere flow of outer and inner experience. By demanding adequate understanding, it involves man in the self-correcting process of learning in which further questions yield complementary insights. By moving man to reflect, to seek the unconditioned, to grant unqualified assent only to the unconditioned, it prevents him from being content with hearsay and legend, with unverified hypotheses and untested theories. Finally, by raising still further questions for intelligence and reflection, it excludes complacent inertia; for if the questions go unanswered, man cannot be complacent; and if answers are sought, man is not inert.

Because it differs radically from other desire, this desire has been named pure. It is to be known, not by the misleading analogy of other desire, but by giving free rein to intelligent and rational consciousness It is, indeed, impalpable but also it is powerful. It pulls man out of the solid routine of perception and conation, instinct and habit, doing and enjoying. It holds him with the fascination of problems. It engages him





in the quest of solutions. It makes him aloof to what is not established. It compels assent to the unconditioned. It is the cool shrewdness of common sense, the disinterestedness of science, the detachment of philosophy. It is the absorption of investigation, the joy of discovery, the assurance of judgment, the modesty of limited knowledge. It is the relentless serenity, the unhurried determination, the imperturbable drive of question following appositely on question in the genesis of truth.

This pure desire has an objective. It is a desire to know. As mere desire, it is for the satisfaction of acts of knowing, for the satisfaction of understanding, of understanding fully, of understanding correctly. But as pure desire, as cool, disinterested, detached, it is not for cognitional acts, and the satisfaction they give their subject, but for cognitional contents, for what is to be known. The satisfaction of mistaken understanding, provided one does not know it as mistaken, can equal the satisfaction of correct understanding. Yet the pure desire scorns the former and prizes the latter; it prizes it, then, as dissimilar to the former; it prizes it not because it yields satisfaction but because its content is correct.

The objective of the pure desire is the content of knowing rather than the act. Still, the desire is not itself a knowing, and so its range is not the same as the range of knowing. Initially in each individual, the pure desire is a dynamic orientation to a totally unknown. As knowledge develops, the objective becomes less and less unknown, more and more known. At any time the objective includes both all that is known and all that remains unknown, for it is the goal of the immanent dynamism of cognitional process, and that dynamism both underlies actual attainment and heads beyond it with ever further questions.

What is this objective? Is it limited or unlimited? Is it one or many? Is it material r ideal? Is it phenomenal or real? Is it an immanent content or a transcendent object? Is it a realm of experience, or of thought, or of essences, or of existents? Answers to these and to any other questions have but a single source They cannot be had without the functioning of the pure desire. They cannot be had from the pure desire alone. They are to be had inasmuch as the pure desire initiates and sustains cognitional process. Thus, if it is true that *A* is, that *A* is one, and that there is only *A*, then the objective of the pure desire is one. But if it is true that *A* is, that *B* is, that *A* is not *B*, then the objective is many. Which, you ask, is true? The fact that you ask, results from the pure

desire. But to reach the answer, desiring is not enough; answers come only from inquiring and reflecting.

Now our definition was that being is the objective of the pure desire to know. Being, then, is

(1) all that is known, and
(2) all that remains to be known.

Again, since a complete increment of knowing occurs only in judgment, being is what is to be known by the totality of true judgments. What, one may ask, is that totality? It is the complete set of answers to the complete set of questions. What the answers are, remains to be seen. What the questions are, awaits their emergence. Meaningless or incoherent or illegitimate questions may be possible, but how they are to be defined, is a further question. The affirmation in hand is that there exists a pure desire to know, an inquiring and critical spirit, that follows up questions with further questions, that heads for some objective which has been named being.

Our definition of being, then, is of the second order. Other definitions determine what is meant. But this definition is more remote for it assigns, not what is meant by being, but how that meaning is to be determined. It asserts that if you know, then you know being; it asserts that if you wish to know, then you wish to know being; but it does not settle whether you know or what you know, whether your wish will be fulfilled or what you will know when it is fulfilled.

Still, though our definition is of the second order, it is not simply indeterminate. For neither the desire to know nor knowing itself are indeterminate. Inasmuch as knowing is determinate, we could say that being is what is to be known by true judgments. Inasmuch as the desire to know ever goes beyond actual knowledge, we could say that being is what is to be known by the totality of true judgments. Hence, being has at least one characteristic: it is all-inclusive. Apart from being there is nothing. Again, being is completely concrete and completely universal. It is completely concrete; over and above the being of any thing, there is nothing more of that thing. It is completely universal; apart from the realm of being, there is simply nothing.

## 2. AN UNRESTRICTED NOTION

One may wonder just how all-inclusive being is. That wonder may be formulated in a variety of manners. But no matter how it is formulated, no matter whether it can be formulated, it can serve only to show how all-inclusive being is. For the wonder is inquiry. It is the desire to know. Anything it can discover or invent, by that very fact is included in the notion of being. Hence, the effort to establish that being is not all-inclusive must be self-defeating; for at the root of all that can be affirmed, at the root of all that can be conceived, is the pure desire to know; and it is the pure desire, underlying all judgment and formulation, underlying all questioning and all desire to question, that defines its all-inclusive objective.



None the less, it may not be amiss to illustrate this principle concretely. It will be said that there is much we do not know. No doubt, our ignorance is great, but we know that fact by raising questions that we do not answer; and being is defined not only by the answers we give but also by the questions we ask. Next, it will be said that there is much it would be futile for us to try to learn. No doubt, the proximately fruitful field of inquiry is restricted. But we know that fact by distinguishing between the questions we can hope soon to answer and those that, as yet, we are not prepared to tackle; and being is defined, not only by the questions we can hope to answer, but also by the questions whose answer we have to postpone.

Thirdly, it will be objected by many that they have no desire to know everything about everything. But how do they know that they do not already know everything about everything? It is because so many questions can be asked. Why do they not effectively will to know everything about everything? Because it is so troublesome to reach even a few answers that they are completely disheartened by the prospect of answering all the questions they could ask.

The attack may be made from the opposite flank. The trouble is that the definition of being is too inclusive. Questions can be meaningless, illusory, incoherent, illegitimate. Trying to answer them does not lead to knowledge of anything. Now, no doubt, there are mistaken questions that lead nowhere. But mistaken questions are formulated questions. Being has been defined, not as the objective of formulated questions, but as the objective of the pure desire to know. Just as that desire is prior to any answer and it itself is not the answer, so too it is prior to any formulated question and it itself is not a formulation. Moreover, just as the pure desire is the intelligent and rational basis from which we discern between correct and incorrect answers, so also it is the intelligent and rational basis from which we discern between valid and mis-

taken questions. In brief, the pure desire to know, whose objective is being, is the source not only of answers but also of their criteria, and not only of questions but also of the grounds on which they are screened. For it is intelligent inquiry and reasonable reflection that just as much yield the right questions as the right answers.

More fundamental misgivings may arise. If one pleases, one may define being as what is to be known through the totality of true judgments. But is being really that? Might it not be something entirely different? The questions arise. They may be valid or mistaken. If they are mistaken, they are to be ignored. If they are valid, then our misgivings are without foundation. For the being that might be totally different, turns out to be exactly what we are talking about. If we ask whether it might be, we ask; and the being we are talking about, is whatever we ask about.

Again, might there not be an unknowable? If the question is invalid, it is to be ignored. If the question is valid, the answer may be 'Yes' or 'No'. But the answer, 'Yes', would be incoherent, for then one would be knowing that the unknowable is; and the answer, 'No', would leave everything knowable and within the range of being.

Other doubts may arise, but instead of chasing after them one by one, it will be better to revert to our initial theorem. Every doubt that the pure desire is unrestricted serves only to prove that it is unrestricted. If you ask whether *X* might not lie beyond its range, the fact that you ask proves that *X* lies within its range. Or else, if the question is meaningless, incoherent, illusory, illegitimate, then *X* turns out to be the mere nothing that results from aberration in cognitional process.

Not only, then, is judgment absolute, not only does it rest upon a grasp of the unconditioned, not only does reflection set the dichotomy, Is it or is it not? But at the root of cognitional process there is a cool, detached, disinterested desire to know and its range is unrestricted. Being is the anything and everything that is the objective of that desire.

### 3. A SPONTANEOUS NOTION

If we have explained what we mean by being, we must now ask what the notion of being is.

In the first place, a distinction has to be drawn between the spontaneously operative notion and, on the other hand, theoretical accounts of its genesis and content. The spontaneously operative notion is invariant; it is common to all men; it functions in the same manner no matter what theoretical account of it a man may come to accept. On the other hand, theoretical accounts of the content and genesis of the notion are numerous; they vary with philosophic contexts, with the completeness of a thinker's observations, with the thoroughness of his analysis. First, we shall give our account of the spontaneously operative notion, and then we shall add a few notes on other theoretical accounts of it.



On the supposition of our analysis of cognitional process, it is easy enough to conclude that the spontaneously operative notion of being has to be placed in the pure desire to know. For, first of all, men are apt to agree that things are, whether or not we know them and, moreover, that there are many things that are known only incompletely or even not at all. The notion of being, then, extends beyond the known. Secondly, being is known in judgment. It is in judgment that we affirm or deny and, until we are ready to affirm or deny, we do not yet know whether or not any *X* happens to be. Still, though being is known only in judging, the notion of being is prior to judging. For prior to any judgment there is reflection, and reflection is formulated in the question, Is it? That question supposes some notion of being and, strangely enough, it is prior to each instance of our knowing being. Not only, then, does the notion of being extend beyond the known but also it is prior to the final component of knowing when being is actually known. Thirdly, there are objects of thought. I can think of a horse and, no less, I can think of a centaur. I can think of the best available scientific opinion on any subject and, no less, I can think of all the previous opinions that in their day were the best available on the same subject. In one sense, they are all equivalent, for as long as one is merely thinking, merely considering, merely supposing, one deals merely with the conditioned and it makes no difference whether or not its conditions are fulfilled. Thinking, then, prescinds from existing. But if it prescinds from existing, does it not prescind from being; and if it prescinds from being, is not all thinking about nothing? The trouble with this argument is that thinking also prescinds from not existing. If I think of a centaur or of phlogiston, I prescind from the fact that they do not exist; hence, if prescinding from existing is prescinding from being, prescinding from non-existence is prescinding from not being; if prescinding from being proves that I am thinking of nothing, then prescinding from not being proves that I am thinking of something.

Now this type of consideration has led many thinkers to suppose that

being is one thing and existing is another, that horses and centaurs, electrons and phlogiston, equally are, but horses and electrons exist while centaurs and phlogiston do not exist. Still that conclusion does not satisfy the facts, for apart from the oddity of asserting that the non-existent is, there is the oversight of the dynamism of cognitional process. In a sense, thinking prescinds from existing and not existing, for it is not thinking but judging that determines whether or not anything exists. In another sense, thinking does not prescind from existing and not existing, for thinking is purposive; we think to get our concepts straight; we wish to get our concepts straight that we may be able to judge; so far from prescinding from existing and not existing, thinking is for the purpose of determining whether or not what is thought does exist.

It follows that the notion of being goes beyond the merely thought, for we ask whether or not the merely thought exists. No less, it follows that the notion of being is prior to thinking, for were it not, then thinking could not be for the purpose of judging, for the purpose of determining whether or not the merely thought exists. The notion of being, then, is prior to conception and goes beyond it; and it is prior to judgment and goes beyond it. That notion must be the immanent, dynamic orientation of cognitional process. It must be the detached and unrestricted desire to know as operative in cognitional process. Desiring to know is desiring to know being; but it is merely the desire and not yet knowing. Thinking is thinking being; it is not thinking nothing; but thinking being is not yet knowing it. Judging is a complete increment in knowing; if correct, it is a knowing of being; but it is not yet knowing being, for that is attained only through the totality of correct judgments.

Still, how can an orientation or a desire be named a notion. A foetal eye is orientated towards seeing; but a foetal eye does not see and it has no notion of seeing; a notion arises only in so far as understanding discerns future function in present structure. Hunger is orientated towards food and eating; it is a desire; it lies within empirical consciousness; but a notion arises only in so far as the orientation of hunger is understood. Purposive human action is orientated towards some end or product; cognitional elements provide the rule and guide of such action; but the cognitional elements are prior to the action; they are constituted, not by the action itself, but by the planning that precedes it.

It remains that none of these instances is exactly parallel to the relation between the desire to know and cognitional process. For the desire to know is not unconscious, as is the foetal eye, nor empirically conscious, as is hunger, nor a consequence of intellectual knowledge, as are deliberation and choice. The desire to know is conscious intelligently and rationally; it is inquiring intelligence and reflecting reasonableness. Simply as desire, it is orientation without, as yet, involving any cognitional content or notion. Still intelligence, as obverse, looks for the intelligible, as reverse. Reasonableness, as obverse, looks for the grounded, as reverse. More fundamentally, the looking for, the desiring, the inquiring-and-reflecting is an obverse that intelligently and rationally heads for an unrestricted objective named being. Were that heading unconscious, there would be an orientation towards being but there would be no desire to know being and no notion of being. Were that heading empirically conscious, there would be an orientation towards being and a felt desire to know being, but there would be no notion of being. In fact, the heading is intelligent and rational, and so there is not only an orientation towards being, not only a pure desire to know being, but also a notion of being.



Let us try to catch this notion, this intention of being, in the act. We speak of abstraction, and commonly we mean a direction of attention to some aspects of the given with a concomitant neglect of other aspects. The geometer considers the circle as a plane figure obeying a certain rule; he disregards the size, the colour, the inexactitude of the figure he draws or imagines; still more does he disregard other and more loosely connected aspects of the given. But that is not all. He disregards all other questions in geometry, all other departments of mathematics, all other fields of science, all other human occupations to which he could turn his hand. He considers only the circle. He abstracts from everything else. He does so intelligently, for though the objective of his desire is unrestricted, still he can move towards it only by concentrating on one element at a time. Again, as intelligence abstracts, so reflection prescinds. If I am to judge whether or not this is a typewriter, I have to prescind from all that is not relevant to that issue. I have to know all that is relevant. If I were a relativist, I would have to know the universe to know all that is relevant to that single judgment. Even though I am not a relativist, even though I find that many conditioned propositions become virtually unconditioned on the fulfilment of a manageable number of conditions, still this restriction of the relevant is accompanied by an acknowledgement of a universe of irrelevances.

Finally, as intelligence concentrates on the significant to abstract from all else, as reflection concentrates on the relevant to prescind from all else, so further questions and further issues arise neither as a surprise nor as a new beginning. The abstracting and the prescinding were provisional; they were only moments in a larger process. Nor is that larger process merely the object of introspective analysis. Immanent within it and operative of it lies an intelligent and rational consciousness that unrestrictedly intends a correspondingly unrestricted objective named being, or the all, or everything about everything, or the concrete universe. Just as the notion of the intelligible is involved in the actual functioning of intelligence, just as the notion of the grounded is involved in the actual functioning of reasonableness, so the notion of being is involved in the unrestricted drive of inquiring intelligence and reflecting reasonableness.

### 4. An All-Pervasive Notion

Hence it is that the notion of being is all-pervasive. It underpins all cognitional contents. It penetrates them all. It constitutes them as cognitional.

It underpins all cognitional contents. Without the pure desire to know, sensitive living would remain in its routine of perception and conation, instinct and habit, emotion and action. What breaks that circuit and releases intellectual activity is the wonder Aristotle described as the beginning of all science and philosophy. But that wonder is intelligent inquiry. It selects data for insight and by that selecting it underpins even the empirical component in our knowing. Still more obviously all ideas and all concepts are responses to the desire to understand, and all judgments are responses to the demand for the unconditioned.

Secondly, the notion of being penetrates all cognitional contents. It is the supreme heuristic notion. Prior to every content, it is the notion of the to-be-known through that content. As each content emerges, the 'to-be-known through the content' passes without residue into the 'known through that content'. Some blank in universal anticipation is filled in, not merely to end that element of anticipation, but also to make the filler a part of the anticipated. Hence, prior to all answers, the notion of being is the notion of the totality to be known through all answers. But, once all answers are reached, the notion of being becomes the notion of the totality known through all answers.



Thirdly, the notion of being constitutes all contents as cognitional. Experiencing is only the first level of knowing; it presents the matter to be known. Understanding is only the second level of knowing; it defines the matter to be known. Knowing reaches a complete increment only with judgment, only when the merely experienced has been thought and the merely thought has been affirmed. But the increment of knowing is always completed in the same fashion. Experience is a kaleidoscopic flow. Objects of thought are as various as the inventiveness of human intelligence. But the contribution of judgment to our knowing is ever a mere 'Yes' or 'No', a mere 'is' or 'is not'. Experience is for inquiring into being. Intelligence is for thinking out being. But by judgment being is known, and in judgment what is known is known as being. Hence knowing is knowing being, yet the known is never mere being, just as judgment is never a mere 'Yes' apart from any question that 'Yes' answers.

### 5. The Core of Meaning

As the notion of being underpins all contents, and penetrates them, and constitutes them as cognitional, so also it is the core of meaning.

For present purposes it will suffice to distinguish

(1) sources of meaning,
(2) acts of meaning,
(3) terms of meaning, and
(4) the core of meaning.

Any element of knowledge may serve as a source of meaning. Hence, sources of meaning include data and images, ideas and concepts, the grasp of the unconditioned and judgment and, no less, the detached and unrestricted desire to know.

Acts of meaning are of three kinds. They are

(1) formal,
(2) full,
(3) instrumental.

The formal act of meaning is an act of conceiving, thinking, considering, defining, supposing, formulating. The full act of meaning is an act of judging. The instrumental act of meaning is the implementation of a formal or of a full act by the use of words or symbols in a spoken, written, or merely imagined utterance.

Terms of meaning are what is meant. They are formal or full.